Linearized Optimal Power Flow with Regards to Grid & Load Characteristics

(work in progress)

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Introduction

- Panayiotis (Panos) Moutis
- Recent activity:
  - Distributed optimization based on Consensus+Innovation
  - PMU requirements & state estimation for distribution systems
  - IEEE 2330.10 & 2330.10 standard WGs
- Main research focus: Virtual Power Plants

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Outline of presentation

- The AC Optimal Power Flow (OPF) problem, practice and challenges
- Review of linearized OPF formulations
- A straightforward new proposal
- Tests & Results
- Discussion

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The AC OPF problem

min $\quad f(x) = P \cdot c_2 \cdot P^T + c_1 \cdot P + c_0$

s.t. $\quad P_{i,g,min} \leq P_{i,g} \leq P_{i,g,max} \qquad (P^{ub/lb})$

$Q_{i,g,min} \leq Q_{i,g} \leq Q_{i,g,max} \qquad (Q^{ub/lb})$

BIM model

$$\left.\begin{aligned} p_{ij} &= g_{ij} \cdot |v_i|^2 - |v_i| \cdot |v_j| \cdot \left[g_{ij} \cdot \cos(\theta_i - \theta_j) + b_{ij} \cdot \sin(\theta_i - \theta_j) \right] \\ q_{ij} &= -b_{ij} \cdot |v_i|^2 - |v_i| \cdot |v_j| \cdot \left[g_{ij} \cdot \sin(\theta_i - \theta_j) - b_{ij} \cdot \cos(\theta_i - \theta_j) \right] \end{aligned}\right\}$$

$\sum_i P_{ij} = P_i = P_{i,g} - P_{i,d} \qquad (P^{sum})$

$\sum_i Q_{ij} = Q_i = Q_{i,g} - Q_{i,d} \qquad (Q^{sum})$

$V_{i,min} \leq V_i \leq V_{i,max} \qquad (V^q)$

$p_{ij}^2 + q_{ij}^2 \leq s_{ij,M}^2 \qquad (F^{ub})$

- Non-linear & non-convex => NP-hard to solve

Relaxing the AC OPF problem

- Semidefinite (SD) relaxations (SDP, SOCP)

BIM model => $tr(V\,Y\,V^*)$ => $W=VV^*$ => $W \succcurlyeq 0$

- For *most* practical cases in *radial networks*
  - **Exact**
  - **Global optimum** retrieved if *f(x) is convex*
- Otherwise, no guarantees
- SDP & SOCP not *very* efficient $[O(n^3), O(n^8)]$

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OPF in practice

- Non-linear solvers (good but elaborate)
- System Operators (SO) use linearized OPF
- Most ‘industry’ software based on linearized OPF
- Practical OPF-based problems much larger (security-constrained OPF, planning)

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OPF challenges ahead

- OPF at distribution systems
- Market restructure seeks maximization of value of existing infrastructure
- Numerous Distributed Generation (DG) units
- Practice heavily favoring linearized methods

… hence, linearized OPF formulations are still of much interest

Classic linearized OPF set-ups

$$p_{ij} = g_{ij} \cdot |v_i|^2 - |v_i| \cdot |v_j| \cdot [g_{ij} \cdot \cos(\theta_i - \theta_j) + b_{ij} \cdot \sin(\theta_i - \theta_j)]$$

$$q_{ij} = -b_{ij} \cdot |v_i|^2 - |v_i| \cdot |v_j| \cdot [g_{ij} \cdot \sin(\theta_i - \theta_j) - b_{ij} \cdot \cos(\theta_i - \theta_j)]$$

- DC OPF:

s.t. $\{P^{ub/lb}, P^{sum}, p_{ij} = -b_{ij} \cdot (\theta_i - \theta_j), -s_{ij,M} \le p_{ij} \le s_{ij,M}\}$ ***(DC set)***

- Decoupled OPF:

s.t. $\{DC\ set, Q^{ub/lb}, Q^{sum}, q_{ij} = -b_{ij} \cdot (|v_i| - |v_j|), F^{ub}\}$

