Achieving Robust Power System Operations using Convex Relaxations of the Power Flow Equations

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Joint work with:

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Seminar at ETH Zürich

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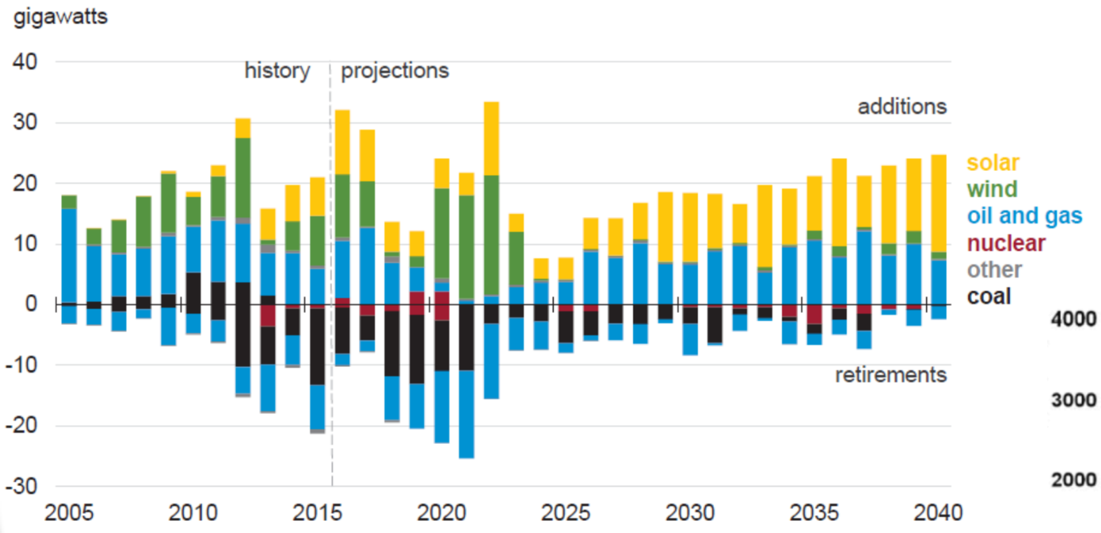
Support from U.S. Department of Energy, GMLC Control Theory (1.4.10) project

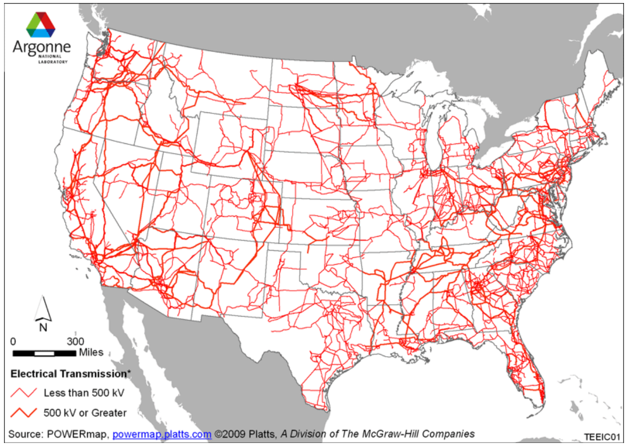
Motivation

US Annual Electricity Generating Capacity Additions and Retirements



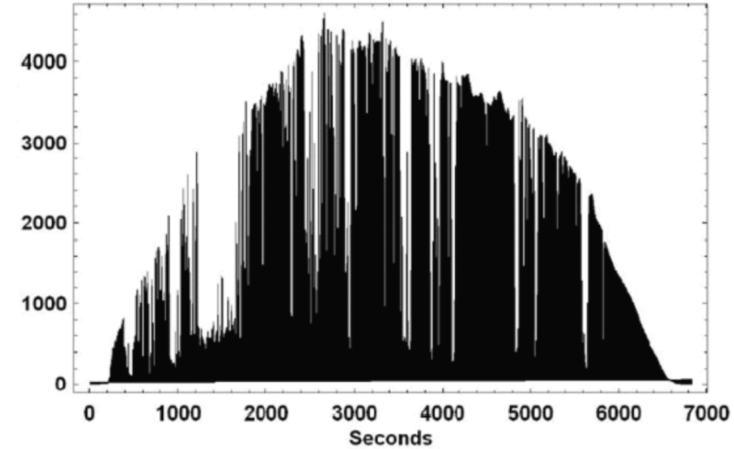
U.S. Energy Information Administration, Annual Energy Outlook 2017.

The Legacy Grid



http://teeic.anl.gov/er/transmission/restech/dist/index.cfm

Variability and Uncertainty



A Typical Day of Solar PV Generation, [Apt and Curtright '08]

New computational tools are needed to economically and reliably operate electric grids with significant renewable generation.

Overview

- Solving robust AC optimal power flow problems.

> D.K. Molzahn, and L.A. Roald, "Towards and AC Optimal Power Flow Algorithm with Robust Feasibility Guarantees," *20th Power Systems Computation Conference (PSCC)*, June 11-15, 2018.

- Certifying engineering constraint satisfaction with limited measurements and controllability.

> D.K. Molzahn, and L.A. Roald, "Grid-Aware versus Grid-Agnostic Distribution System Control: A Method for Certifying Engineering Constraint Satisfaction," *52nd Hawaii International Conference on Systems Sciences (HICSS)*, January 8-11, 2019.

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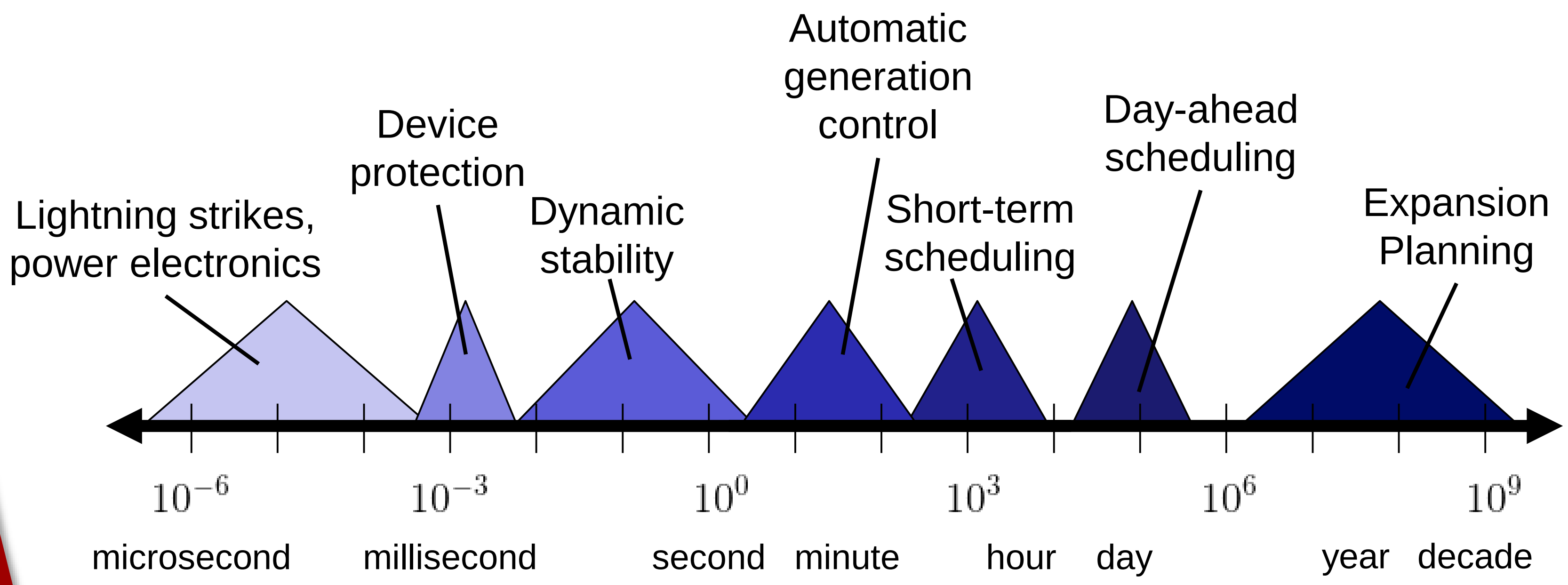
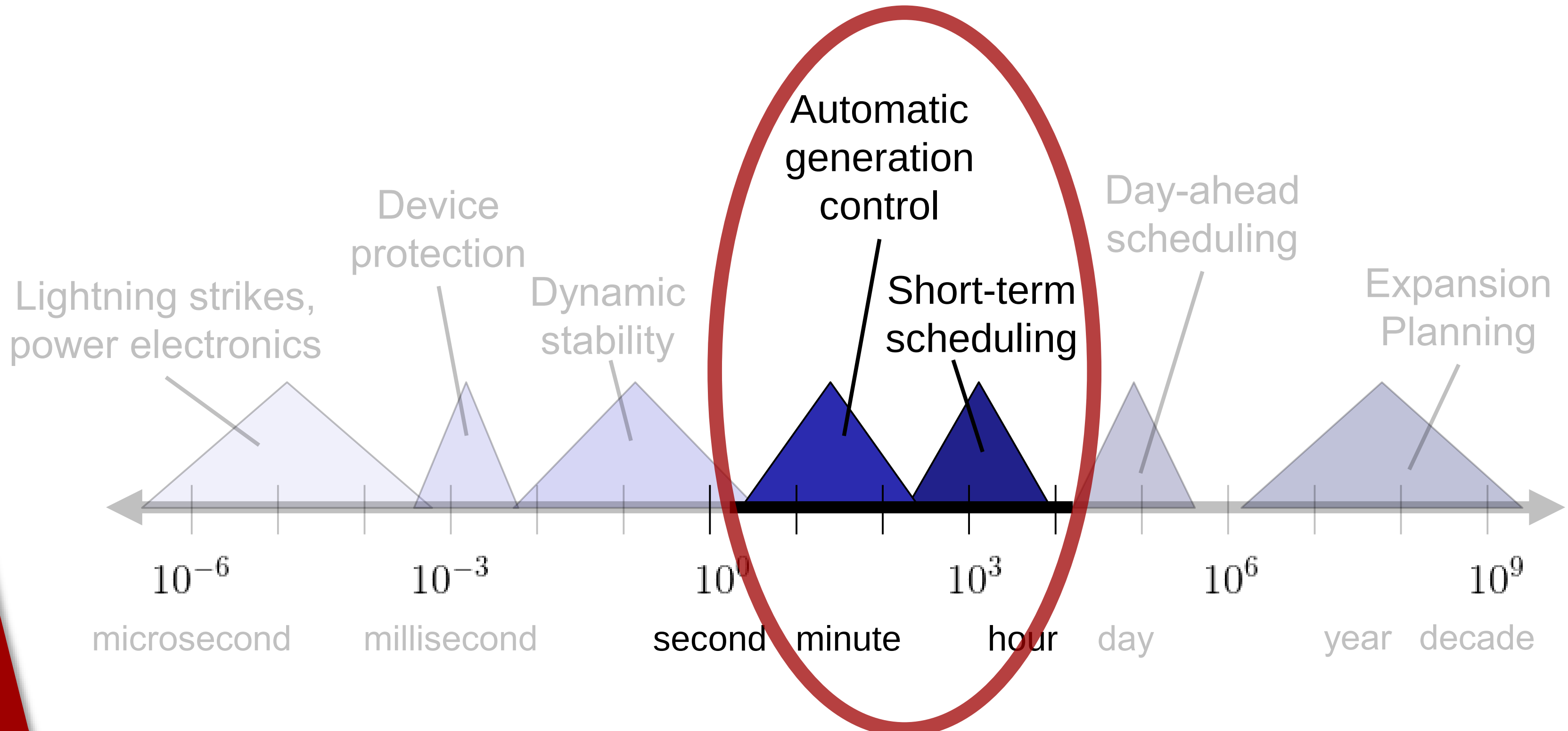
Time Scales in Power Systems



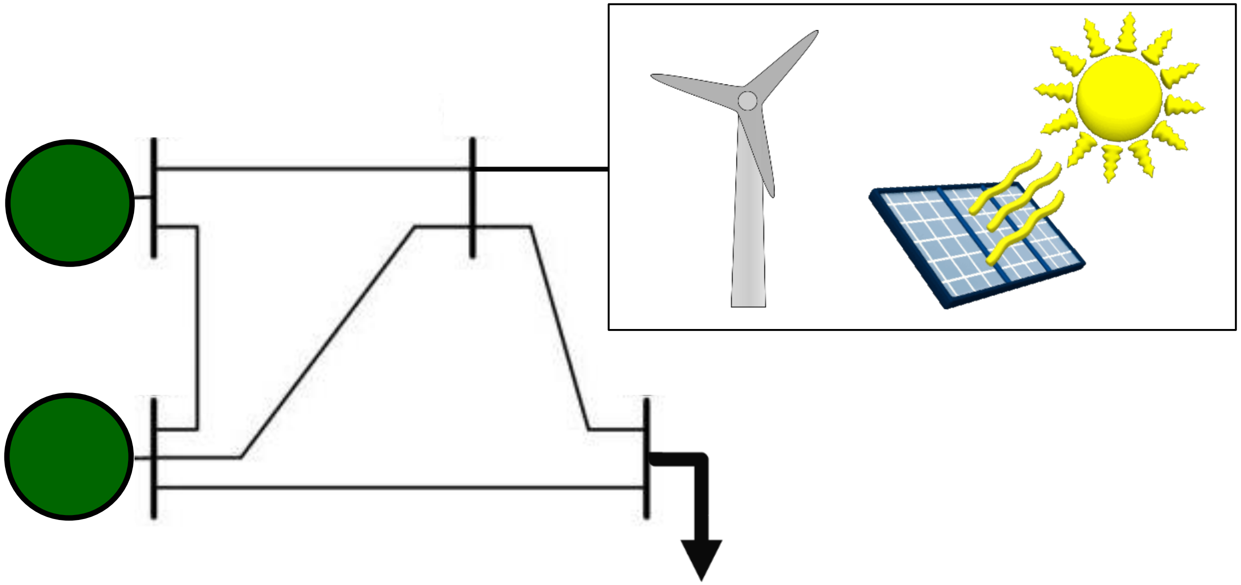
Time Scales in Power Systems



How to handle uncertainties from renewable generators and loads in this time frame?

Problem Overview

How to dispatch the generators such that the system is robust to any realization within a specified uncertainty set?



- Other stochastic approaches for power system optimization:

[Capitanescu, Fliscounakis, Panciatici, & Wehenkel '12], [Vrakopoulou, Katsampani, Margellos, Lygeros, & Andersson '13], [Phan & Ghosh '14], [Nasri, Kazempour, Conejo, & Ghandhari '16], [Louca & Bitar '17], [Venzke, Halilbasic, Markovic, Hug, & Chaitzivasileiadis '17], [Roald & Andersson '17], [Roald, Molzahn & Tobler '17], [Marley, Vrakopoulou, & Hiskens '17], [Lorca & Sun '18]

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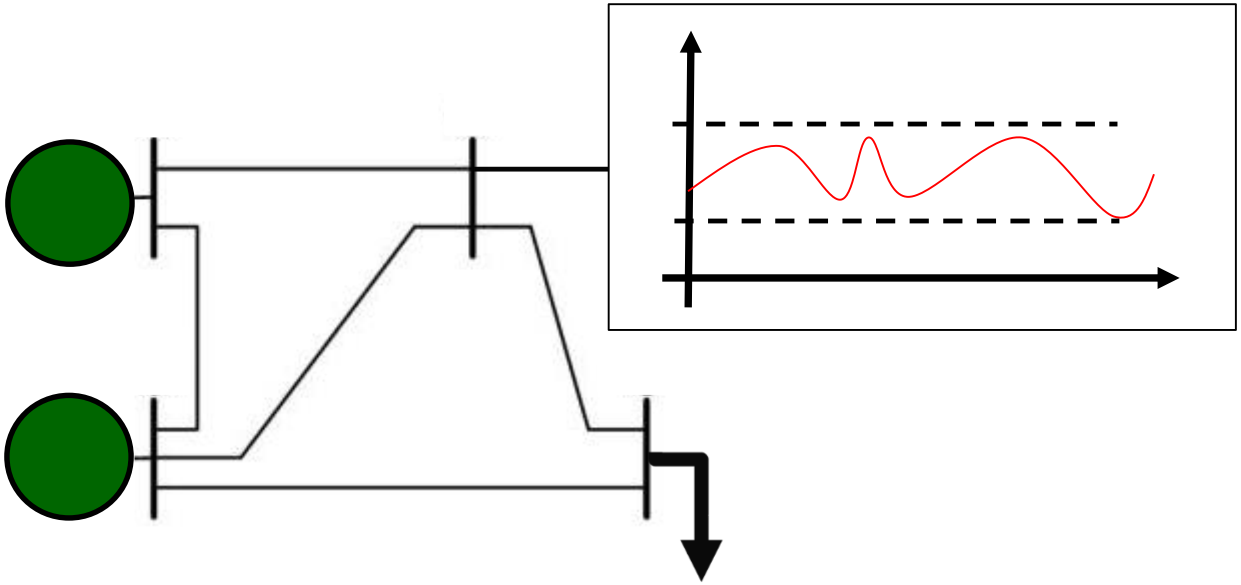

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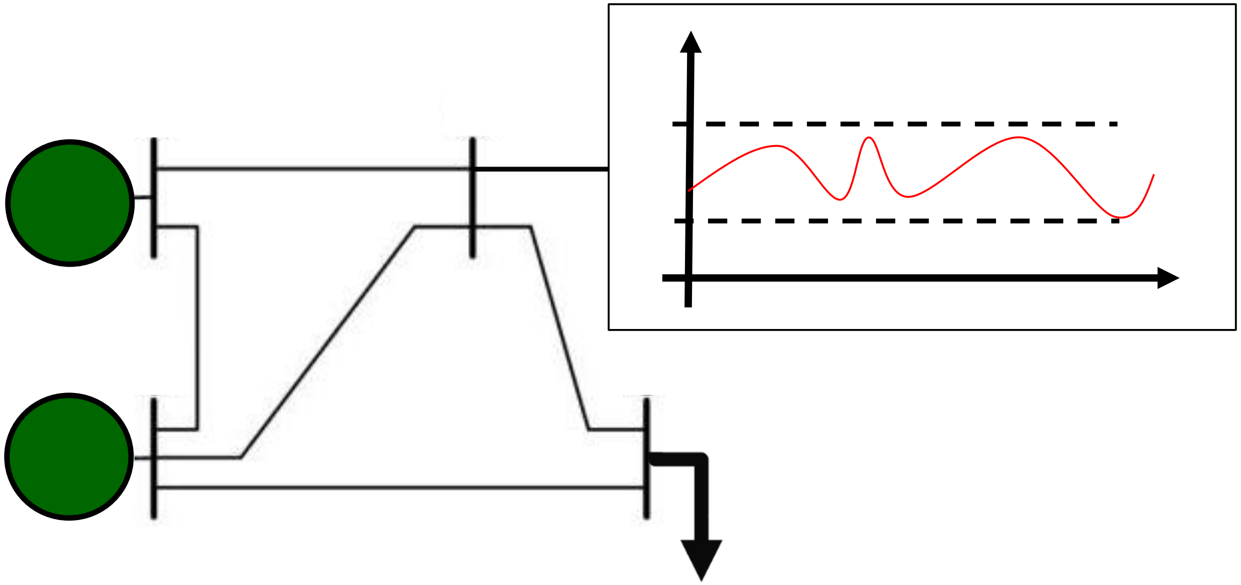

