Administrative issues

- Lecture evaluation:
  - Please participate
  - Your constructive feedback is important!

- Two presentations today

On the menu today

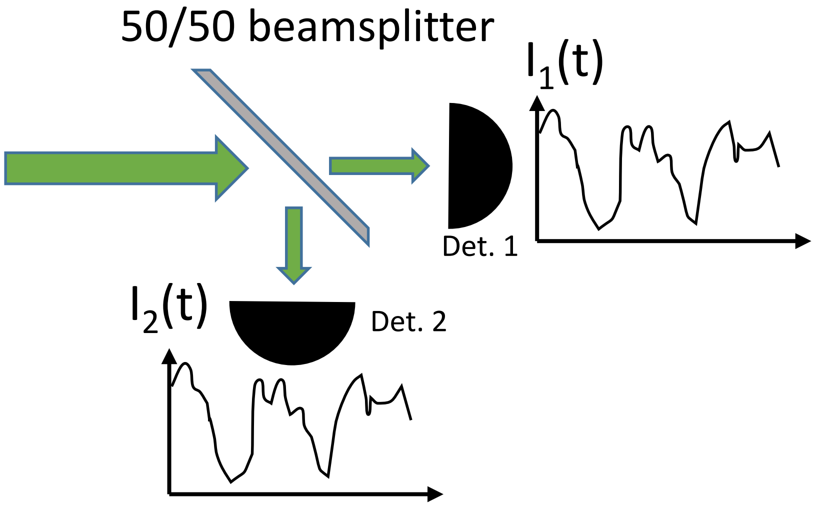
Photon-photon correlations

- The second-order correlation function
- Bunching and anti-bunching

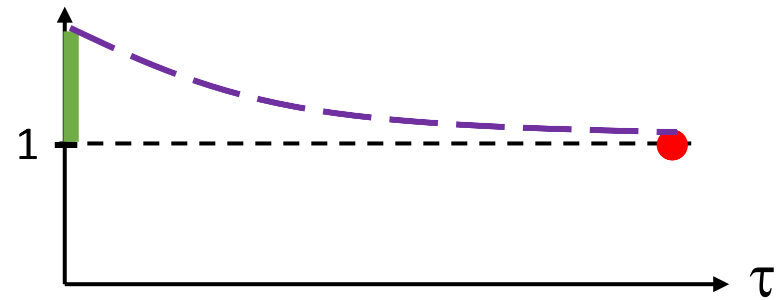
Optical forces

- Radiation pressure
- The Maxwell stress tensor
- Force on a dipolar scatterer
- Optical traps and optical tweezers

Intensity autocorrelation - the classical case

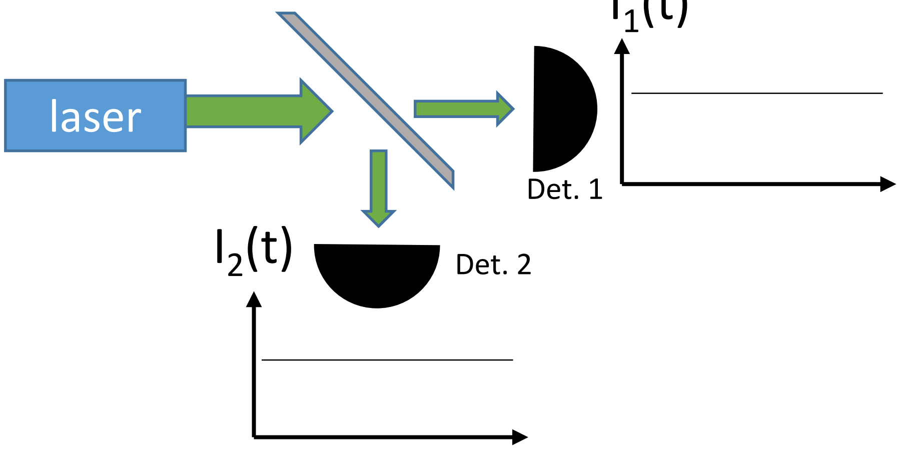


$$g^{(2)}(\tau) = \frac{\langle I(t) I(t+\tau) \rangle}{\langle I(t) \rangle^2}$$