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Recent linearized OPF set-ups

With focus on resistive-aware methods

BCs

- Bolognani & Zampieri 2016

s.t. $\{P^{ub/lb}, P^{sum}, Q^{ub/lb}, Q^{sum}, F^{ub},$

$$\left.\begin{aligned} |v| &= diag(|v_0|) + \frac{1}{|v_0|} \cdot Re(Z \cdot s^*) \\ \theta &= diag(\theta_0) + \frac{1}{|v_0|^2} \cdot Im(Z \cdot s^*) \end{aligned}\right\}$$

- Gan & Low 2014

s.t. $\{BCs, |v_i|^2 = |v_j|^2 - (s_{ij} \cdot z_{ij}* + z_{ij} \cdot s_{ij}*)\}$

- Dhople et al. 2015

s.t. $\{BCs, \begin{matrix} |v| - 1 = R \cdot p \\ \theta = -R \cdot q \end{matrix}\}$

- Guggilam et al. 2016

s.t. $\{BCs, \begin{matrix} |v| - 1 = R \cdot p \\ \theta = X \cdot p \end{matrix}\}$

Two new linearized OPF proposals

- Straight-forward
- Simple
- Resistive-aware
- Building on the DC and Decoupled OPF understandings

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Linearized for angles OPF (LA OPF)

$$\left.\begin{aligned} p_{ij} &= g_{ij} \cdot \underbrace{|v_i|}_{\approx 1}^2 - \underbrace{|v_i|}_{\approx 1} \cdot \underbrace{|v_j|}_{\approx 1} \cdot \left[g_{ij} \cdot \underbrace{\cos(\theta_i - \theta_j)}_{\approx 1} + b_{ij} \cdot \underbrace{\sin}_{\approx}(\theta_i - \theta_j) \right] \\ q_{ij} &= -b_{ij} \cdot \underbrace{|v_i|}_{\approx 1}^2 - \underbrace{|v_i|}_{\approx 1} \cdot |v_j| \cdot \left[g_{ij} \cdot \underbrace{\sin}_{\approx}(\theta_i - \theta_j) - b_{ij} \cdot \underbrace{\cos(\theta_i - \theta_j)}_{\approx 1} \right] \end{aligned}\right\}$$

- LA OPF:

$$\text{s.t. } \{P^{ub/lb}, P^{sum}, Q^{ub/lb}, Q^{sum}, F^{ub}, p_{ij} - q_{ij} = \left(g_{ij} - b_{ij}\right) \cdot (\theta_i - \theta_j)\}$$

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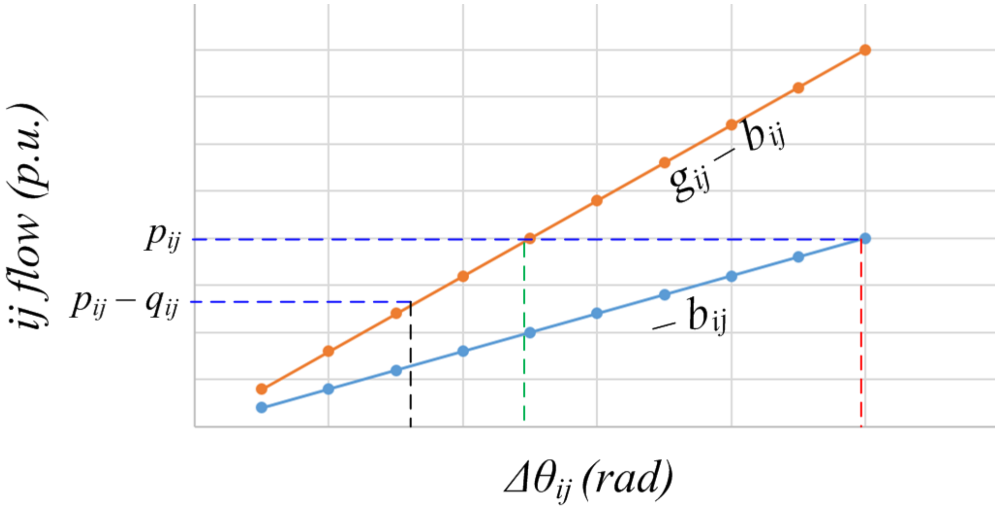
The value of LA OPF