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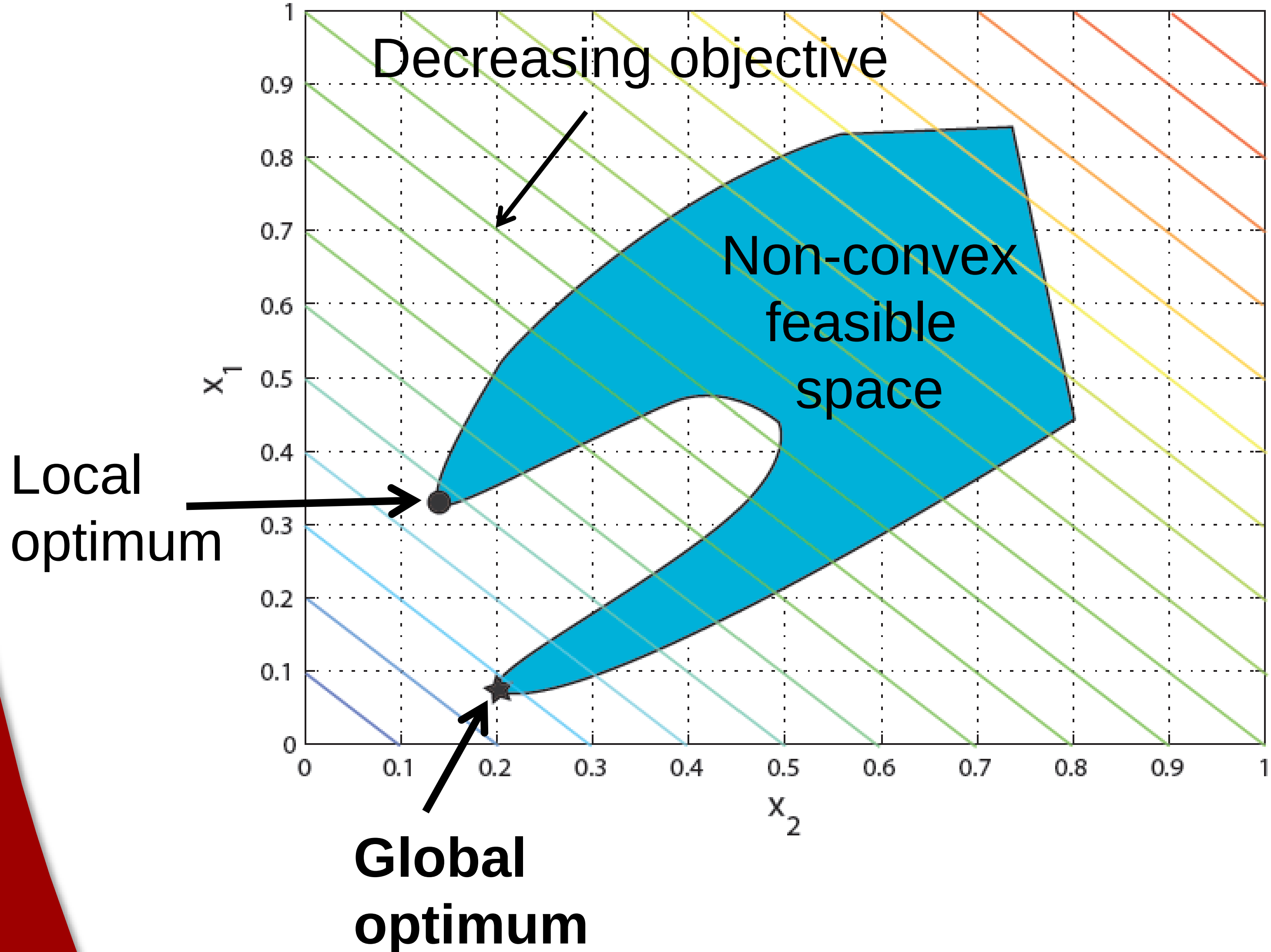
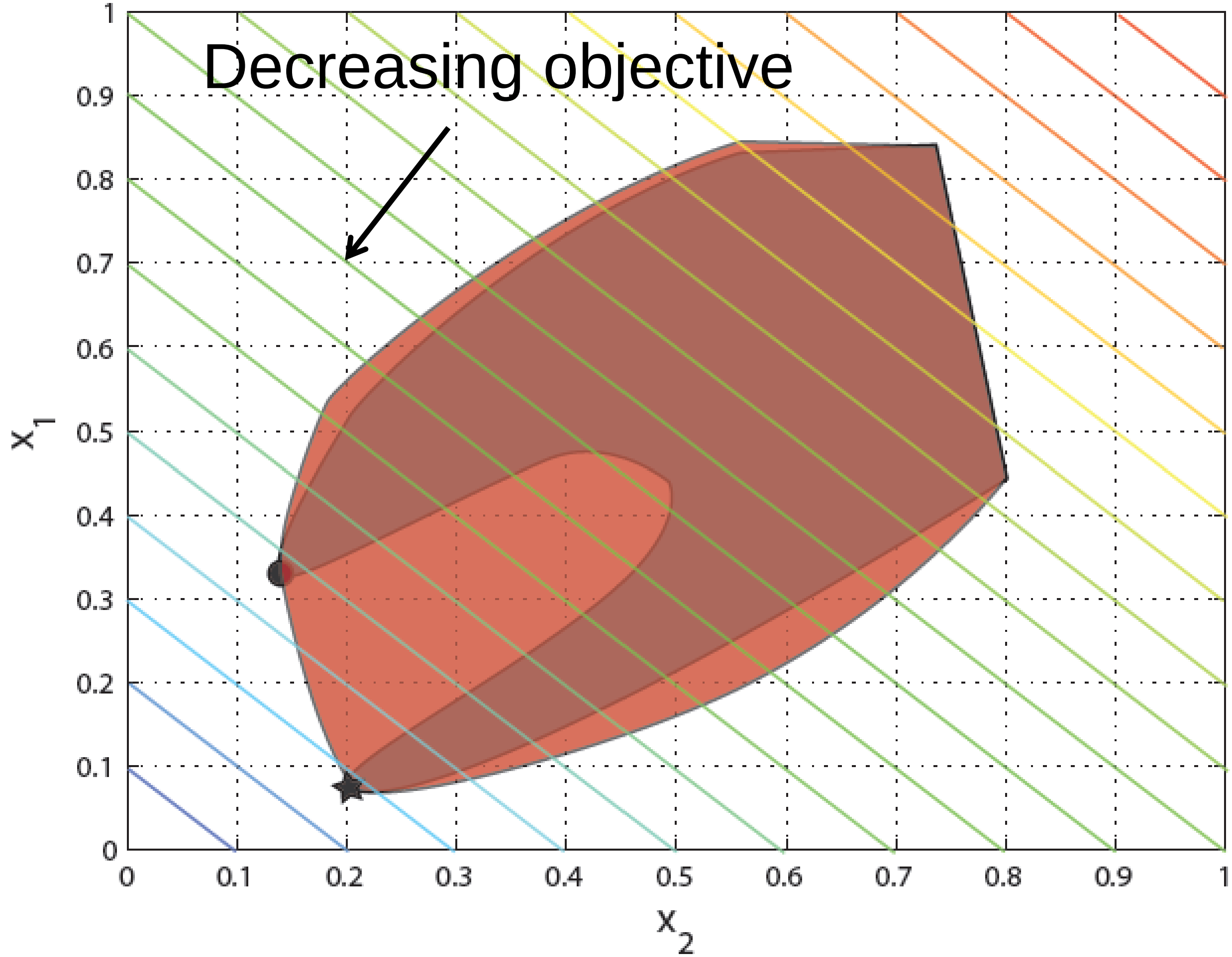
Convex Relaxation

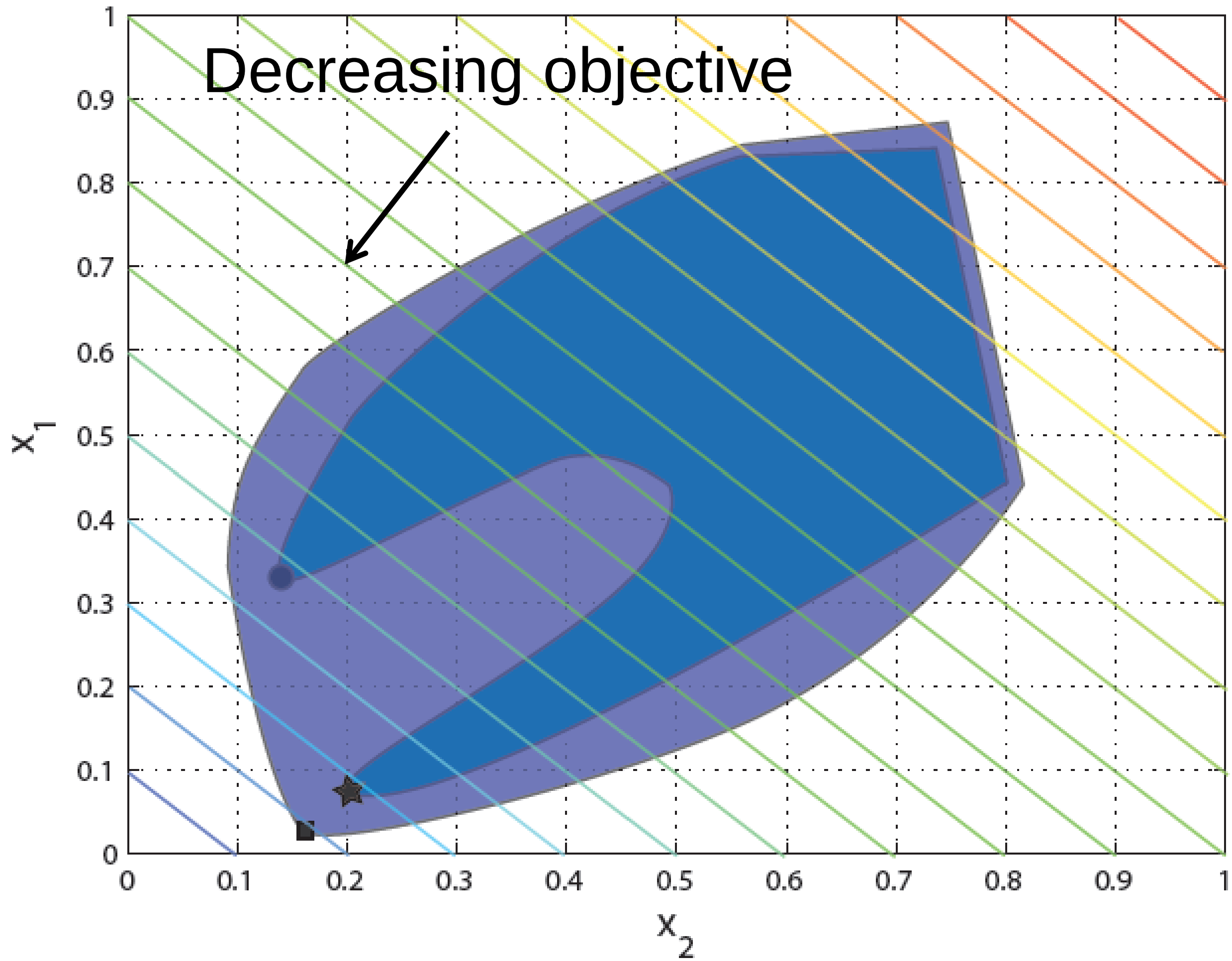


Convex Relaxation



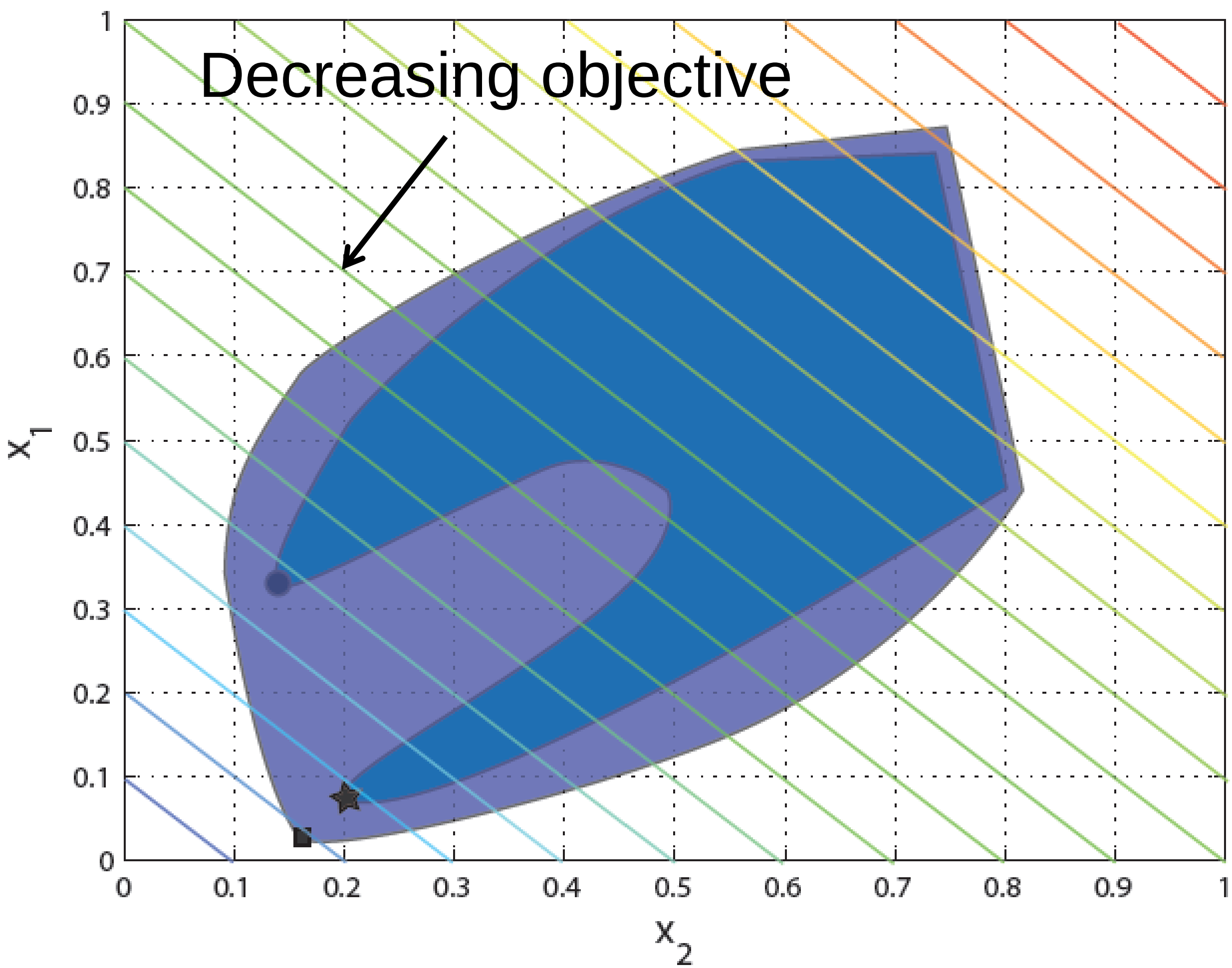
Relaxation finds global optimum
(**zero relaxation gap**)

Convex Relaxation



Relaxation does not find global optimum
(**non-zero relaxation gap**)

Convex Relaxation

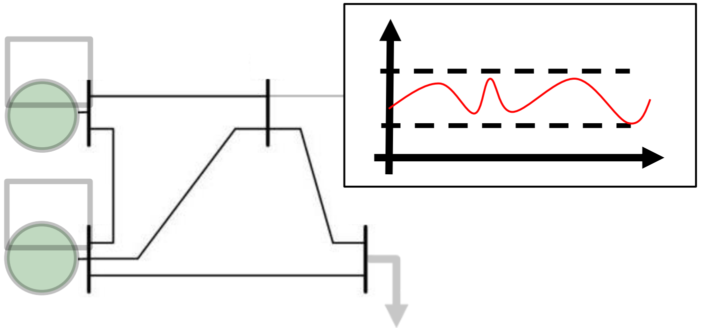


We use a combination of the sparse semidefinite programming relaxation and the QC relaxation [Lavaei & Low '12], [Jabr '11], [Molzahn, Holzer, Lesieutre, & DeMarco '13], [Coffrin, Hijazi, & Van Hentenryck '16]

Problem Formulation

- Uncertainty model
- Generator model
- Network model

Uncertainty Model



- Variations in active power injections, ω, within a specified uncertainty set, $\mathcal{W}$, around a forecast injection, $\hat{p}_{inj}$:

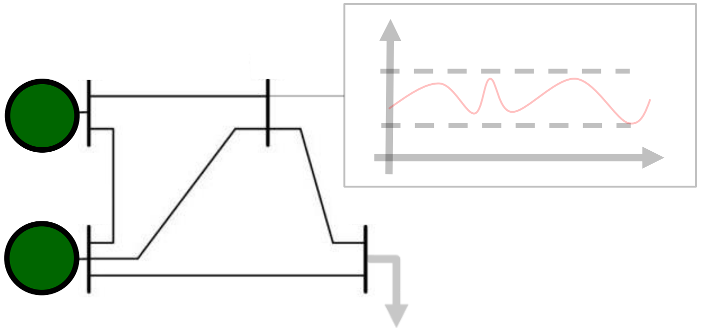
$$p_{inj}(\omega) = \hat{p}_{inj} + \omega$$

- Fixed power factor: $q_{inj}(\omega) = \hat{q}_{inj} + \gamma\,\omega$

- Box uncertainty set: $\mathcal{W} = \{\omega \in [\omega^{min}, \omega^{max}]\}$

- Many possible generalizations

Generator Model



- Automatic Generation Control (AGC):

$$p_{G,i}(\omega) = p_{G0,i} - \alpha_i \left(\sum_{k \in \mathcal{N}} \omega_k - \delta p(\omega) \right)$$

Scheduled generation

Actual generation

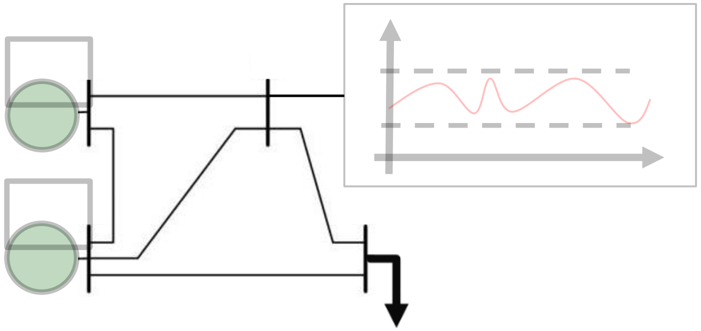
Specified participation factors

Change in active power injections (uncertainty realization plus change in losses)

- Scheduled voltage magnitudes supported by varying reactive power generation:

$$v_G(\omega) = v_{G0}, \qquad q_G^{min} \leq q_G(\omega) \leq q_G^{max}$$

Network Model



- Nonlinear AC power flow equations:

$$p_{inj,i}(\omega) = v_i(\omega) \sum_{k=1}^{n} v_k(\omega) \Big[\mathbf{G}_{ik} \cos\Big(\theta_i(\omega) - \theta_k(\omega)\Big) + \mathbf{B}_{ik} \sin\Big(\theta_i(\omega) - \theta_k(\omega)\Big) \Big]$$

$$q_{inj,i}(\omega) = v_i(\omega) \sum_{k=1}^{n} v_k(\omega) \Big[\mathbf{G}_{ik} \sin\Big(\theta_i(\omega) - \theta_k(\omega)\Big) - \mathbf{B}_{ik} \cos\Big(\theta_i(\omega) - \theta_k(\omega)\Big) \Big]$$

$$\theta_{ref}(\omega) = 0$$

Robust Optimal Power Flow

$$\min \sum_{i\in\mathcal{G}} \left(c_{2,i}\, (p_{G0,i})^2 + c_{1,i}\, p_{G0,i} + c_{0,i} \right)$$

Minimize scheduled generation cost

$$\text{subject to} \quad (\forall i \in \mathcal{N},\ \forall (\ell,m) \in \mathcal{L},\ \forall \omega \in \mathcal{W})$$

$$p_{G,k}(\omega) = p_{G0,k} - \alpha \left(\sum_{i\in\mathcal{N}} \omega_i - \delta p\,(\omega) \right),$$

$$v_k(\omega) = v_{G0,k}, \qquad \forall k \in \mathcal{G}$$

Generator model

$$p_{G,i}^{min} \leq p_{G,i}\,(\omega) \leq p_{G,i}^{max}$$
$$q_{G,i}^{min} \leq q_{G,i}(\omega) \leq q_{G,i}^{max}$$
$$v_i^{min} \leq v_i(\omega) \leq v_i^{max}$$
$$|i_{\ell m}(\omega)| \leq i_{\ell m}^{max}, \quad |i_{m\ell}(\omega)| \leq i_{\ell m}^{max}$$

Engineering constraints

$$p_{G,i}(\omega) + \hat{p}_{inj,i}(\omega) + \omega_i$$
$$= v_i(\omega) \sum_{k=1}^{n} v_k(\omega) \left[\mathbf{G}_{ik} \cos\left(\theta_i(\omega) - \theta_k(\omega)\right) + \mathbf{B}_{ik} \sin\left(\theta_i(\omega) - \theta_k(\omega)\right) \right]$$
$$q_{G,i}(\omega) + \hat{q}_{inj,i}(\omega) + \gamma\,\omega_i$$
$$= v_i(\omega) \sum_{k=1}^{n} v_k(\omega) \left[\mathbf{G}_{ik} \sin\left(\theta_i(\omega) - \theta_k(\omega)\right) - \mathbf{B}_{ik} \cos\left(\theta_i(\omega) - \theta_k(\omega)\right) \right]$$

Network model

Robust Optimal Power Flow

$$\min \sum_{i \in \mathcal{G}} \left(c_{2,i} \, (p_{G0,i})^2 + c_{1,i} \, p_{G0,i} + c_{0,i} \right)$$

subject to $(\forall i \in \mathcal{N}, \; \forall (\ell, m) \in \mathcal{L}, \; \forall \omega \in \mathcal{W})$

$$p_{G,k}(\omega) = p_{G0,k} - \alpha \left(\sum_{i \in \mathcal{N}} \omega_i - \delta p\,(\omega) \right),$$

Infinite dimensional problem!

$$v_k(\omega) = v_{G0,k}, \qquad \forall k \in \mathcal{G}$$

$$p_{G,i}^{min} \leq p_{G,i}\,(\omega) \leq p_{G,i}^{max}$$

$$q_{G,i}^{min} \leq q_{G,i}(\omega) \leq q_{G,i}^{max}$$

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$$p_{G,i}(\omega) + \hat{p}_{inj,i}(\omega) + \omega_i = v_i(\omega) \sum_{k=1}^{n} v_k(\omega) \left[\mathbf{G}_{ik} \cos\left(\theta_i(\omega) - \theta_k(\omega)\right) + \mathbf{B}_{ik} \sin\left(\theta_i(\omega) - \theta_k(\omega)\right) \right]$$

$$q_{G,i}(\omega) + \hat{q}_{inj,i}(\omega) + \gamma\, \omega_i = v_i(\omega) \sum_{k=1}^{n} v_k(\omega) \left[\mathbf{G}_{ik} \sin\left(\theta_i(\omega) - \theta_k(\omega)\right) - \mathbf{B}_{ik} \cos\left(\theta_i(\omega) - \theta_k(\omega)\right) \right]$$

Two Key Challenges

- Satisfying the engineering constraints for all uncertainty realizations

$$p_{G,i}^{min} \leq p_{G,i}(\omega) \leq p_{G,i}^{max}$$
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$$v_i^{min} \leq v_i(\omega) \leq v_i^{max}$$
$$\left|i_{\ell m}(\omega)\right|^2 \leq \left(i_{\ell m}^{max}\right)^2, \quad \left|i_{m\ell}(\omega)\right|^2 \leq \left(i_{\ell m}^{max}\right)^2$$

- Certifying power flow feasibility for all uncertainty realizations

$$p_{G,i}(\omega) + \hat{p}_{inj,i}(\omega) + \omega_i = v_i(\omega) \sum_{k=1}^{n} v_k(\omega) \left[\mathbf{G}_{ik} \cos\left(\theta_i(\omega) - \theta_k(\omega)\right) + \mathbf{B}_{ik} \sin\left(\theta_i(\omega) - \theta_k(\omega)\right)\right]$$
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- Certifying power flow feasibility for all uncertainty realizations

$$p_{G,i}(\omega) + \hat{p}_{inj,i}(\omega) + \omega_i$$
$$= v_i(\omega) \sum_{k=1}^{n} v_k(\omega) \Big[\mathbf{G}_{ik} \cos \big(\quad \big) + \mathbf{B}_{ik} \sin \big(\theta_i(\omega) - \theta_k(\omega) \big) \Big]$$
$$q_{G,i}(\omega) + \hat{q}$$
$$= \quad \sum_{k=1}^{n} v_k(\omega) \Big[\mathbf{G}_{ik} \sin \big(\theta_i(\omega) - \theta_k(\omega) \big) - \mathbf{B}_{ik} \cos \big(\theta_i(\omega) - \theta_k(\omega) \big) \Big]$$

Ongoing work!