- Beam of light impinging on a 50/50 beamsplitter (BS)
- Record intensity I(t) in each arm after BS
- For a classical field $I_1(t) = I_2(t)$, so $g^{(2)}$ is intensity autocorrelation
- For long delay times $g^{(2)}(\tau \to \infty) = 1$
- correlation at zero delay $g^{(2)}(\tau = 0) \geq 1$
- global maximum at zero delay $g^{(2)}(0) \geq g^{(2)}(\tau)$

Intensity autocorrelation - the coherent case



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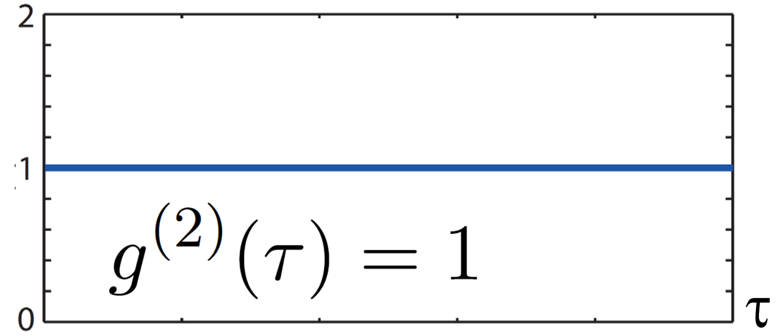
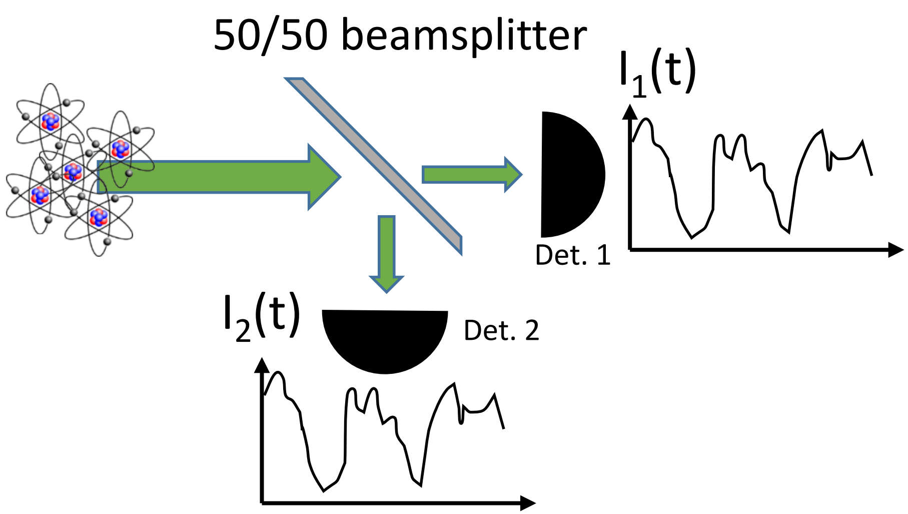


$$g^{(2)}(\tau) = 1$$

- Perfectly monochromatic field $E(t) \propto \cos(\omega t)$
- Intensity is therefore $I(t) = \text{const.}$

Intensity autocorrelation - the chaotic case



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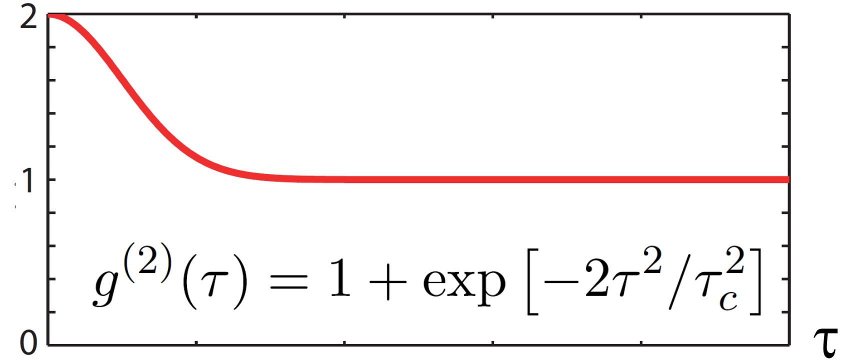