- Comparing DC to LA OPF



- DC constraint: $p_{ij} = -b_{ij} \cdot (\theta_i - \theta_j)$
- LA constraint: $p_{ij} - q_{ij} = (g_{ij} - b_{ij}) \cdot (\theta_i - \theta_j)$

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Coupled OPF (COPF)

$$\left.\begin{aligned} p_{ij} &= g_{ij}\cdot|v_i|^2 - |v_i|\cdot|v_j|\cdot\left[g_{ij}\cdot\cos(\theta_i-\theta_j)+b_{ij}\cdot\sin(\theta_i-\theta_j)\right] \\ q_{ij} &= -b_{ij}\cdot|v_i|^2 - |v_i|\cdot|v_j|\cdot\left[g_{ij}\cdot\sin(\theta_i-\theta_j)-b_{ij}\cdot\cos(\theta_i-\theta_j)\right] \end{aligned}\right\}$$

- COPF:

$$\text{s.t. } \{LA\ OPF,\ q_{ij} = \frac{p_{ij}\cdot g_{ij}}{b_{ij}} + \frac{|v_i|}{b_{ij}}\left(-g_{ij}^2 - b_{ij}^2\right) + \frac{|v_j|}{b_{ij}}\cdot\left(g_{ij}^2 + b_{ij}^2\right)\ \}$$

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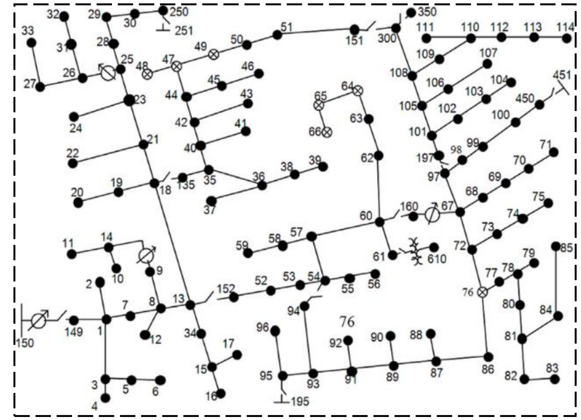
Testing on IEEE-123 (+48 DG units, no Q_g)