Ensure Engineering Constraint Satisfaction for All Uncertainty Realizations

$$p_{G,i}^{min} \leq p_{G,i}(\omega) \leq p_{G,i}^{max}$$
$$q_{G,i}^{min} \leq q_{G,i}(\omega) \leq q_{G,i}^{max}$$
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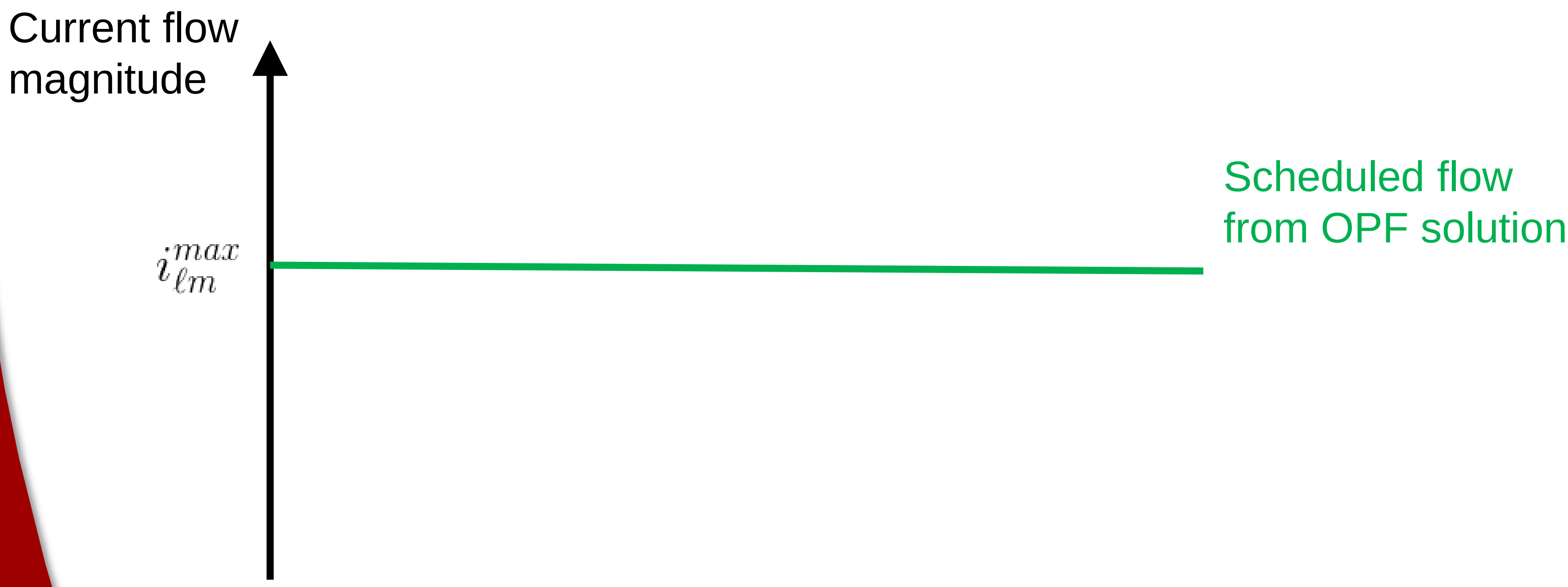
$$|i_{\ell m}(\omega)|^2 \leq (i_{\ell m}^{max})^2, \quad |i_{m\ell}(\omega)|^2 \leq (i_{\ell m}^{max})^2$$

$$\forall\, \omega \in \mathcal{W}$$

Robustness via Constraint Tightening

- Avoid constraint violations by enforcing a security margin, interpreted as tightened constraints



Engineering Constraints

Robustness via Constraint Tightening

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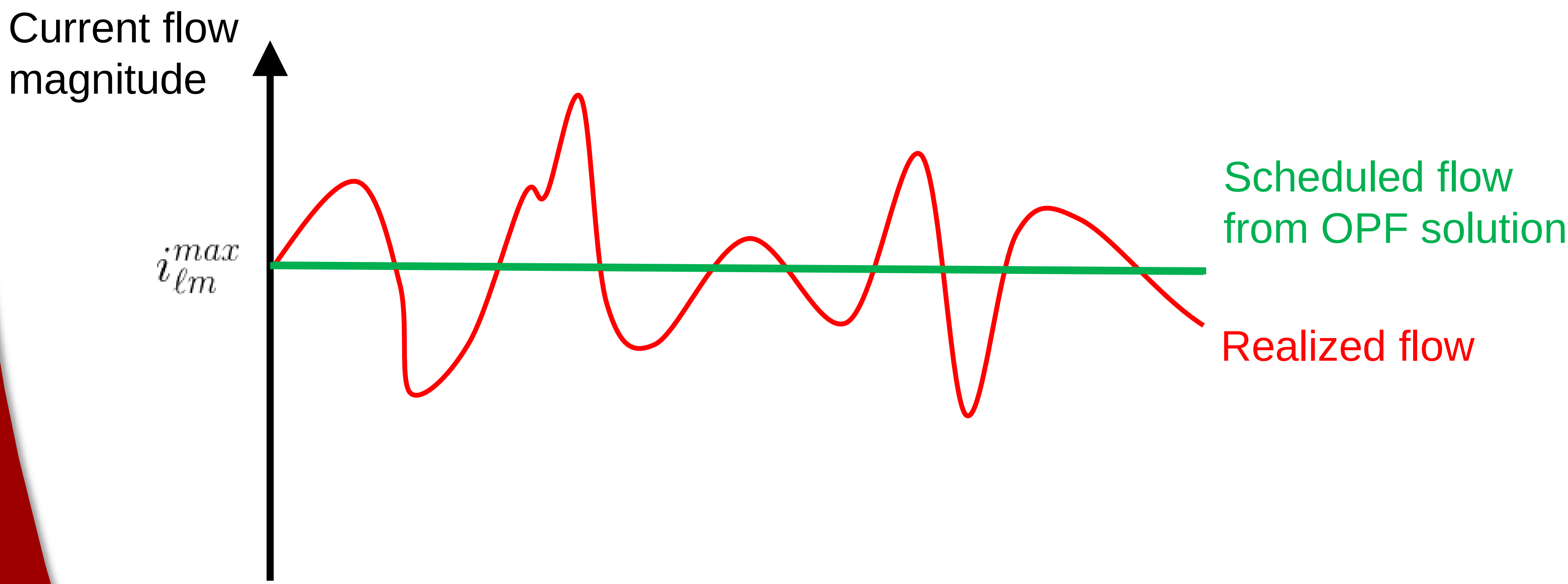

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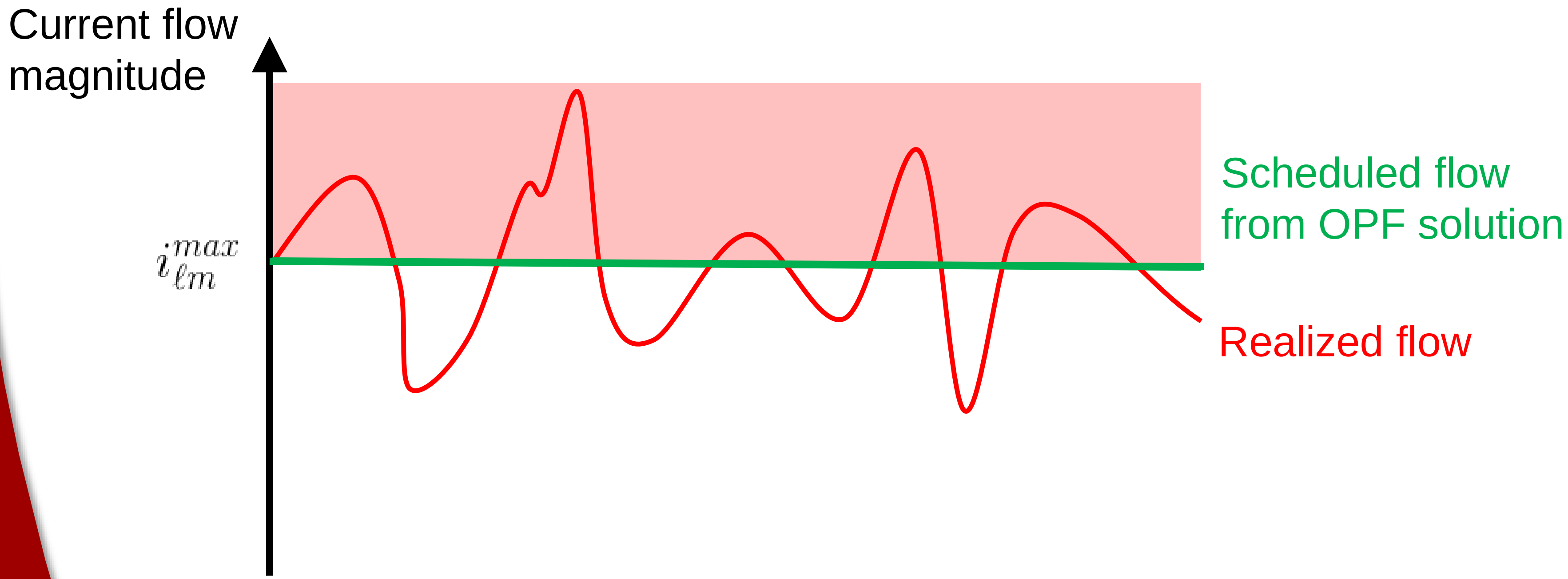

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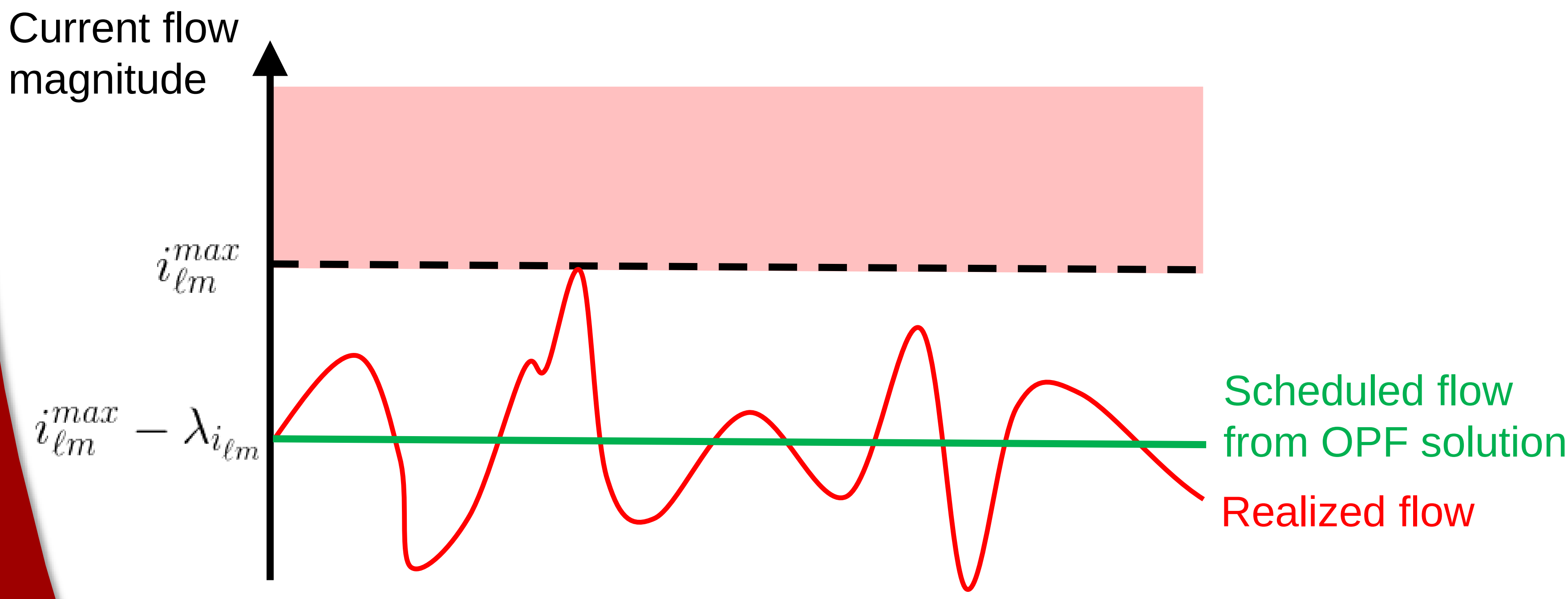

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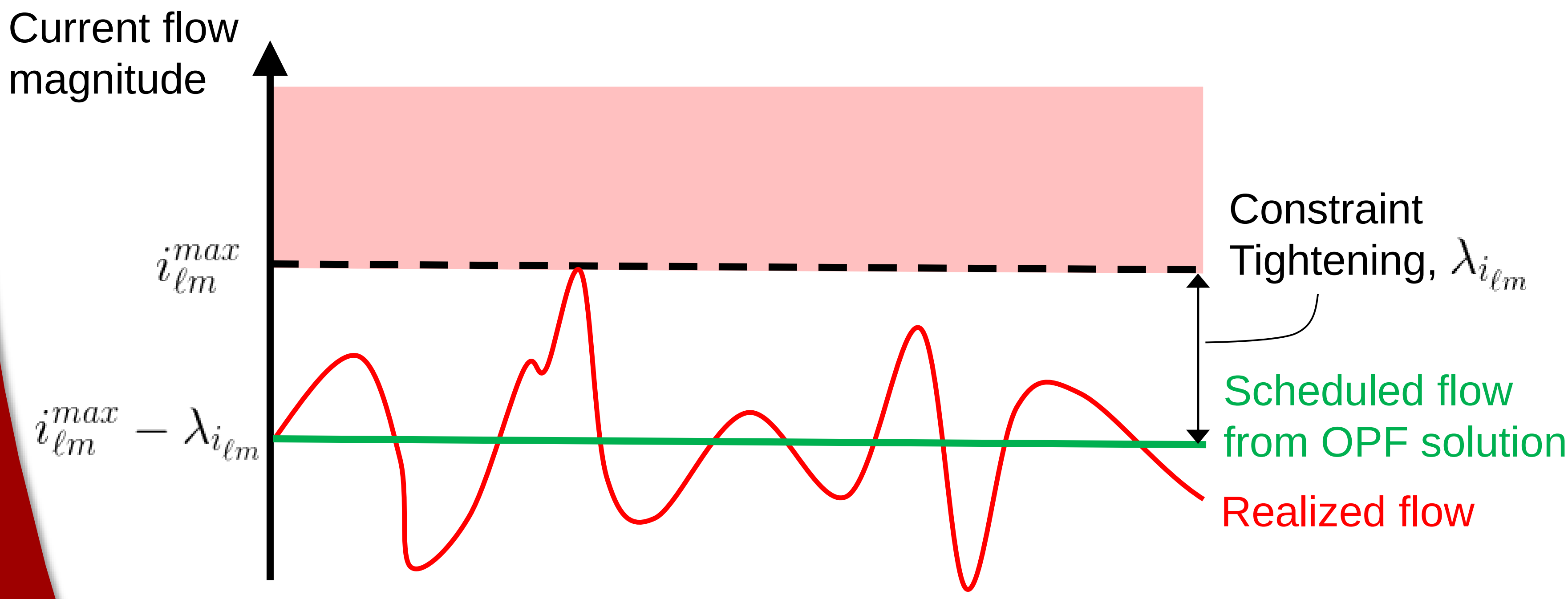

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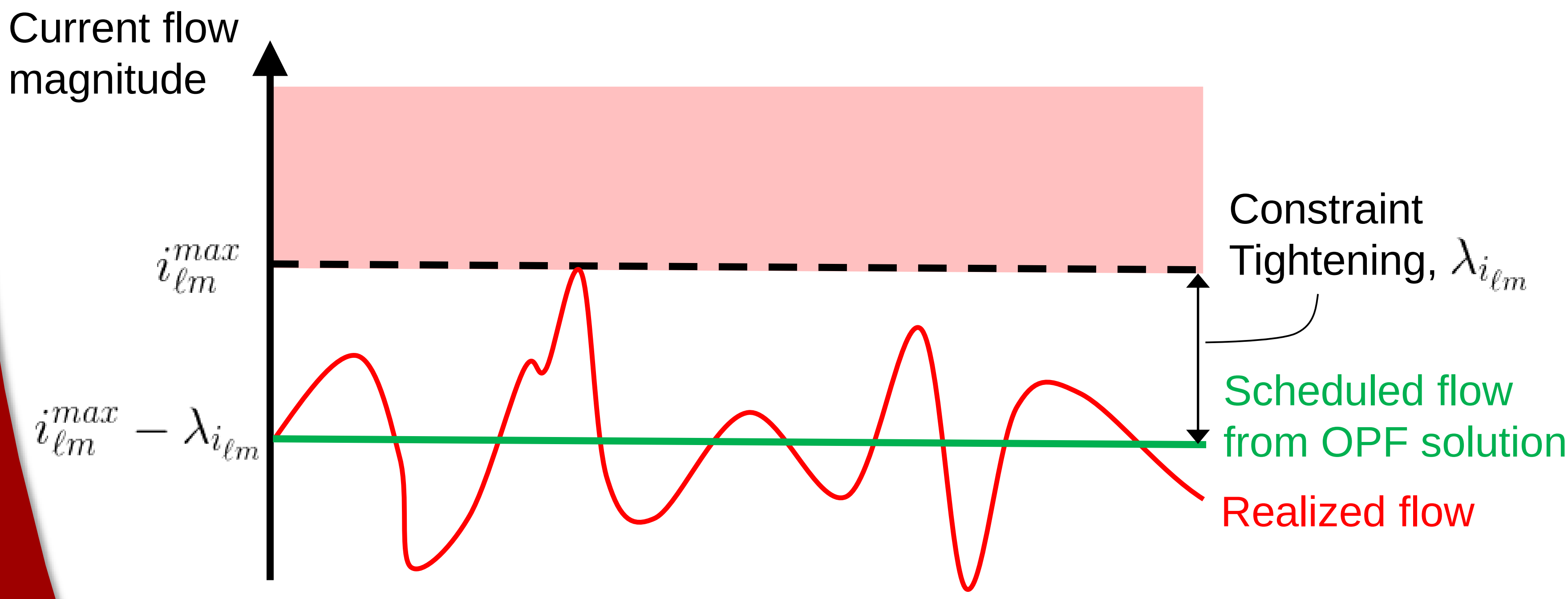

Challenge: How to define appropriate constraint tightenings?

Computing Constraint Tightenings

- Tightenings are functions of the scheduled operating point

$$\lambda_{i_{\ell m}} = \Bigg\{ \max_{\omega \in \mathcal{W}} \quad |i_{\ell m}(\omega)| \qquad \text{subject to}$$

$$p_{G,k}(\omega) = p_{G0,k} - \alpha \left(\sum_{i \in \mathcal{N}} \omega_i - \delta p(\omega) \right),$$

$$v_k(\omega) = v_{G0,k}, \qquad \forall k \in \mathcal{G}$$

Generator model

$$p_{G,i}(\omega) + \hat{p}_{inj,i}(\omega) + \omega_i$$
$$= v_i(\omega) \sum_{k=1}^{n} v_k(\omega) \left[\mathbf{G}_{ik} \cos\Big(\theta_i(\omega) - \theta_k(\omega)\Big) + \mathbf{B}_{ik} \sin\Big(\theta_i(\omega) - \theta_k(\omega)\Big) \right]$$

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Network model

$$v(\omega) \geq \underline{v}$$

High-voltage power flow solution

$$\Bigg\} - |i_{0,\ell m}|$$

Difference between maximum and scheduled flows

Computing Constraint Tightenings

- Tightenings are functions of the scheduled operating point

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Convex Relaxation

$$v(\omega) \geq \underline{v}$$

High-voltage power flow solution

$$\left. \right\} - |i_{0,\ell m}|$$

Difference between maximum and scheduled flows

Engineering Constraints

Reformulation as a Bi-Level Problem

$$\min \sum_{i \in \mathcal{G}} \left(c_{2,i} \left(p_{G0,i} \right)^2 + c_{1,i}\, p_{G0,i} + c_{0,i} \right)$$

$$\text{subject to} \quad \left(\forall i \in \mathcal{N},\ \forall \left(\ell, m \right) \in \mathcal{L} \right)$$

Network Constraints ~~for all $\omega \in \mathcal{W}$~~

$$p_{G,i}^{min} + \lambda_{p_{G,i}^{min}} \leq p_{G0,i} \leq p_{G,i}^{max} - \lambda_{p_{G,i}^{max}}$$

$$q_{G,i}^{min} + \lambda_{q_{G,i}^{min}} \leq q_{G0,i} \leq q_{G,i}^{max} - \lambda_{q_{G,i}^{max}}$$

$$v_{i}^{min} + \lambda_{v_{i}^{min}} \leq v_{0,i} \leq v_{i}^{max} - \lambda_{v_{i}^{max}}$$

$$\left| i_{0,\ell m} \right| \leq i_{\ell m}^{max} - \lambda_{i_{\ell m}}$$

$$\left| i_{0,m \ell} \right| \leq i_{\ell m}^{max} - \lambda_{i_{m \ell}}$$

Ensuring power flow feasibility for all realizations is the subject of ongoing work.