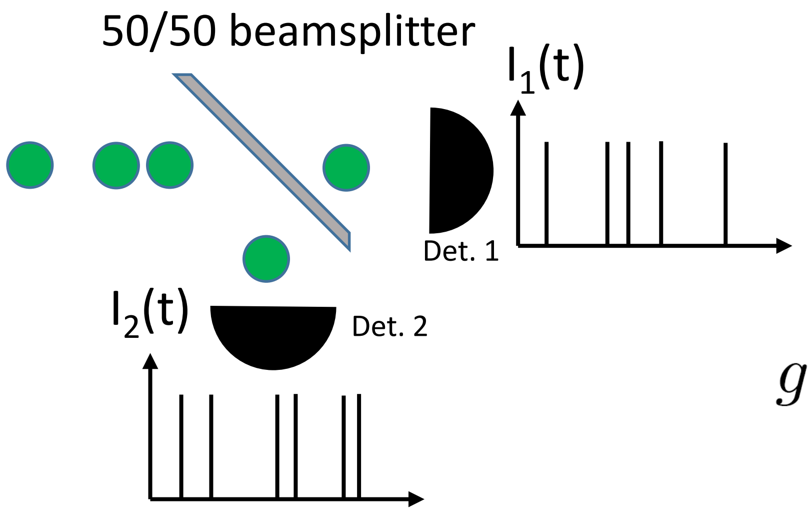
$$g^{(2)}(\tau) = 1 + \exp\left[-2\tau^2/\tau_c^2\right]$$

- Collection of sources
- Random phase ϕ_a
- Gaussian distribution of emission frequencies

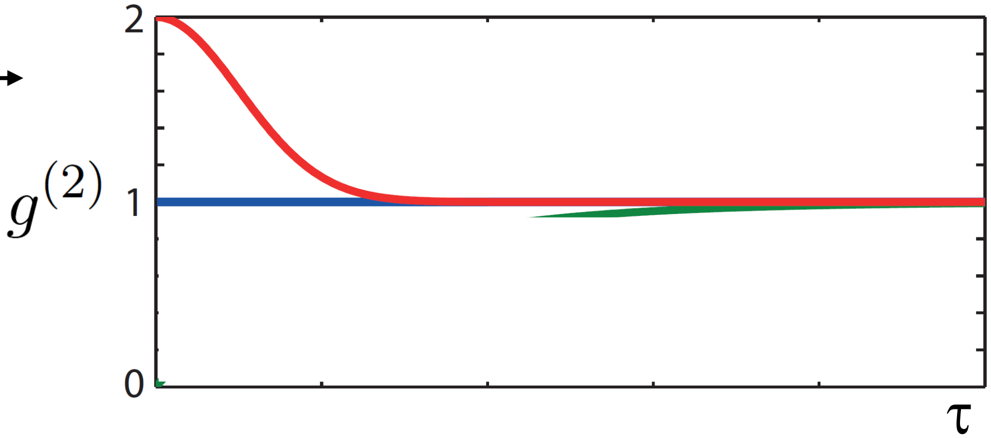
$$E(t) = E_0 \sum_{\text{atoms}} \exp\left[-\mathrm{i}\Omega_a t - \phi_a\right]$$