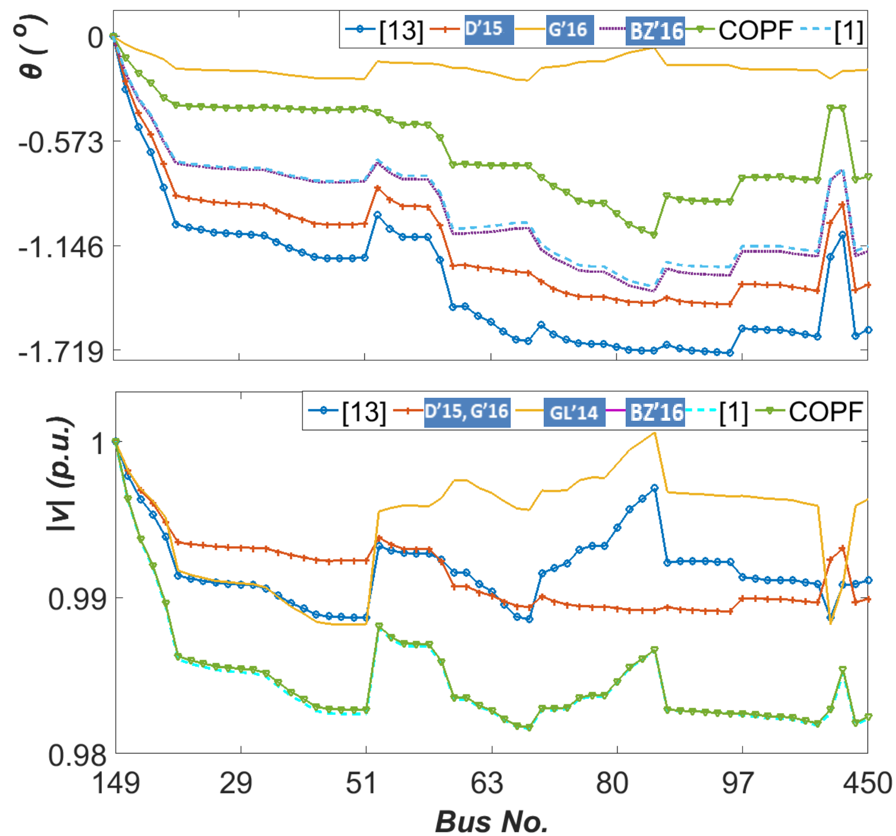


	d/c = [13]	BZ'16	GL'14	D'15, G'16	COPF	SDP = [1]
Dispatch Cost ($)	6.047	6.047	6.047	6.047	6.047	6.172

P_i (kW)	d/c	BZ'16	GL'14	D'15, G'16	COPF	SDP
Slack	2740	2740	2740	2740	2740	2695
DG@44	10	10	10	10	10	10
DG@66	40	40	40	40	40	40
DG@93	0	0	0	0	0	40
DG@151	30	30	30	30	30	30

θ (⁰)	d/c	BZ'16	D'15	G'16	COPF	SDP
Slack	0	0	0	0	0	0
Bus 49	-1.215	-0.802	-0.229	-1.031	-0.401	-0.791
Bus 79	-1.684	-1.289	-0.138	-1.427	-0.911	-1.261
Bus 300	-1.644	-1.203	-0.189	-1.392	-0.785	-1.175

|v| (pu)	d/c	BZ'16	GL'14	D'15, G'16	COPF	SDP
Slack	1	1	1	1	1	1
Bus 49	0.989	0.983	0.988	0.992	0.983	0.983
Bus 79	0.993	0.984	0.998	0.989	0.984	0.984
Bus 300	0.991	0.982	0.996	0.990	0.982	0.982



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Testing on IEEE-123 not enough…

- One arbitrary loading
- Assuming DG raises questions about injections
- Not clear view of effect of resistive/reactive nature of grid

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Statistical, multi-scenario testing

- One 10-bus radial system (exactness guarantees)
- Three system profiles $r_{ij}>x_{ij}$, $r_{ij}<x_{ij}$, arbitrary
- Three injection profiles
  - i. Unrestricted $P_{g,i}$ & $Q_{g,i}$ (emulating transmission)
  - ii. Unrestricted $P_{g,i}$ & restricted $Q_{g,i}$ (current code)
  - iii. 50% DG penetration & restricted $Q_{g,i}$ (limited DG)
- Introducing feasibility gap…

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Feasibility Gap

- Linearized formulations to initialize ‘closer’ to optimal for non-linear, SDP, SOCP, etc methods
- Geometric distance of a solution of a linearized OPF set-up (~) from the AC OPF feasible region

$$\varepsilon_f = \sqrt{\sum_{i \in N} \left(\varepsilon_{Pi}^2 + \varepsilon_{Qi}^2 \right)} \quad \left\{ \begin{aligned} \varepsilon_{Pi} &= \tilde{p}_{g,i} - \tilde{p}_{l,i} - |\tilde{v}_i|^2 \sum_{j \in NBi} g_{ij} + \\ &\quad + |\tilde{v}_i| \sum_{j \in NBi} |\tilde{v}_j| \cdot [g_{ij} \cdot cos(\tilde{\theta}_i - \tilde{\theta}_j) + b_{ij} \cdot sin(\tilde{\theta}_i - \tilde{\theta}_j)] \\ \varepsilon_{Qi} &= \tilde{q}_{g,i} - \tilde{q}_{l,i} + |\tilde{v}_i|^2 \sum_{j \in NBi} b_{ij} + \\ &\quad + |\tilde{v}_i| \sum_{j \in NBi} |\tilde{v}_j| \cdot [g_{ij} \cdot sin(\tilde{\theta}_i - \tilde{\theta}_j) - b_{ij} \cdot sin(\tilde{\theta}_i - \tilde{\theta}_j)] \end{aligned} \right.$$