Tightened engineering constraints on scheduled variables

If tightenings are fixed, $\lambda = \hat{\lambda}$, this problem is **deterministic,** containing only the scheduled variables!

Engineering Constraints

Alternating Algorithm

Initialize:

$$k = 1, \quad \hat{\lambda}^k = 0$$

Solve deterministic OPF problem:

$$\left(p_{G0}^k, q_{G0}^k, v_0^k, i_{0,\ell m}^k, i_{0,m\ell}^k\right) =$$

$$\arg\min \sum_{i \in \mathcal{G}} \left(c_{2,i}\left(p_{G0,i}\right)^2 + c_{1,i}\, p_{G0,i} + c_{0,i}\right)$$

subject to $(\forall i \in \mathcal{N},\ \forall (\ell, m) \in \mathcal{L})$

Network constraints for scheduled operating point

$$p_{G,i}^{min} + \hat{\lambda}_{p_{G,i}^{min}}^k \leq p_{G0,i} \leq p_{G,i}^{max} - \hat{\lambda}_{p_{G,i}^{max}}^k$$

$$q_{G,i}^{min} + \hat{\lambda}_{q_{G,i}^{min}}^k \leq q_{G0,i} \leq q_{G,i}^{max} - \hat{\lambda}_{q_{G,i}^{max}}^k$$

$$v_i^{min} + \hat{\lambda}_{v_i^{min}}^k \leq v_{0,i} \leq v_i^{max} - \hat{\lambda}_{v_i^{max}}^k$$

$$\left|i_{0,\ell m}\right| \leq i_{\ell m}^{max} - \hat{\lambda}_{i_{\ell m}}^k$$

$$\left|i_{0,m\ell}\right| \leq i_{\ell m}^{max} - \hat{\lambda}_{i_{m\ell}}^k$$

Update tightenings based on the previous operating point:

$$\hat{\lambda}^{k+1} = \lambda\left(p_{G0}^k, q_{G0}^k, v_0^k, i_{0,\ell m}^k, i_{0,m\ell}^k\right)$$

$$k = k + 1$$

Terminate when tightenings stop changing: $\left|\hat{\lambda}^k - \hat{\lambda}^{k-1}\right| < \epsilon$

Algorithm Summary

- For a given scheduled operating point, use convex relaxations to bound the worst-case impacts of any possible uncertainty realization, interpreted as a tightening of the constraints
- Compute a new scheduled operating point based on the tightened constraints
- Iterate until convergence
  - No guarantee of convergence, but typically converges in a few iterations
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Algorithm Summary

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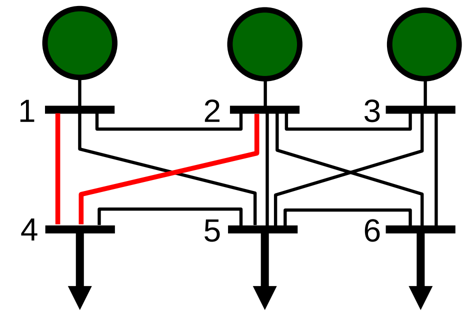
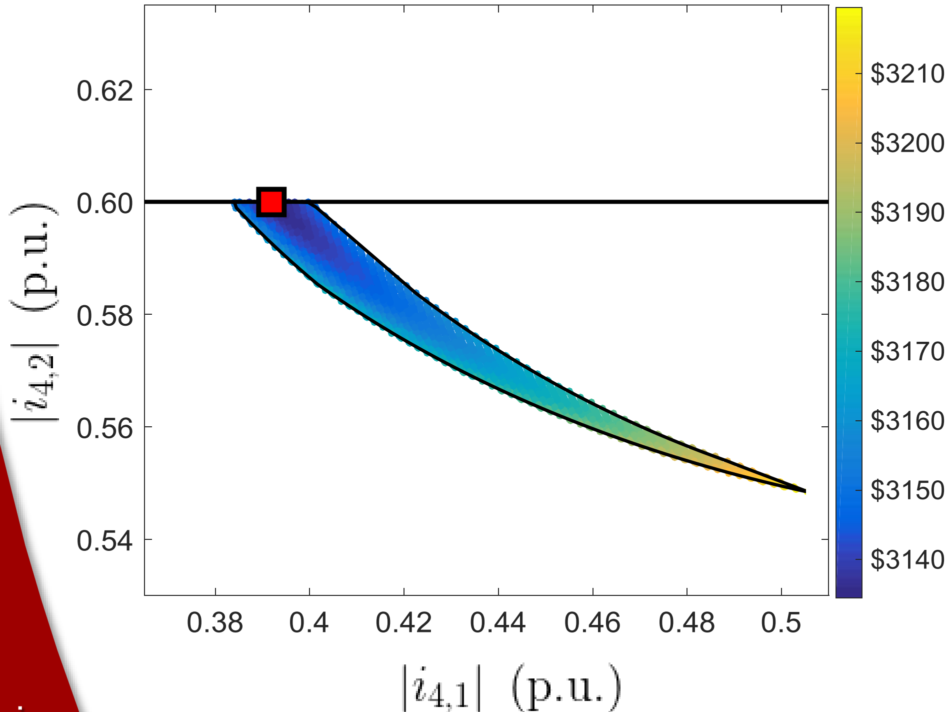
Example

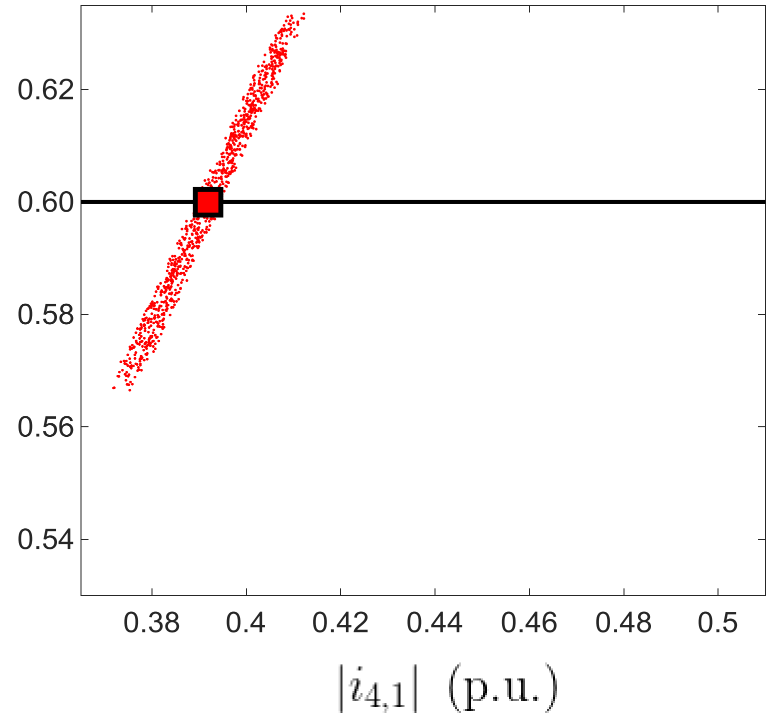
- 6-bus system "case6ww"
  - Equal participation factors
  - $\pm$5% uncertainty in each load demand



Feasible Space



Uncertainty Realizations



Feasible space constructed using the approach in [Molzahn '17]

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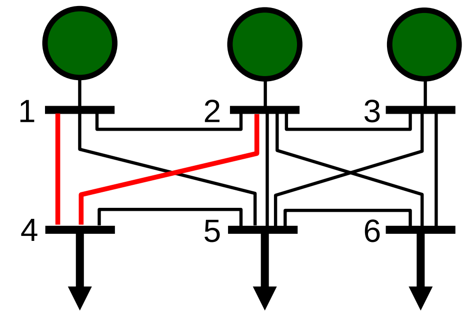

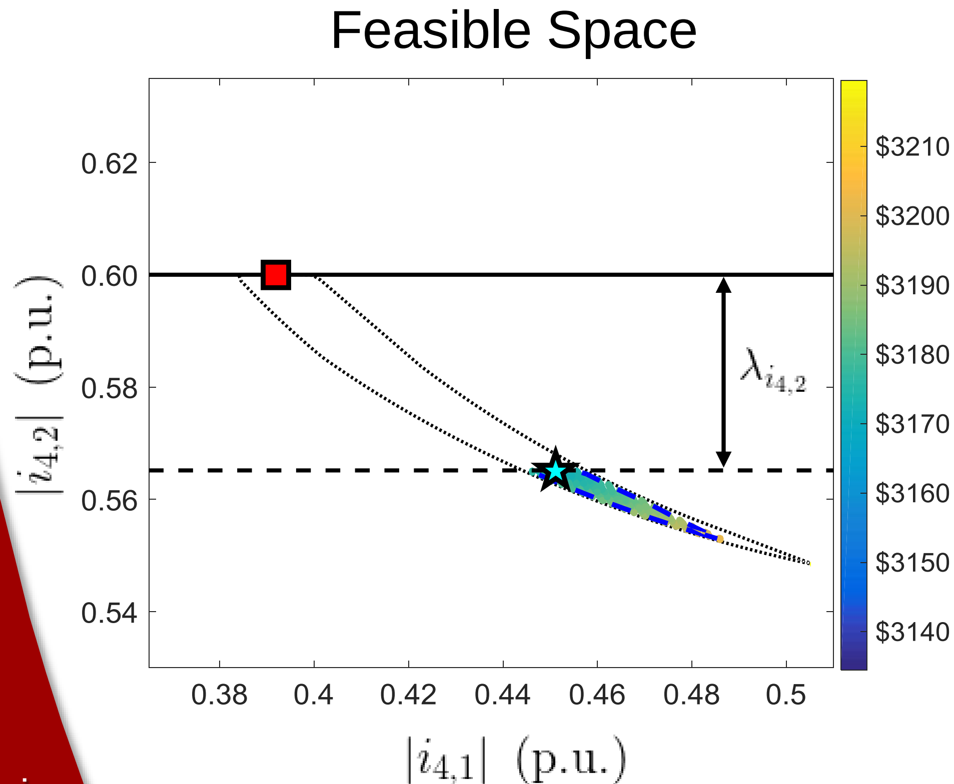

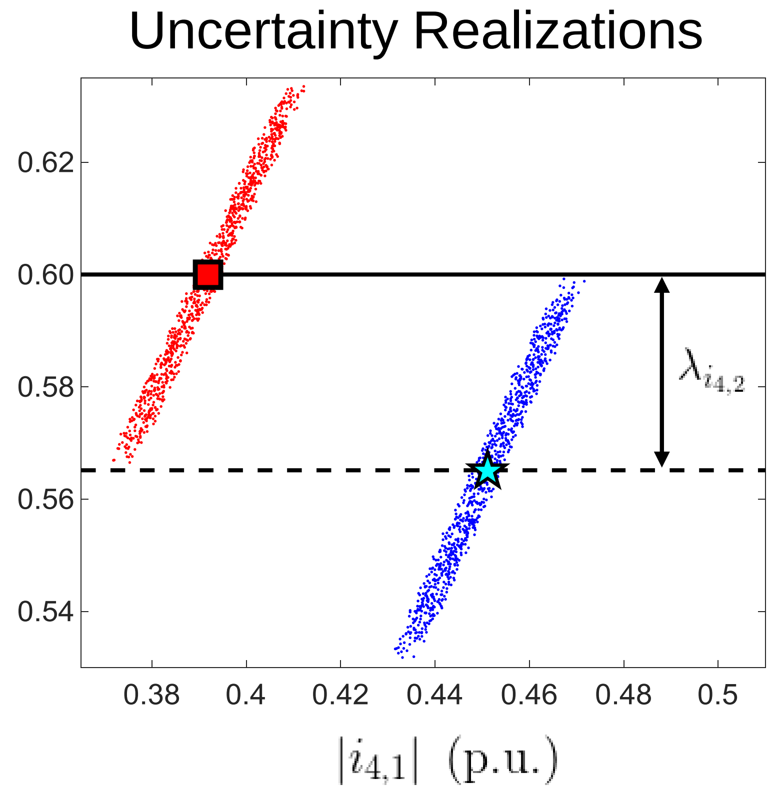

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Overview

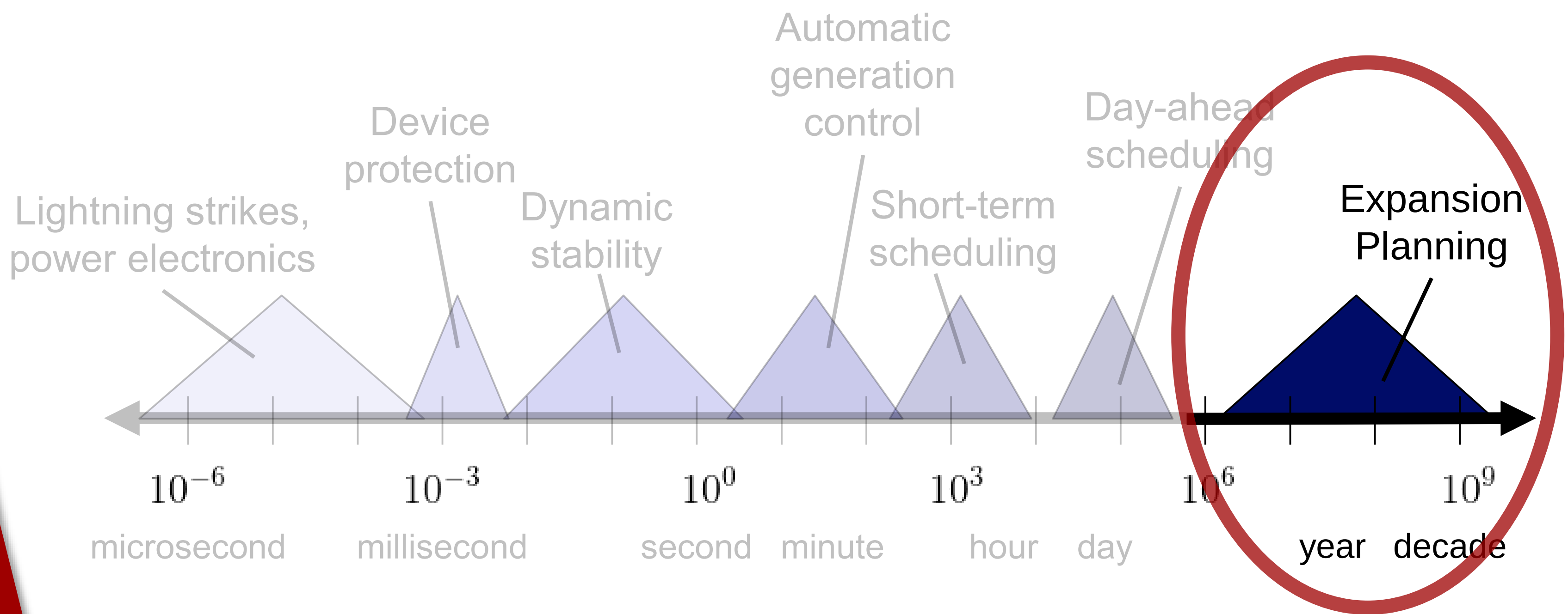
- Solving robust AC optimal power flow problems.

D.K. Molzahn, and L.A. Roald, "Towards and AC Optimal Power Flow Algorithm with Robust Feasibility Guarantees," *20th Power Systems Computation Conference (PSCC)*, June 11-15, 2018.

- Certifying engineering constraint satisfaction with limited measurements and controllability.

D.K. Molzahn, and L.A. Roald, "Grid-Aware versus Grid-Agnostic Distribution System Control: A Method for Certifying Engineering Constraint Satisfaction," *52nd Hawaii International Conference on Systems Sciences (HICSS)*, January 8-11, 2019.

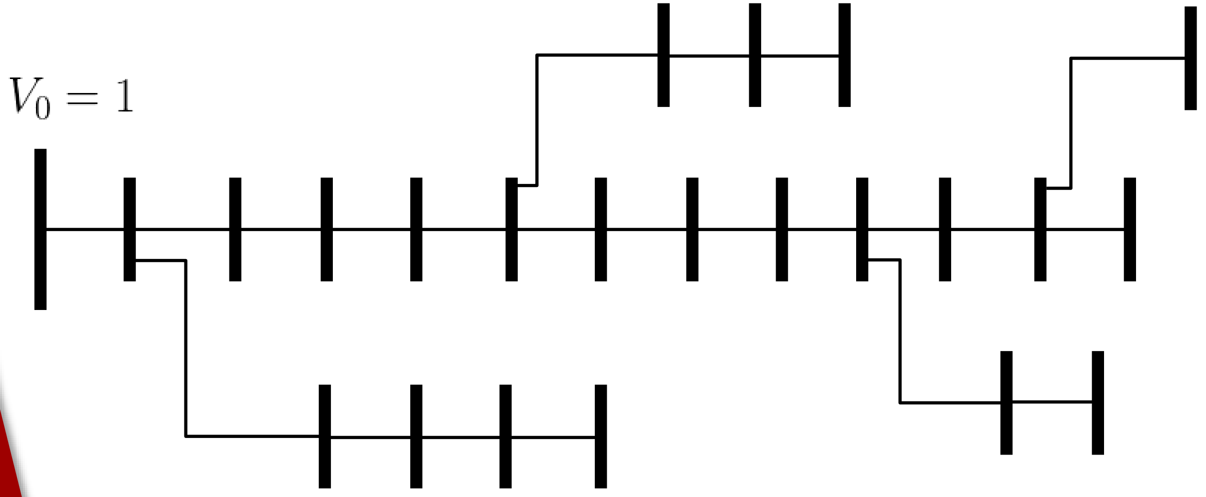
Time Scales in Power Systems



How to handle uncertainties from renewable generators and loads in this time frame?

Problem Statement

- Certify when limited measurements and controllability are sufficient to ensure satisfaction of all engineering constraints for a range of power injection fluctuations.



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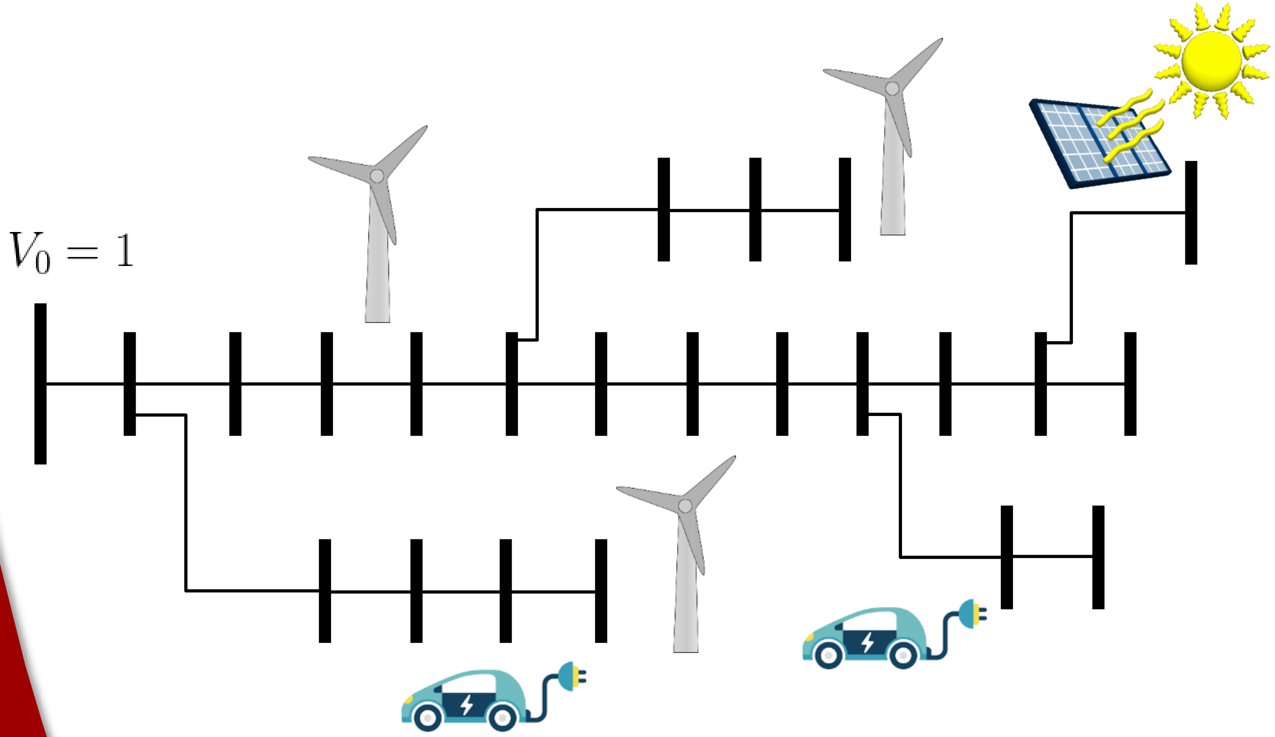