$$P(\Omega_a) \propto \exp\left[-(\Omega_0 - \Omega_a)^2 \tau_c^2\right]$$

Intensity correlation – summary

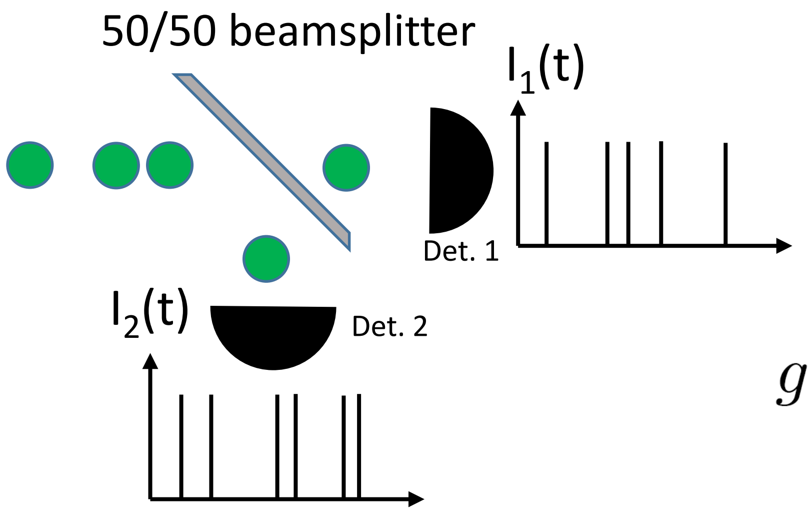


$$g^{(2)}(\tau) = \frac{\langle n_1(t) n_2(t+\tau) \rangle}{\langle n_1(t) \rangle \langle n_2(t) \rangle}$$



- Second-order correlation function measures temporal intensity correlation
- Bunching: photons tend to "arrive together", classically allowed/expected

Intensity correlation – summary



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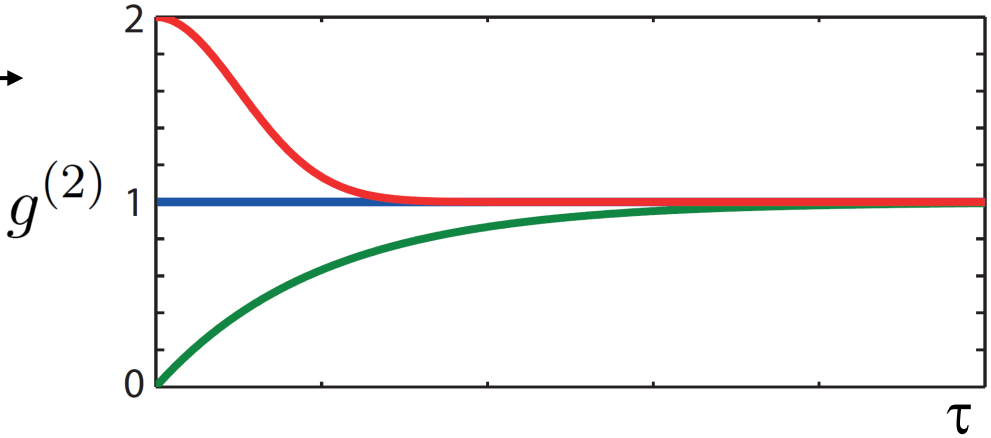
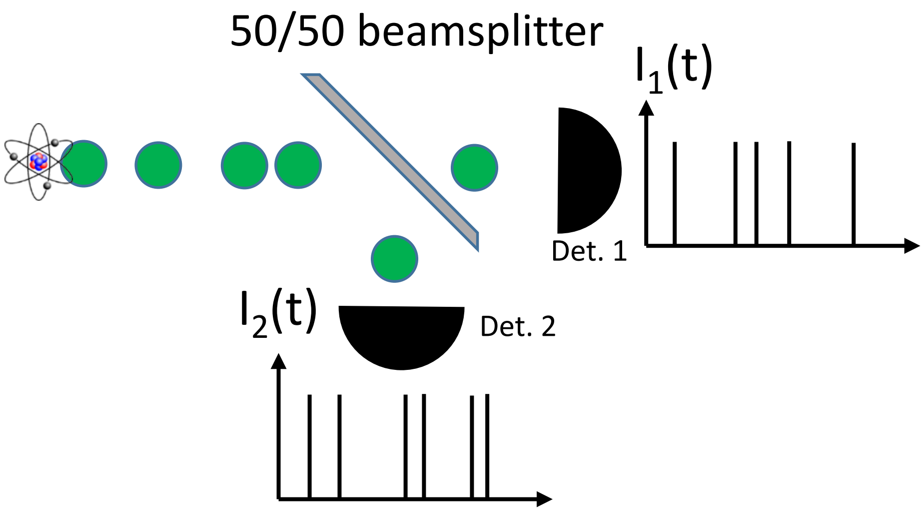