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Voltage Angle Results

Average and Standard Deviation of Error in Voltage Angles (in Degrees) Between Approximate and SDP Relaxed OPF

Line Type	Scenario	DC	BZ'16	D'15	G'16	LADC
$r_{ij} < x_{ij}$	i	0.96±0.80	0.11±0.16	0.60±0.36	0.42±0.40	0.33±0.30
	ii	1.48±0.95	0.65±0.80	0.46±0.37	0.95±0.59	1.07±0.82
	iii	1.91±0.68	0.12±0.14	0.44±0.28	1.10±0.44	0.57±0.20
$r_{ij} > x_{ij}$	i	2.67±2.15	0.08±0.13	0.60±0.53	0.63±0.60	0.39±0.28
	ii	2.71±1.88	1.75±1.25	0.73±0.54	0.95±0.59	1.45±1.13
	iii	3.85±1.49	0.87±0.64	1.14±0.36	0.78±0.56	0.57±0.38
Varied	i	1.47±1.19	0.10±0.15	0.39±0.31	0.49±0.46	0.17±0.16
	ii	2.56±1.82	1.45±1.21	0.74±0.53	1.00±0.59	1.32±1.11
	iii	3.54±1.37	0.38±0.42	1.12±0.37	1.15±0.51	0.38±0.27

Maximum Point Error in Voltage Angles (in Degrees) Between Approximate and SDP Relaxed OPF

Line Type	Scenario	DC	BZ'16	D'15	G'16	LADC
$r_{ij} < x_{ij}$	i	6.83	2.02	3.00	4.11	2.20
	ii	8.39	5.95	2.37	5.93	6.91
	iii	7.18	1.28	1.58	4.89	1.40
$r_{ij} > x_{ij}$	i	14.52	2.42	3.88	3.67	1.88
	ii	10.75	9.10	2.37	4.68	8.48
	iii	10.28	3.60	2.37	2.47	2.48
Varied	i	8.74	2.19	2.42	4.15	1.69
	ii	10.73	8.37	2.64	4.31	8.11
	iii	9.35	2.72	2.60	3.70	2.00

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Voltage Magnitude Results

Average and Standard Deviation of Error in Voltage Magnitude (in %) Between Approximate and SDP Relaxed OPF

Line Type	Scenario	d/c	BZ'16	GL'14	D'15 & G'16	COPF
$r_{ij} < x_{ij}$	i	1.09±0.76	***0.22±0.34***	0.19±0.35	1.10±1.13	***0.22±0.34***
	ii	0.95±0.67	***0.24±0.18***	0.17±0.17	2.99±0.88	***0.24±0.18***
	iii	1.24±0.65	***0.26±0.18***	0.16±0.11	3.03±0.85	***0.26±0.18***
$r_{ij} > x_{ij}$	i	0.87±0.58	***0.25±0.38***	0.22±0.41	0.73±0.72	***0.25±0.38***
	ii	2.06±1.48	***0.27±0.26***	0.22±0.29	1.86±0.63	***0.27±0.26***
	iii	0.89±0.63	***0.26±0.18***	0.19±0.16	1.94±0.58	***0.26±0.18***
Varied	i	1.01±0.63	***0.23±0.35***	0.19±0.35	0.98±1.01	***0.23±0.35***
	ii	1.76±1.30	***0.26±0.24***	0.21±0.27	2.08±0.65	***0.26±0.24***
	iii	0.67±0.41	***0.25±0.17***	0.18±0.15	2.26±0.62	***0.25±0.17***