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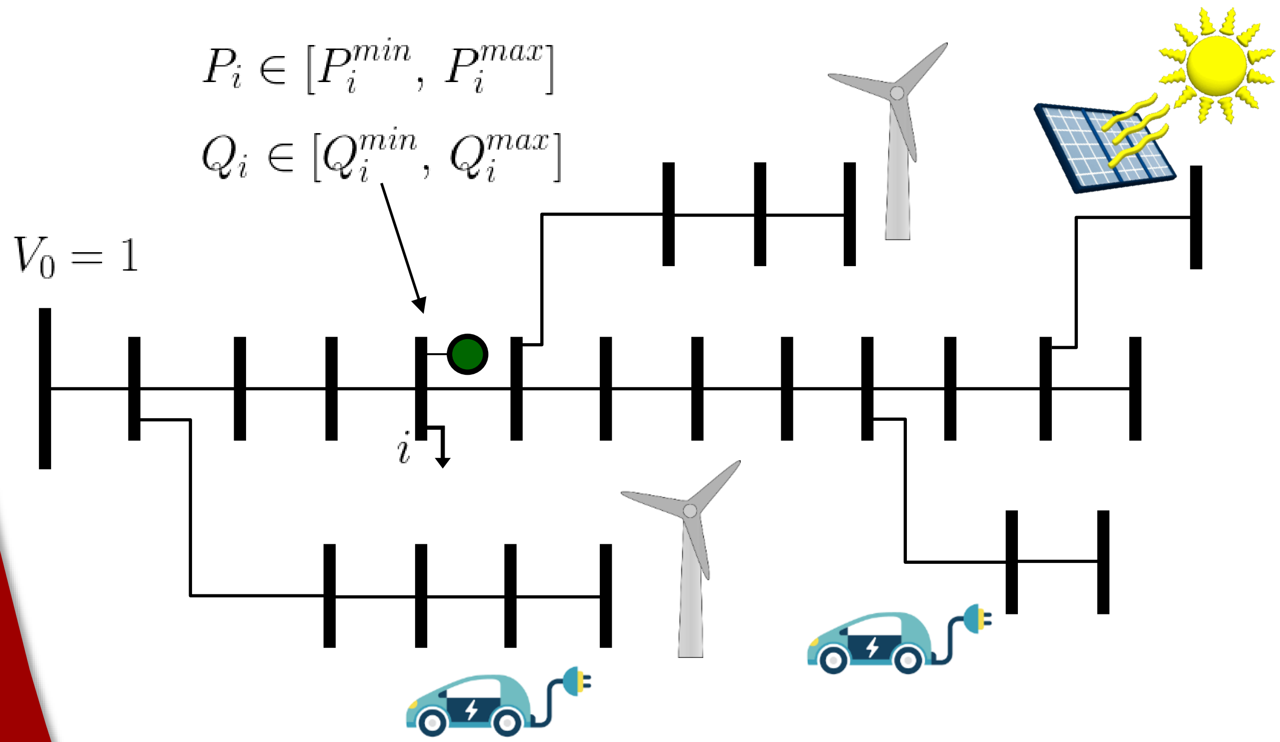

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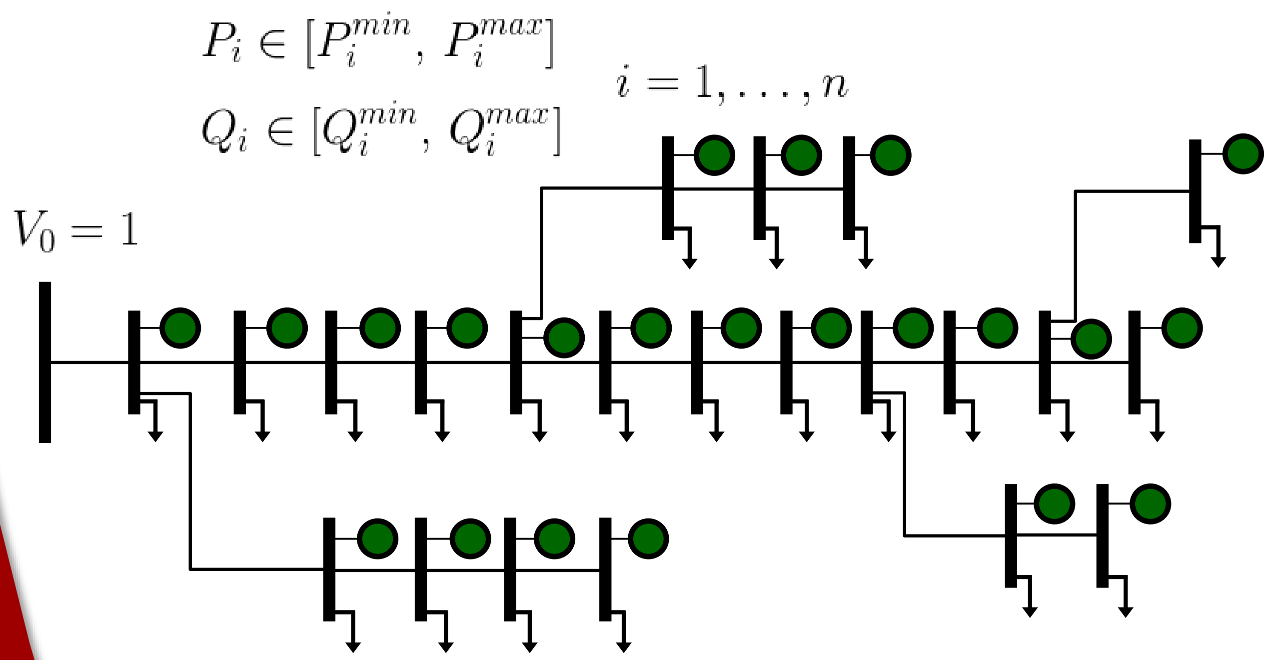

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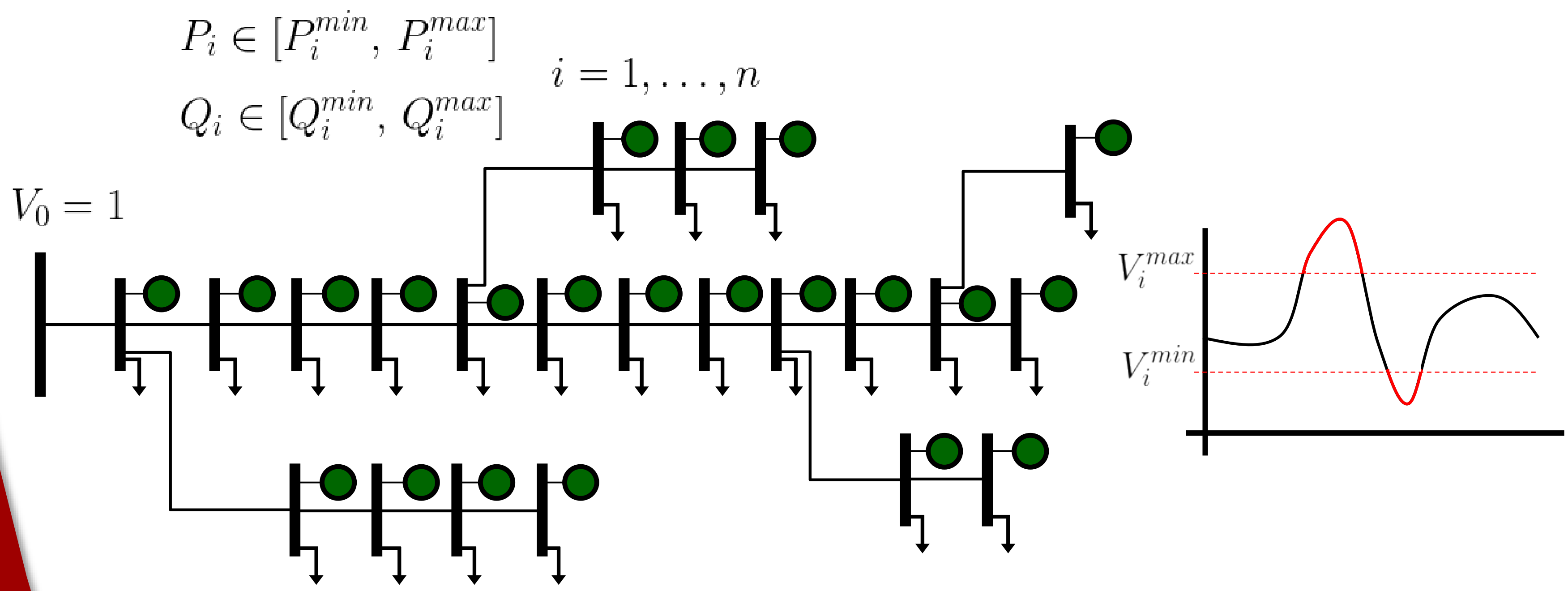

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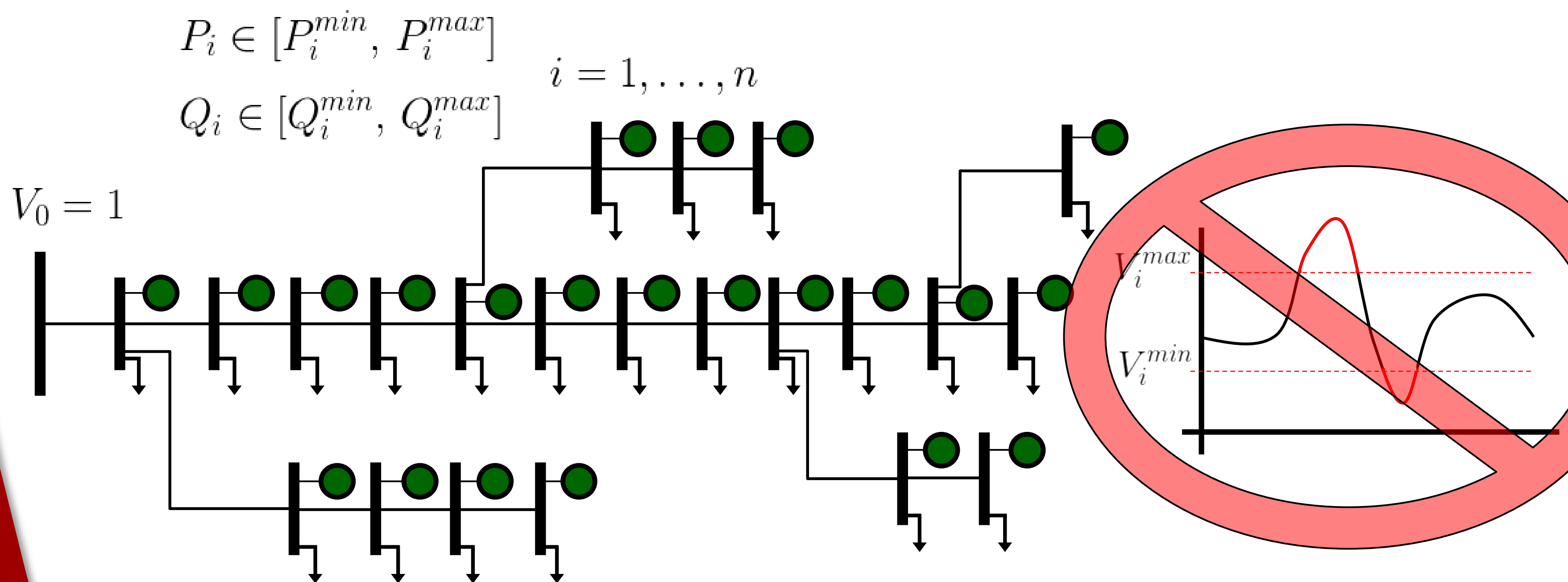

For all $P_i \in [P_i^{min}, P_i^{max}]$ and $Q_i \in [Q_i^{min}, Q_i^{max}]$, attempt to certify that $V_i \in [V_i^{min}, V_i^{max}]$.

Bounds on the Extreme Voltages

$$\underline{V}_n = \min_{P,Q,V,\theta} V_n \quad \textbf{or} \quad \overline{V}_n = \max_{P,Q,V,\theta} V_n$$

subject to $\quad (\forall i \in \mathcal{N})$

$$\underline{P}_k \leq P_k \leq \overline{P}_k, \qquad \forall k \in \mathcal{N} \backslash ref,$$

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$$P_i = V_i \sum_{k \in \mathcal{N}} V_k \left(\mathbf{G}_{ik} \cos\left(\theta_{ik}\right) + \mathbf{B}_{ik} \sin\left(\theta_{ik}\right)\right),$$

$$Q_i = V_i \sum_{k \in \mathcal{N}} V_k \left(\mathbf{G}_{ik} \sin\left(\theta_{ik}\right) - \mathbf{B}_{ik} \cos\left(\theta_{ik}\right)\right),$$

$$\theta_{ref} = 0$$

$$V_{lb} \leq V_i$$

Maximize or minimize the voltage magnitude at a particular bus n

Set of possible power injection realizations

Power flow equations

Avoid low-voltage power flow solutions

Constraint Satisfaction

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Convex relaxation

$$\theta_{ref} = 0$$

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Avoid low-voltage power flow solutions

Constraint Satisfaction

Constraint Satisfaction Certificate

- If the bounds on the achievable voltage magnitudes are less extreme than the voltage limits, no power injection fluctuation can cause constraint violations.

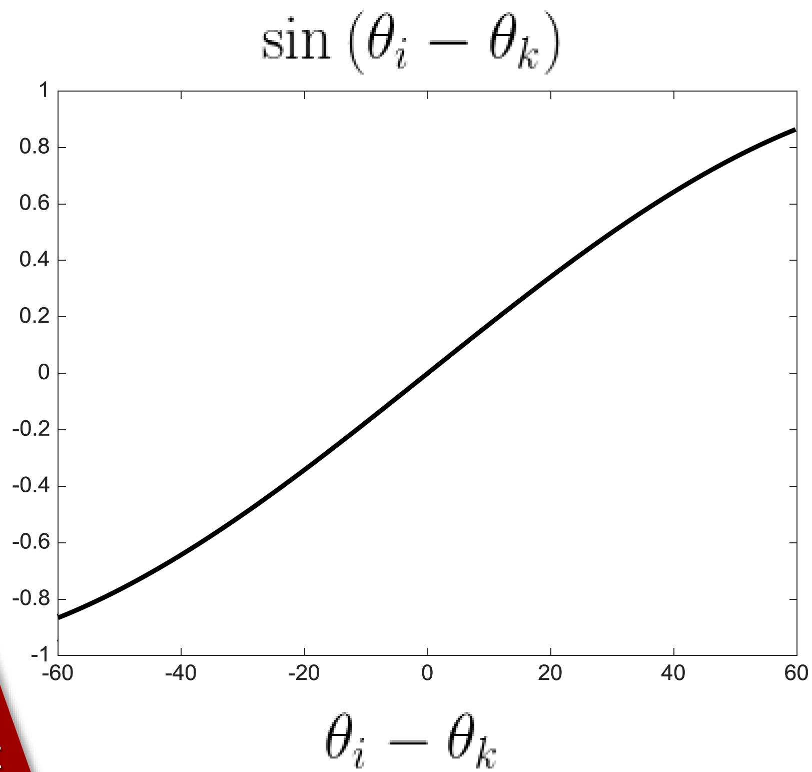
$$\boxed{V_i^{min} \leq \underline{V}_i \ \text{ and } \ \overline{V}_i \leq V_i^{max}, \quad \forall i \in \mathcal{N}}$$

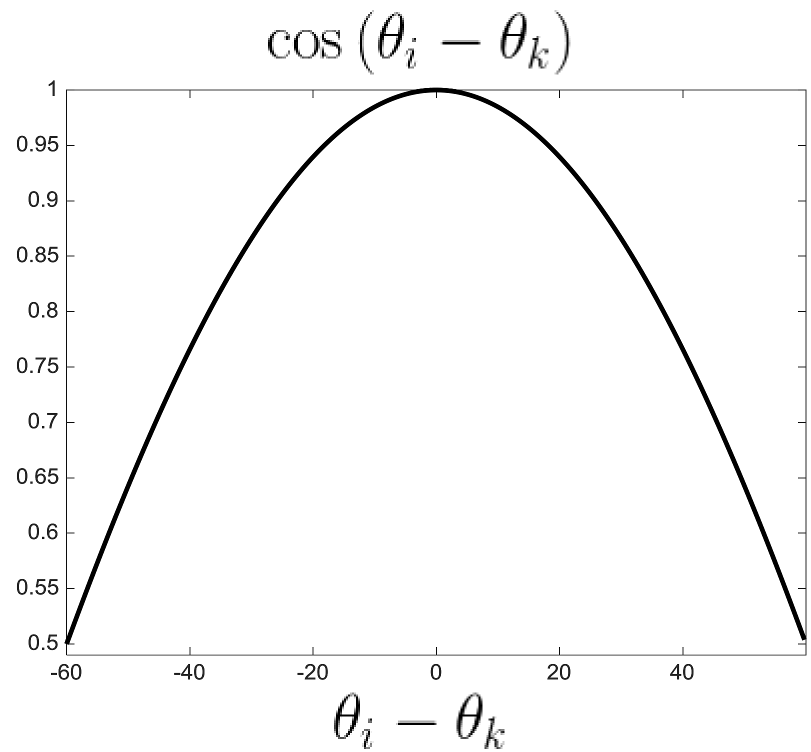
- Repeat for all constrained quantities.

The QC Relaxation

- Construct convex envelopes around the sine and cosine functions in the power flow equations with polar voltages

[Coffrin, Hijazi & Van Hentenryck '15]





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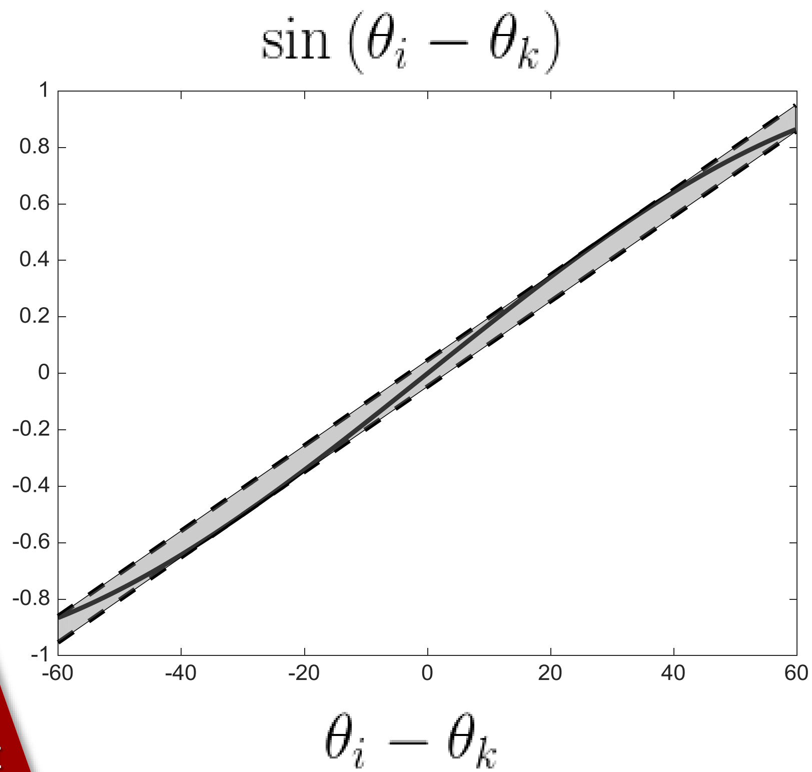
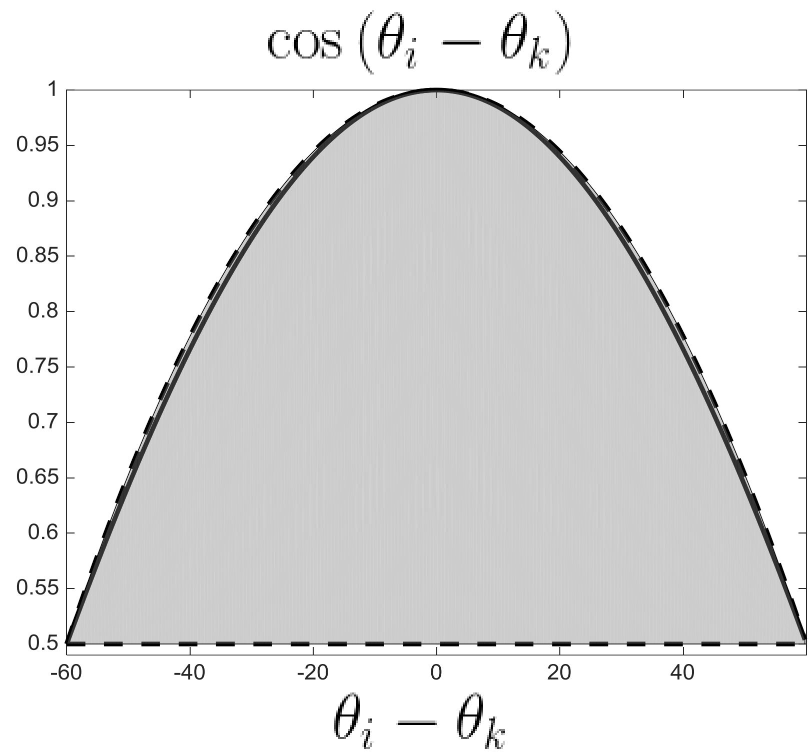
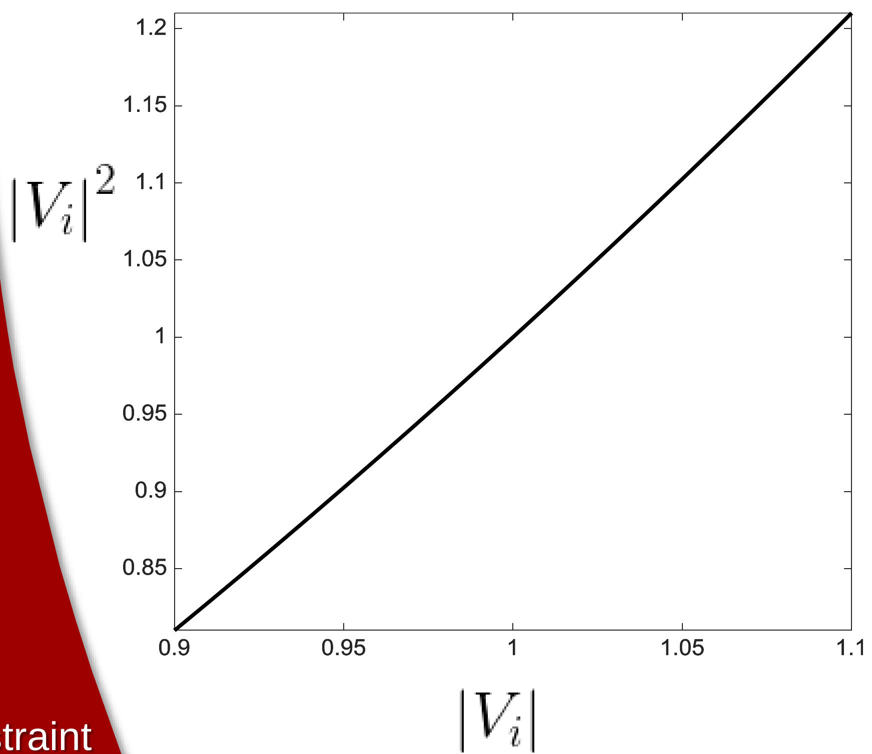
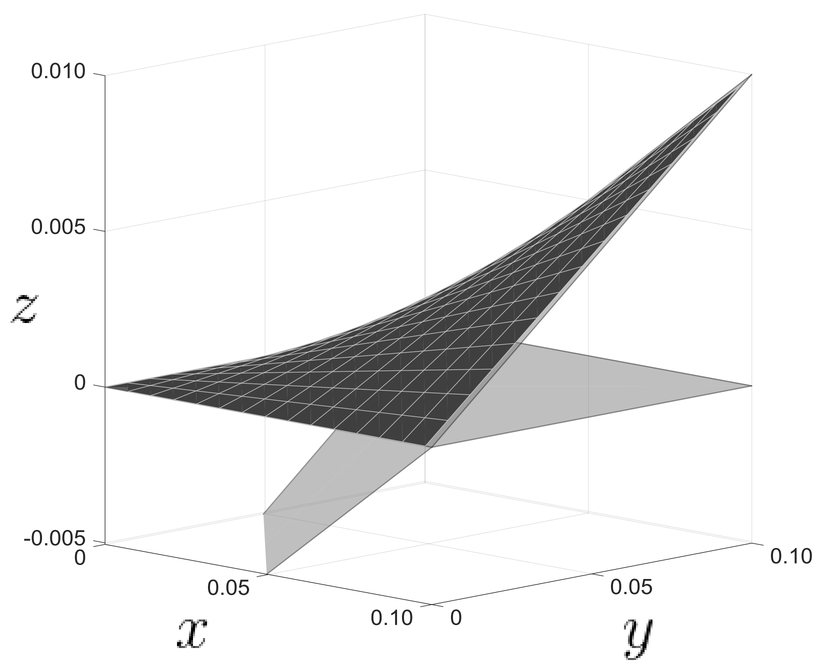




The QC Relaxation

- Convex envelopes for the squared voltage terms
- McCormick envelopes for the bilinear product terms





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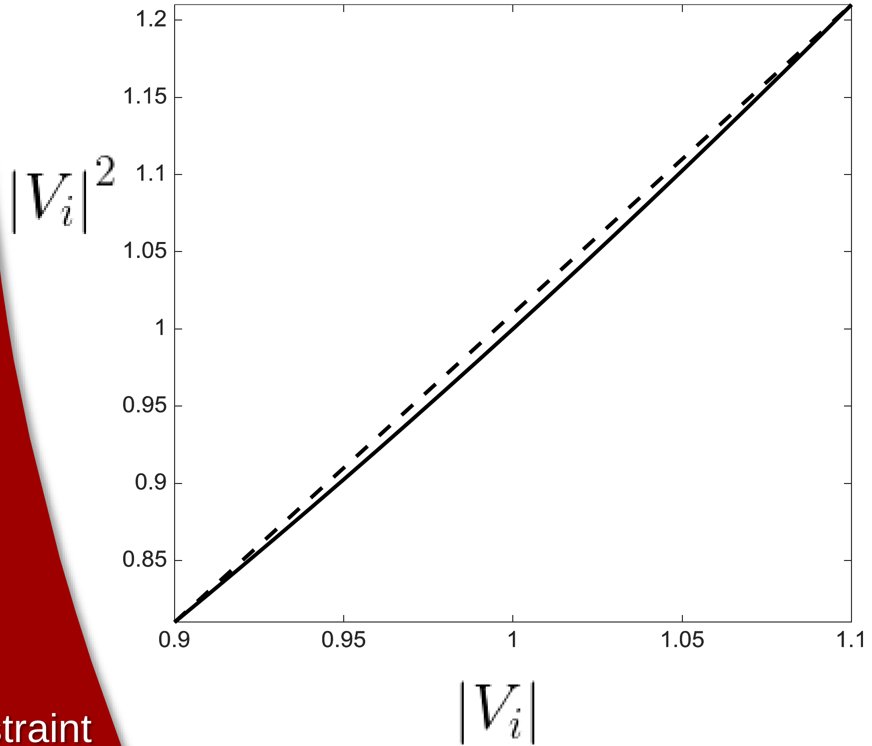

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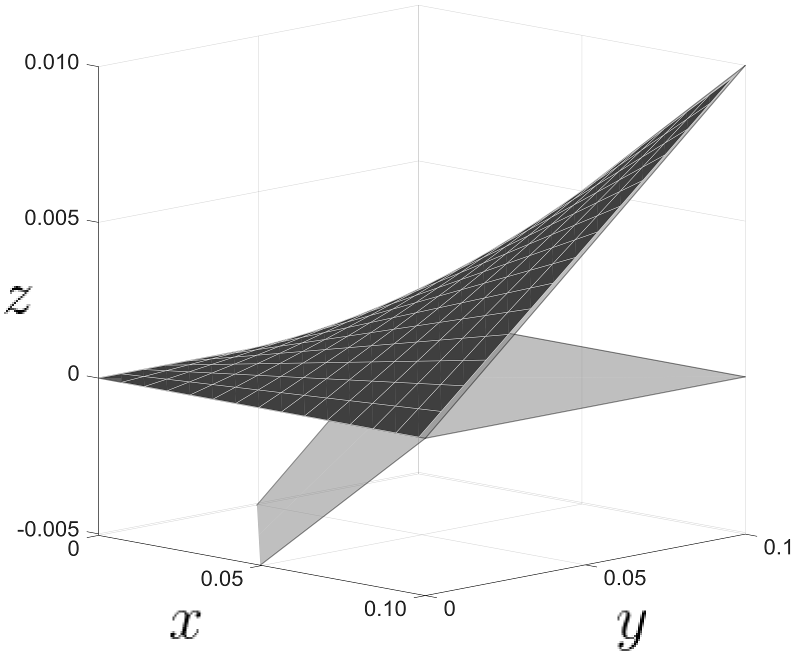

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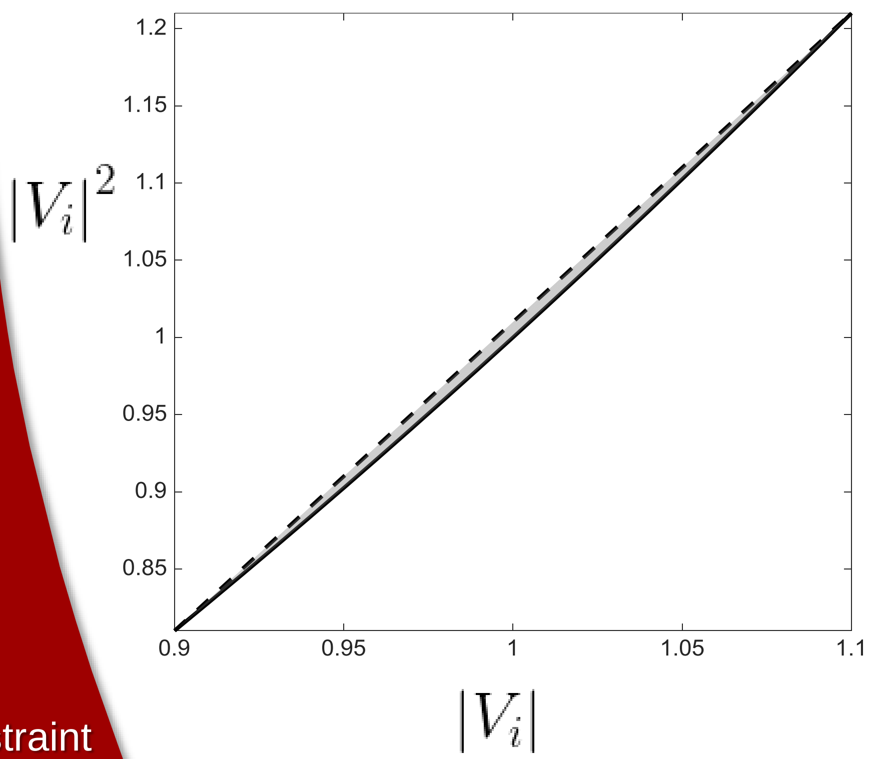

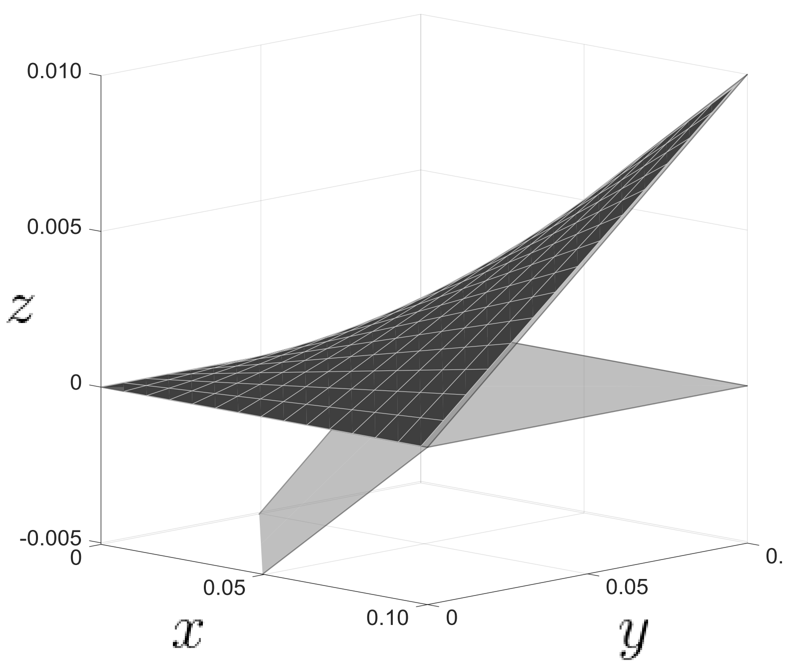

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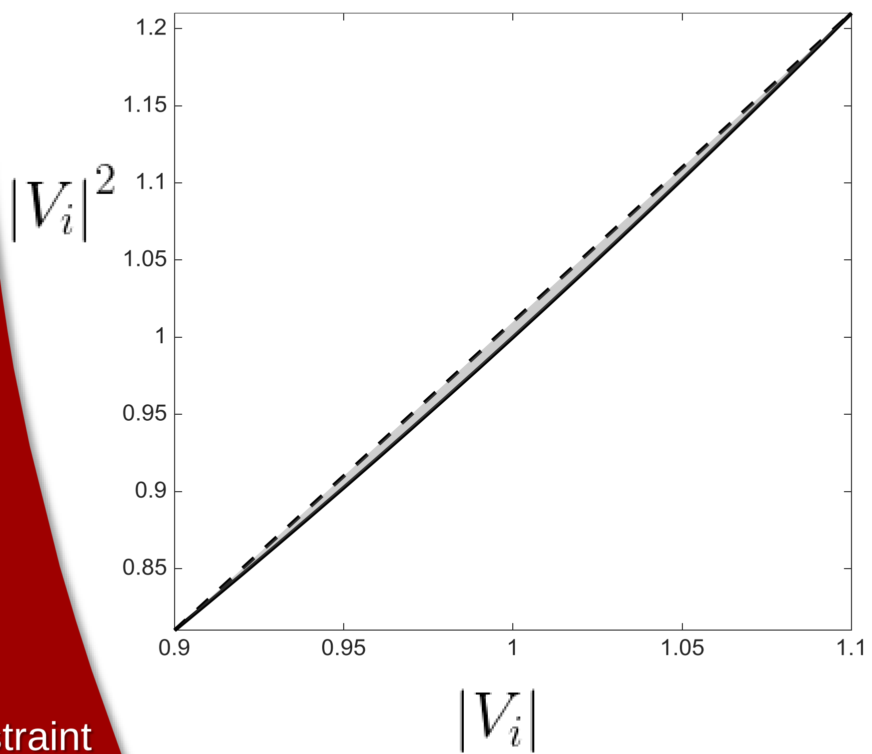

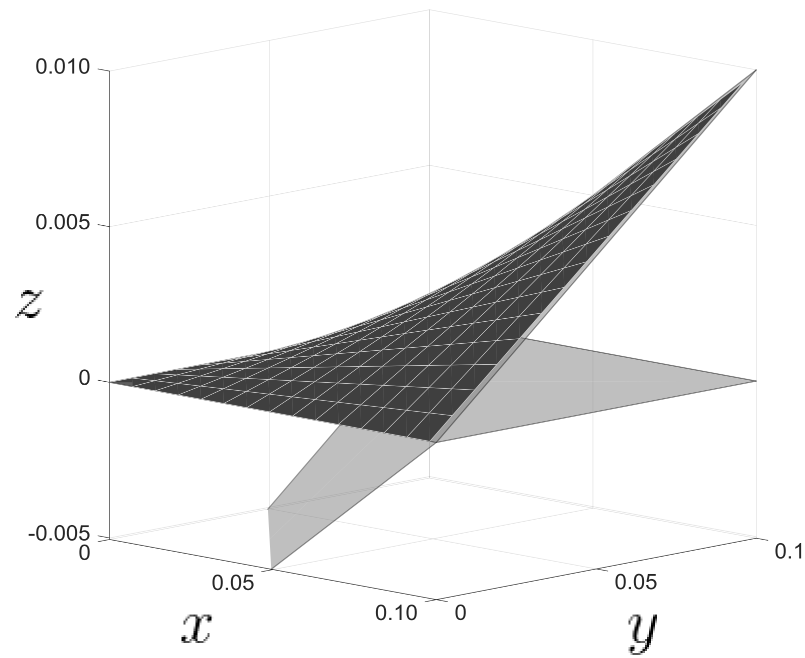

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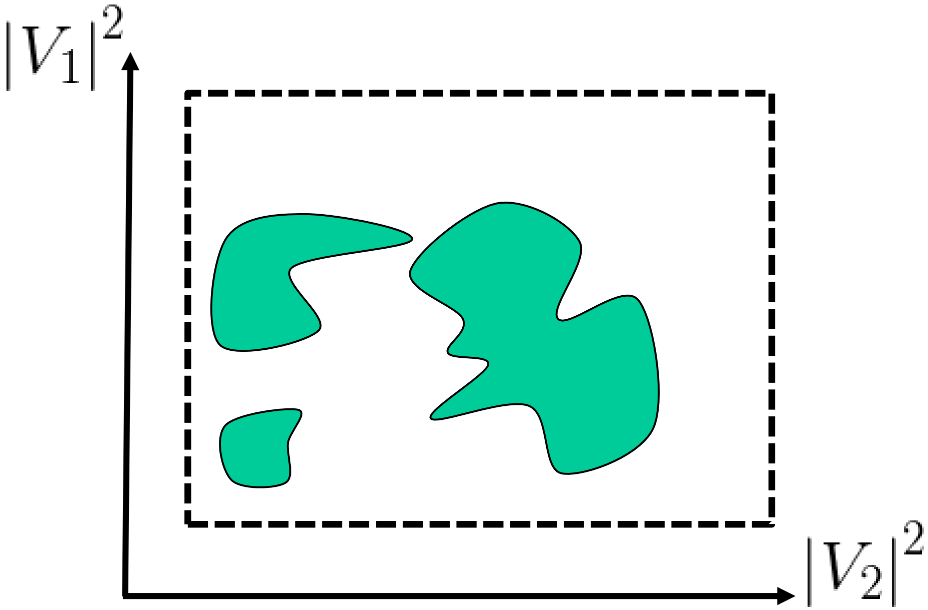

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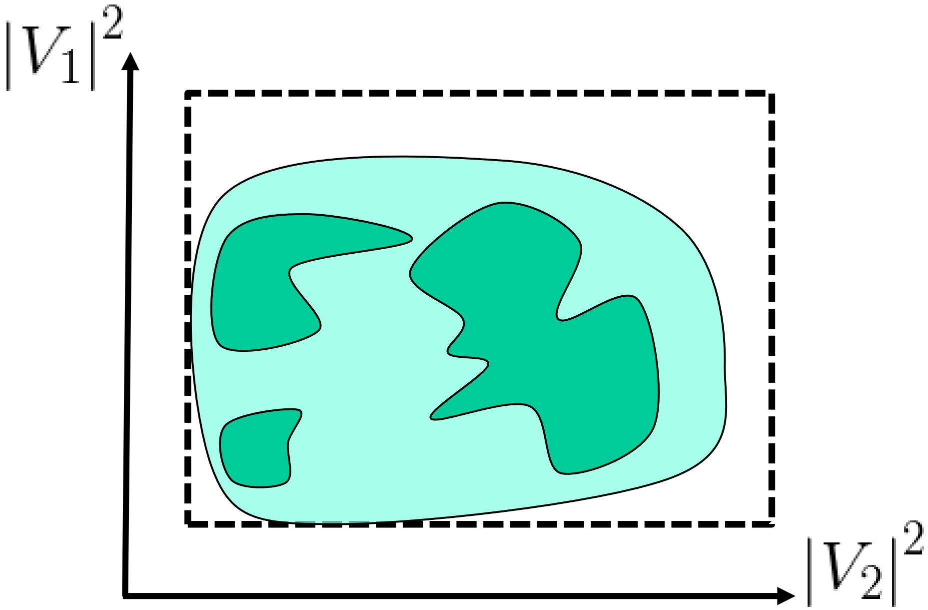

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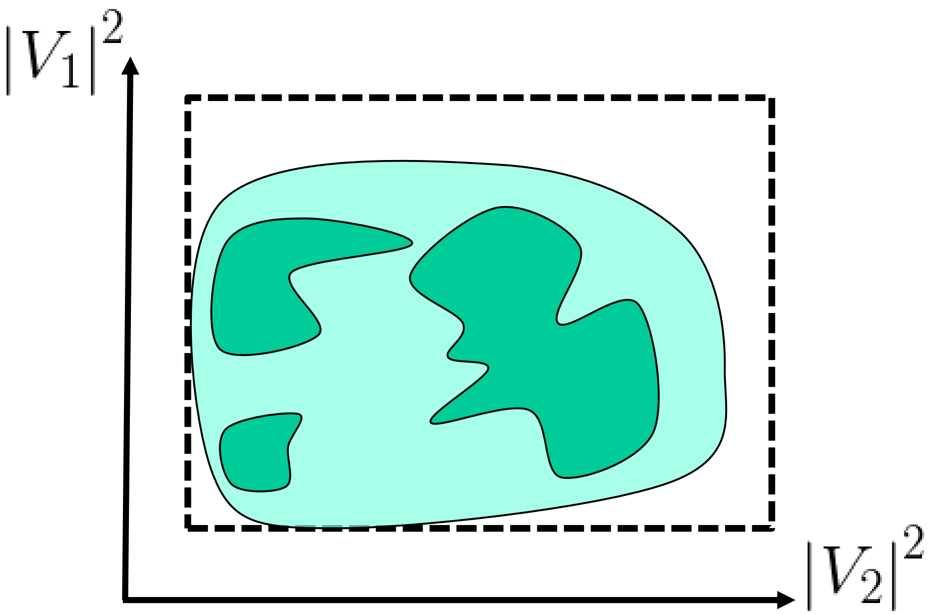

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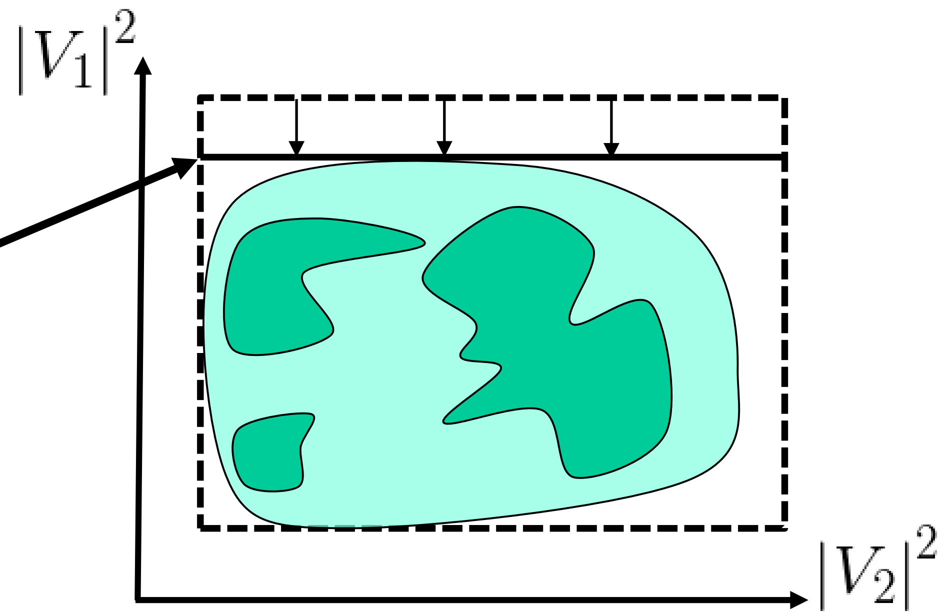