Maximum Point Error in Voltage Magnitude (in %) Between Approximate and SDP Relaxed OPF

Line Type	Scenario	d/c	BZ'16	GL'14	D'15 & G'16	COPF
$r_{ij} < x_{ij}$	i	4.69	4.90	5.30	5.42	4.90
	ii	3.76	1.97	2.41	8.48	1.97
	iii	3.76	1.02	1.30	6.87	1.02
$r_{ij} > x_{ij}$	i	5.01	5.39	5.85	3.85	5.39
	ii	11.95	3.39	3.86	5.99	3.39
	iii	9.01	1.70	2.16	4.92	1.70
Varied	i	4.33	4.94	5.31	5.04	4.94
	ii	9.65	2.95	3.70	6.38	2.95
	iii	9.41	1.52	2.21	5.49	1.52

Feasibility Gap Results

Average and Standard Deviation of Feasibility Gap Between Approximate and SDP Relaxed OPF

Line Type	Scenario	d/c	BZ’16	GL’14
$r_{ij} < x_{ij}$	i	0.31±0.31	0.07±0.10	0.29±0.29
	ii	0.51±0.41	0.16±0.19	0.27±0.24
	iii	0.68±0.35	0.07±0.06	0.26±0.26
		D’15	G’16	COPF
	i	0.29±0.29	0.17±0.17	0.14±0.15
	ii	0.27±0.24	0.37±0.29	0.22±0.18
	iii	0.27±0.26	0.31±0.19	0.21±0.16

Line Type	Scenario	d/c	BZ’16	GL’14
$r_{ij} > x_{ij}$	i	7.58±5.17	***2.13±3.19***	1.87±3.47
	ii	16.82±12.53	***2.18±2.20***	1.82±2.47
	iii	1.76±1.46	0.25±0.20	0.14±0.11
		D’15	G’16	COPF
	i	6.00±5.91	5.97±5.89	***2.13±3.19***
	ii	15.80±5.27	15.70±5.26	2.20±2.20
	iii	0.15±0.11	0.20±0.14	***0.16±0.14***

Line Type	Scenario	d/c	BZ’16	GL’14
Varied	i	7.51±4.61	***1.59±2.40***	1.38±2.55
	ii	5.65±4.29	0.85±0.73	0.93±1.22
	iii	4.18±2.79	1.68±1.15	43.20±20.94
		D’15	G’16	COPF
	i	7.18±7.28	7.13±7.24	***1.60±2.40***
	ii	6.39±2.01	6.32±2.01	0.82±0.73
	iii	15.36±4.05	15.46±4.07	1.64±1.13

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Discussion (1/2)

- No **one** formulation fits all
- The better angle calculation of BZ'16 is not guaranteed by its minimum maximum error proof (unlike D'15)
- Same for magnitudes with GL'14 (maximum errors up to 5%) – but only for distribution
- COPF & BZ'16 (almost guaranteed) voltage magnitude performance

Discussion (2/2)

- GL'14 with small feasibility gaps besides no angle
- Reduced feasibility gaps with COPF & BZ'16
- Combining formulations can tackle concerns, e.g.

$$\text{s.t. } \{BCs, \left|v_i\right|^2 = \left|v_j\right|^2 - \left(s_{ij} \cdot z_{ij}{*} + z_{ij} \cdot s_{ij}{*}\right), \ \theta = diag(\theta_0) + \frac{1}{|v_0|^2} \cdot Im(Z \cdot s^*)\}$$

- Solving the formulation of average best performance & best of the maximum errors one
- Loading characteristics show effect on performance

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Conclusion

- Employing linearized OPF still preferred
- Ample formulations available
- No single formulation as best one
- Grid & loading characteristics can serve best *choice*
- Linear formulations allow for multiple runs

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Thanks for your attention!

Questions, please?

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