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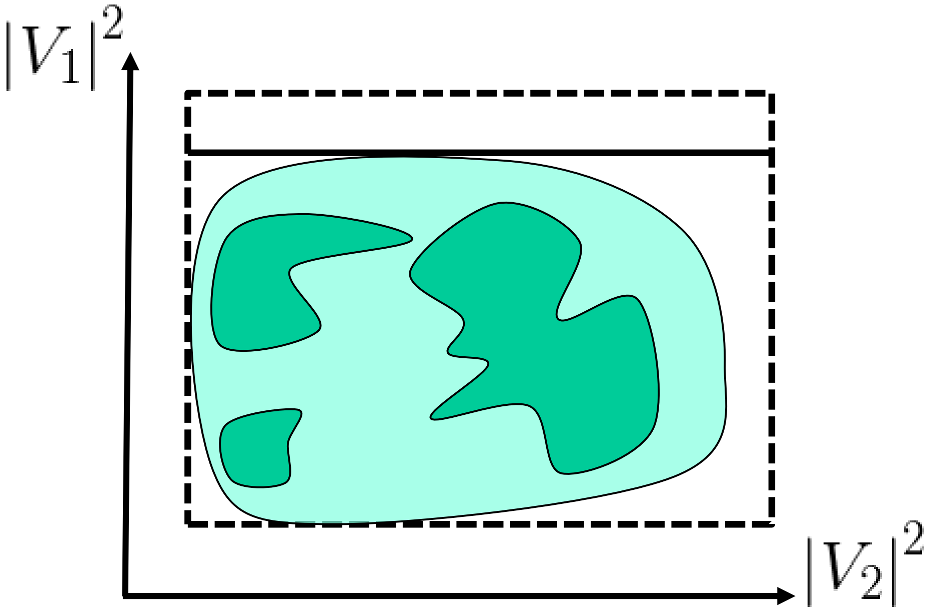

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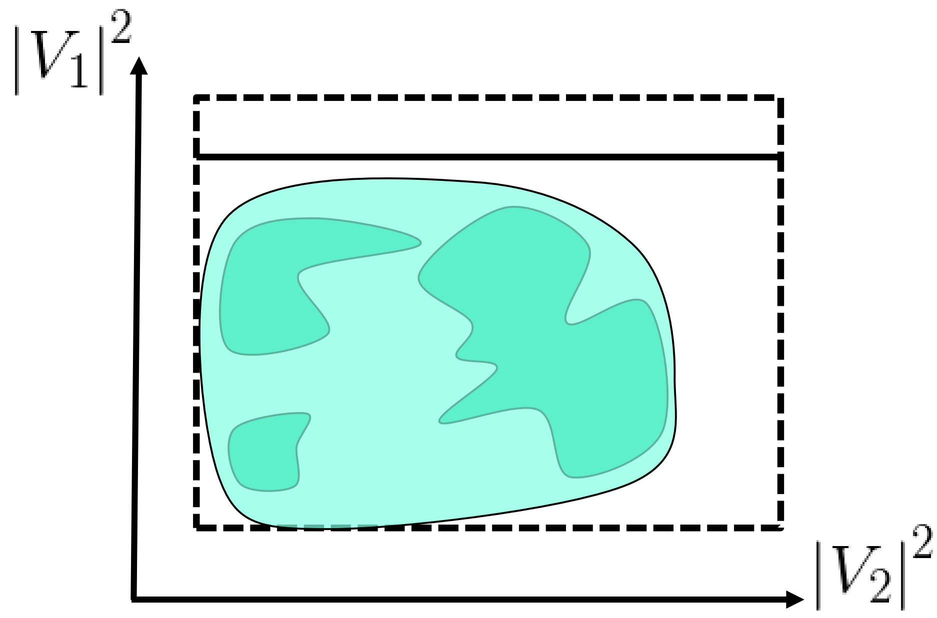

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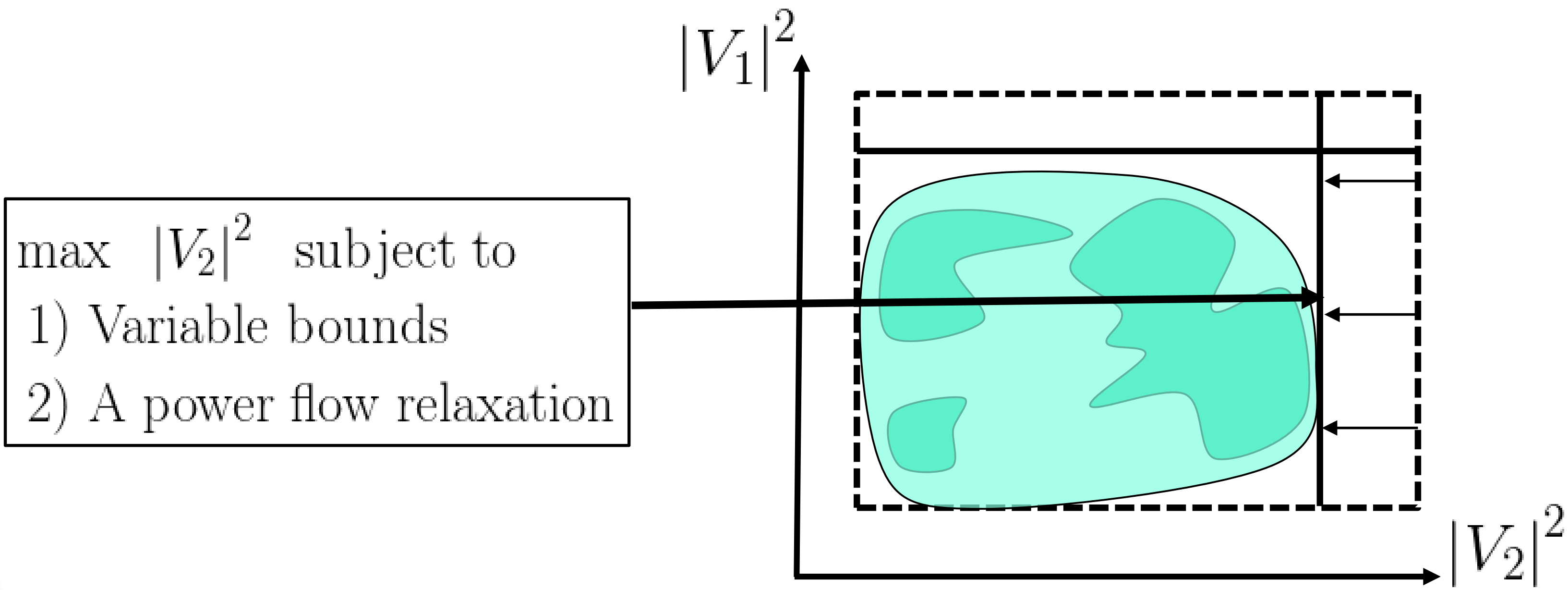

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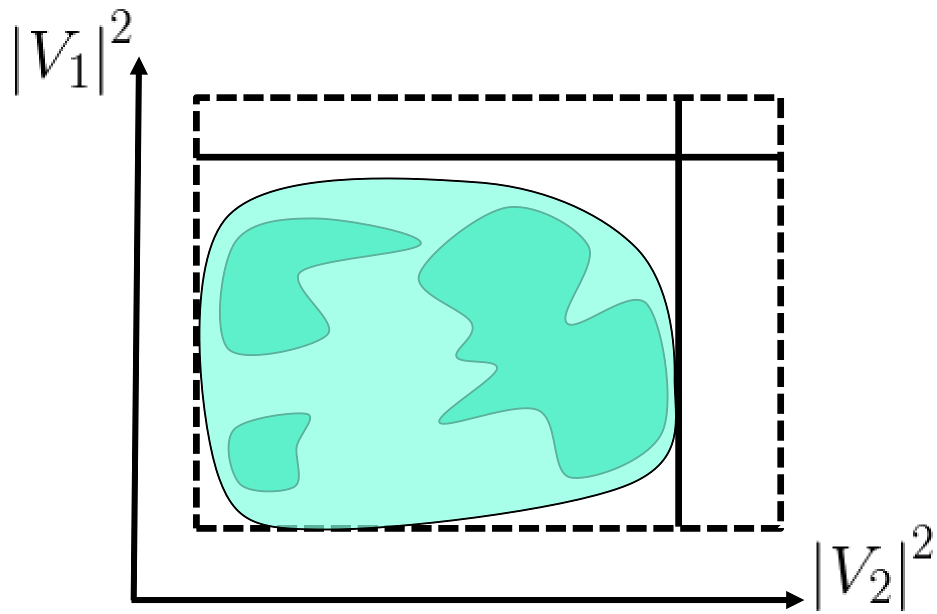

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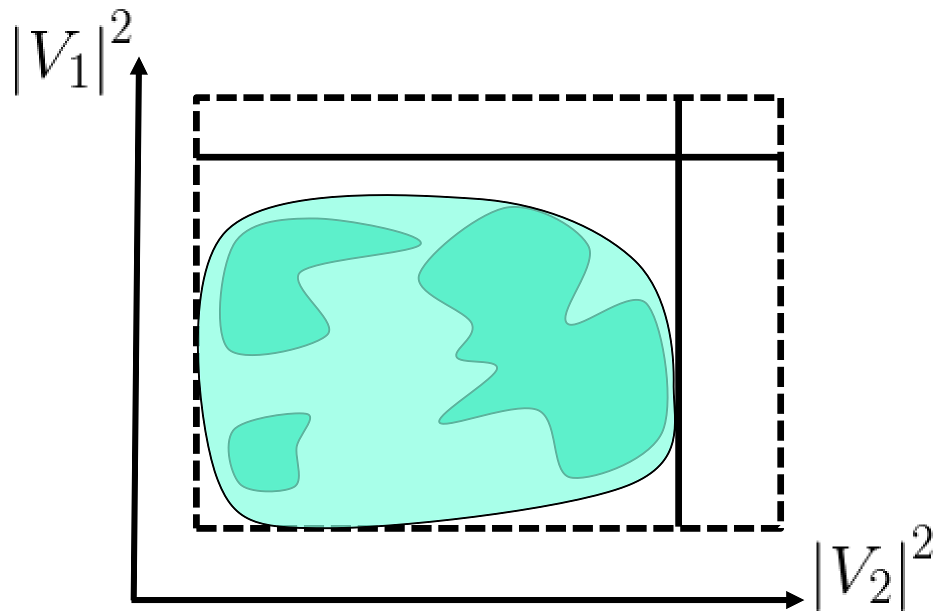

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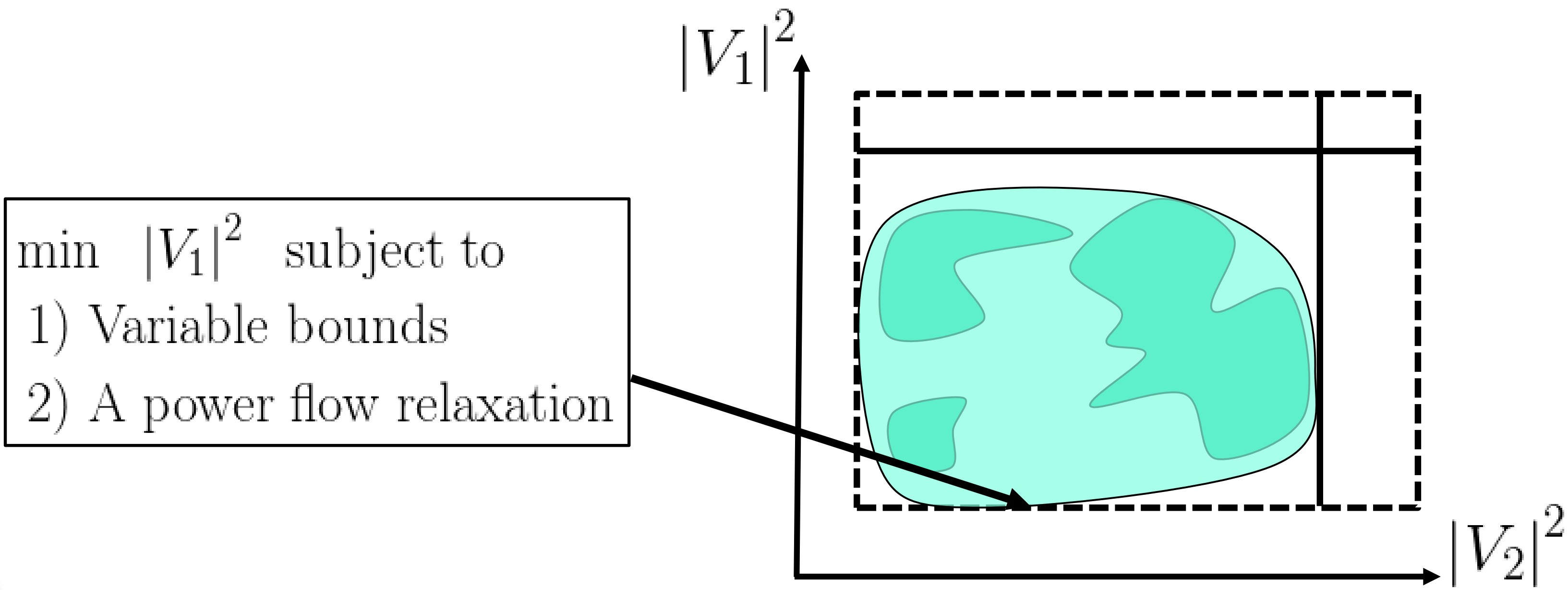

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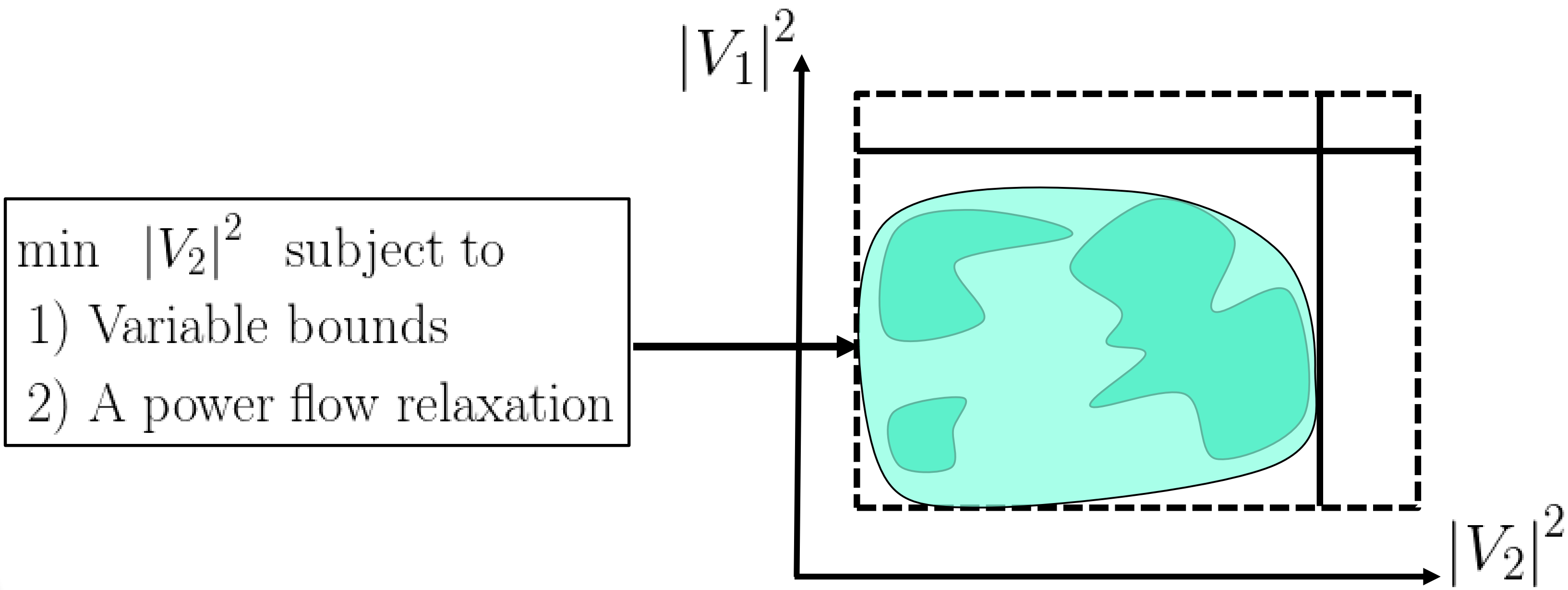

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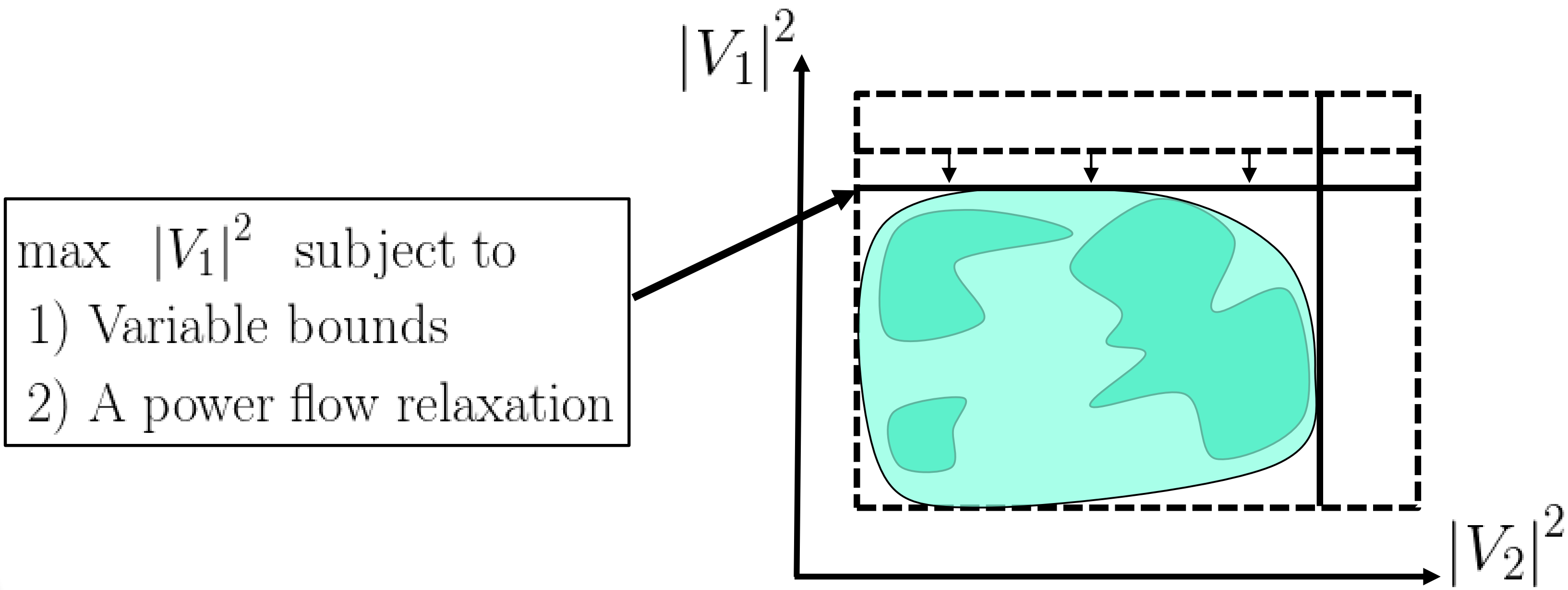

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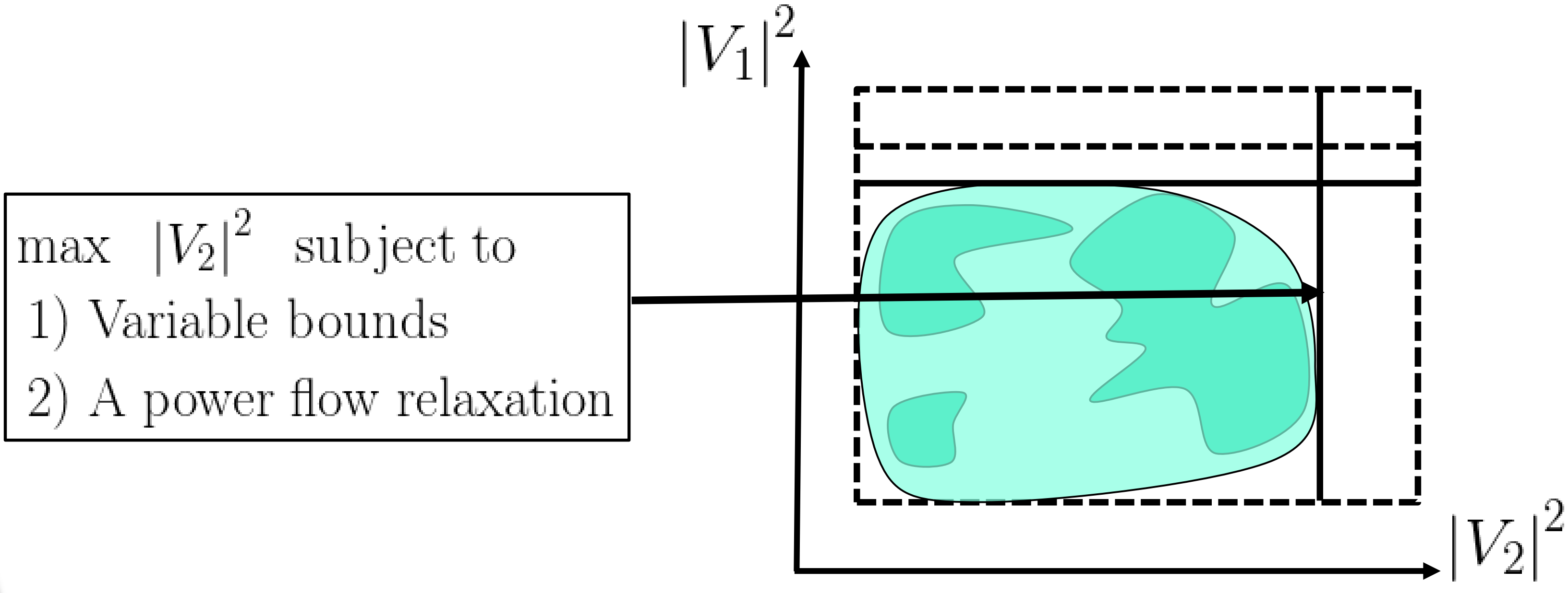

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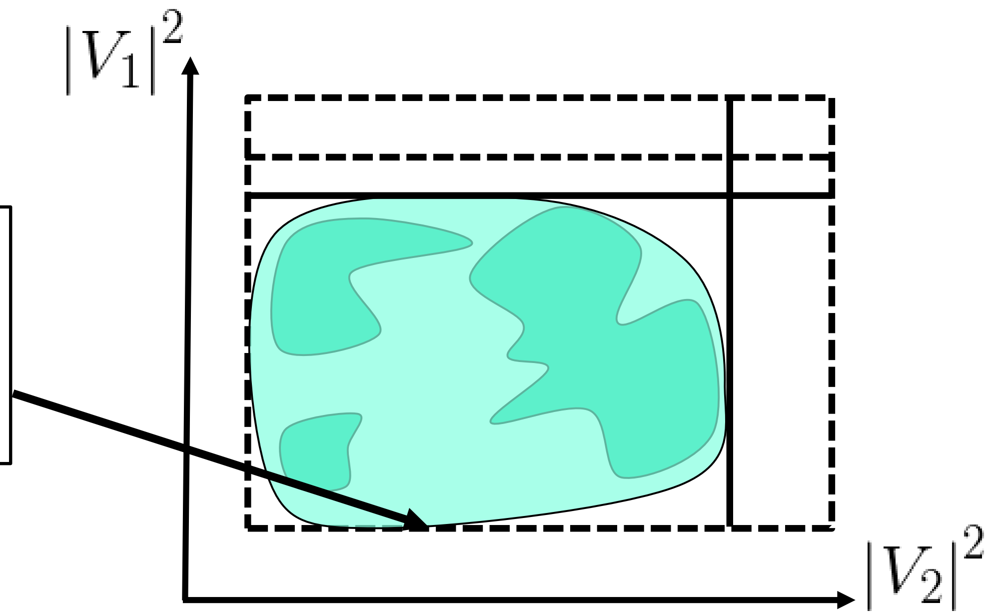

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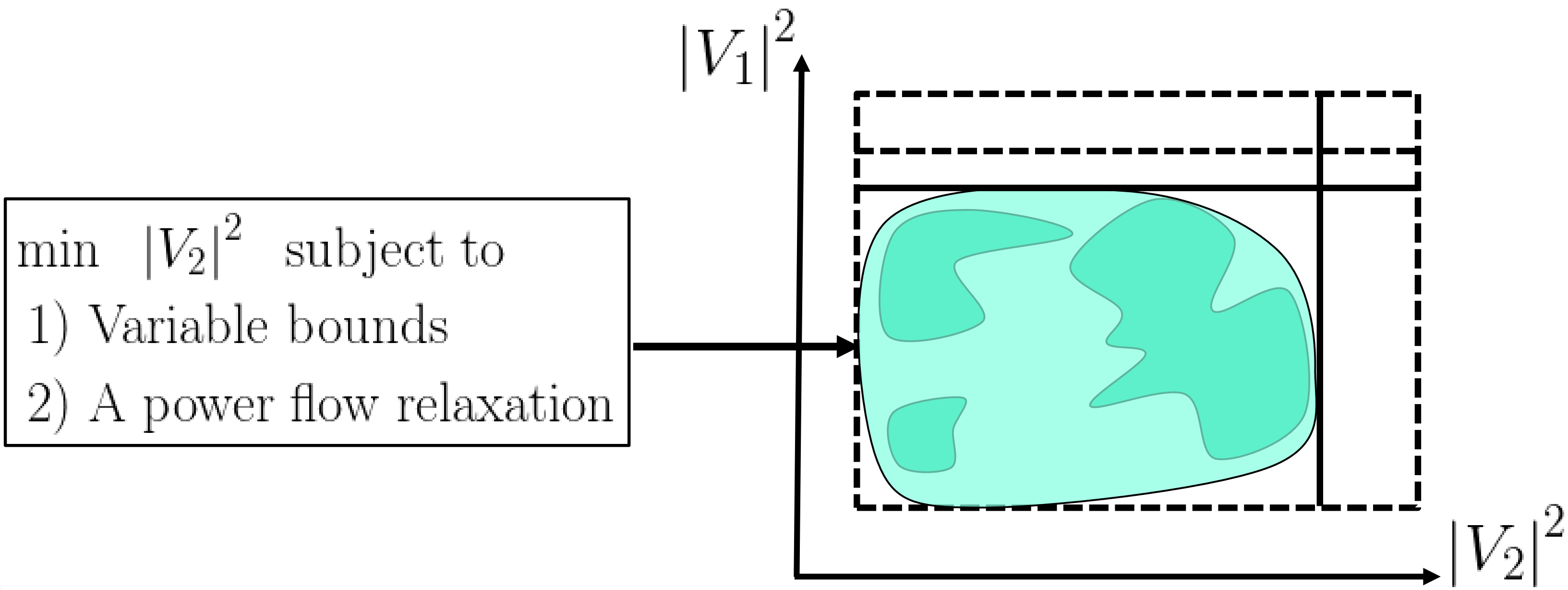

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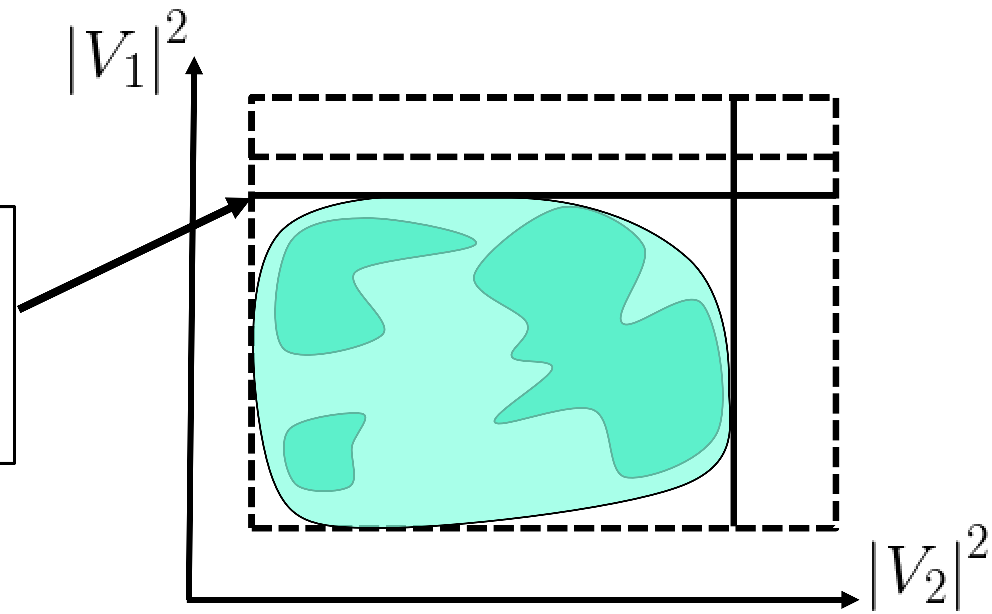

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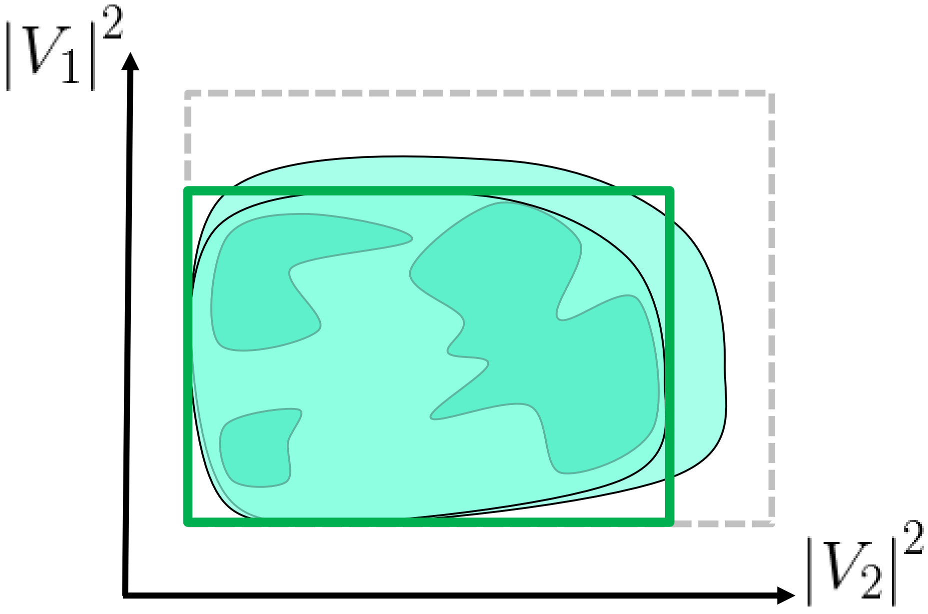

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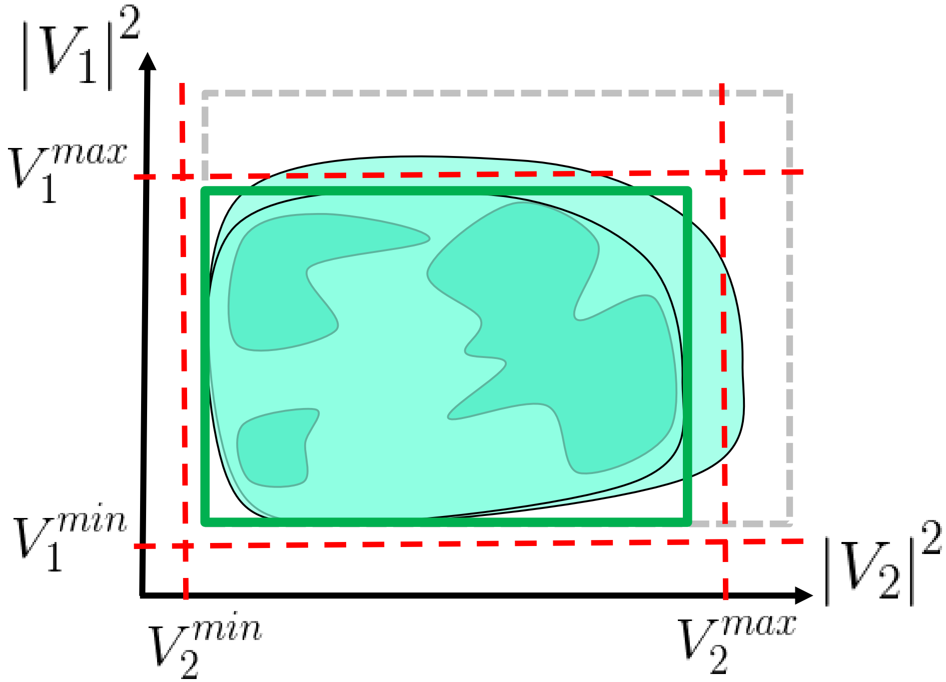

[Coffrin, Hijazi, Van Hentenryck '15], [Kocuk, Dey, & Sun '16], [Chen, Atamturk, & Oren '16], [Schetinin '18]

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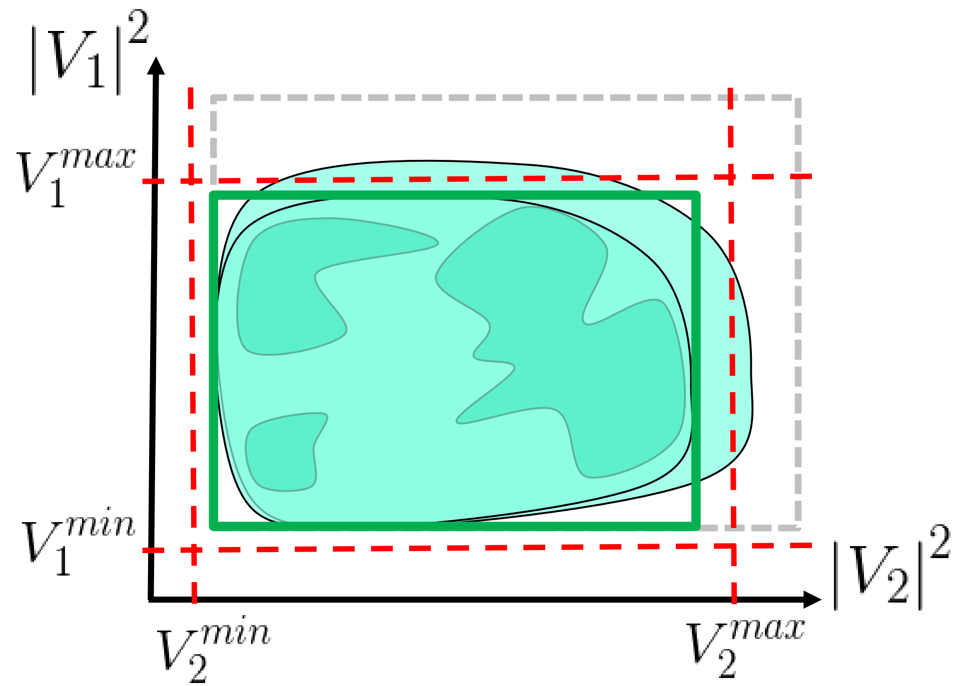
Certify constraint satisfaction!



[Coffrin, Hijazi, Van Hentenryck ’15], [Kocuk, Dey, & Sun ‘16], [Chen, Atamturk, & Oren ‘16], [Schetinin ‘18]

Mitigating Potential Violations

- If we cannot certify constraint satisfaction, model additional controllability:

$$\underline{V}_n = \min_{P,Q,V,\theta} V_n \quad \textbf{or} \quad \overline{V}_n = \max_{P,Q,V,\theta} V_n$$

Maximize or minimize the voltage magnitude

$$\text{subject to} \quad (\forall i \in \mathcal{N})$$

$$\underline{P}_k \leq P_k \leq \overline{P}_k, \qquad \forall k \in \mathcal{N} \backslash ref,$$

$$\underline{Q}_k \leq Q_k \leq \overline{Q}_k, \qquad \forall k \in \mathcal{N} \backslash ref,$$

Set of possible power injection realizations

$$\underset{\sim}{V}_j \leq V_j \leq \tilde{V}_j, \qquad \forall j \in \mathcal{V},$$

Voltage controller model

$$P_i = V_i \sum_{k \in \mathcal{N}} V_k \left(\mathbf{G}_{ik} \cos\left(\theta_{ik}\right) + \mathbf{B}_{ik} \sin\left(\theta_{ik}\right)\right),$$

$$Q_i = V_i \sum_{k \in \mathcal{N}} V_k \left(\mathbf{G}_{ik} \sin\left(\theta_{ik}\right) - \mathbf{B}_{ik} \cos\left(\theta_{ik}\right)\right),$$

Power flow equations

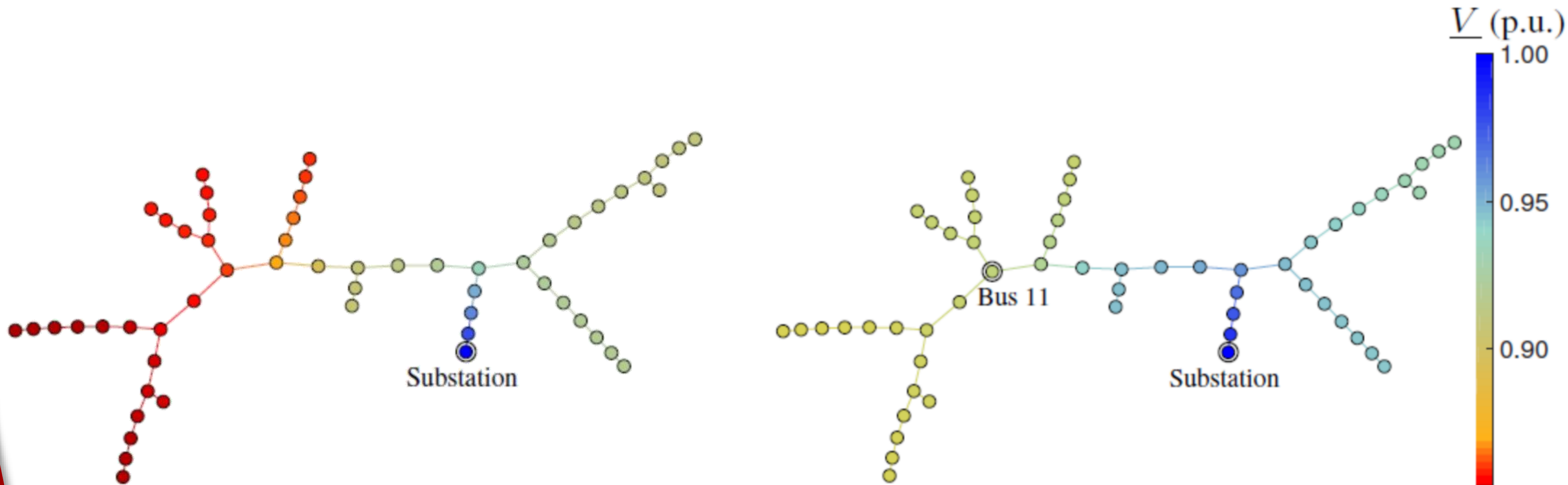
$$\theta_{ref} = 0$$

$$V_{lb} \leq V_i$$

Avoid low-voltage power flow solutions

Illustrative Example

- 56-bus radial test case with $\pm 50\%$ load variability,
  $V_i^{min} = 0.90$ p.u., $V_i^{max} = 1.10$ p.u. [Bolognani & Zampieri '16]



Insufficient measurement and control to certify security

Voltage control at bus 11 modeled as $V_{11} \in [0.92, 1.08]$ p.u. is sufficient to certify security

Constraint Satisfaction

Conclusions

- Algorithm for computing robustly feasible operating points for OPF problems.
- Algorithm for certifying constraint satisfaction with limited measurements and controllability.
- Next steps:
  - Applying sufficient conditions to guarantee power flow solvability.
  - Improving computational tractability.
  - Studying convergence characteristics of robust AC OPF algorithm.
  - Computing least-cost measurement and controllability additions that ensure constraint satisfaction.

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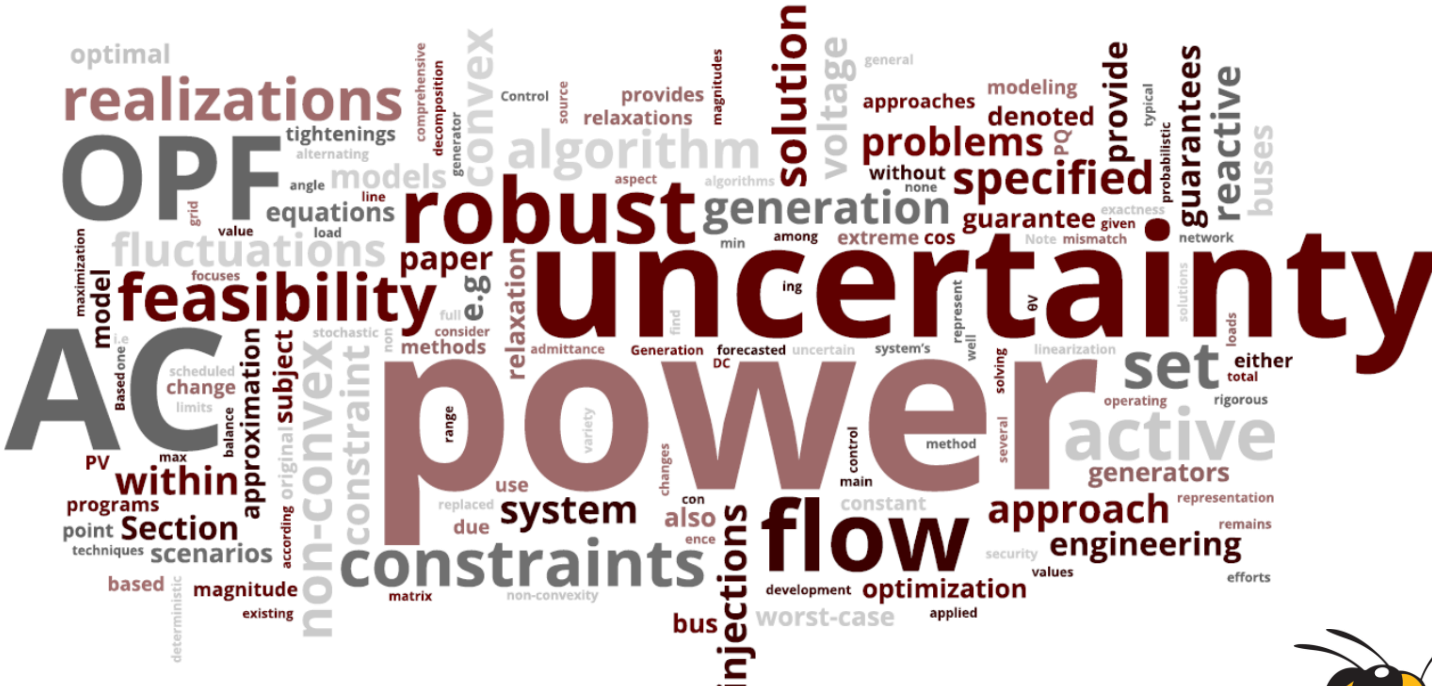
Questions?



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