- Second-order correlation function measures temporal intensity correlation
- Bunching: photons tend to “arrive together”, classically allowed/expected
- Antibunching: photons tend to “arrive alone”, classically forbidden

Counting photons – a single quantum emitter



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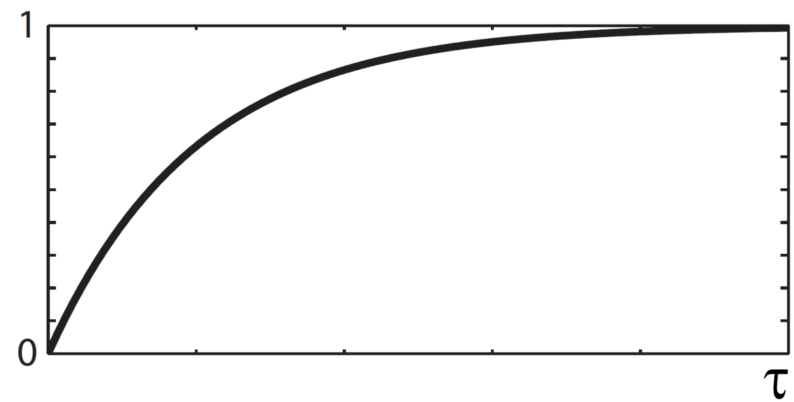
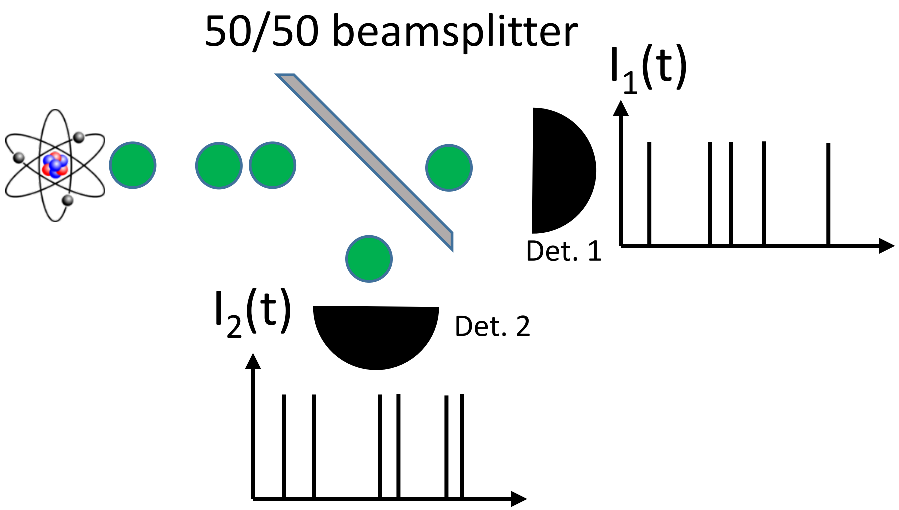
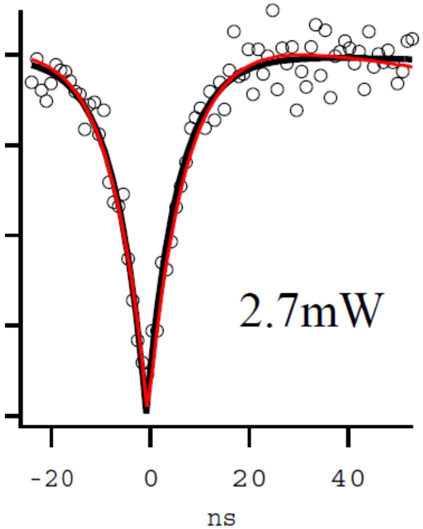


- Assume source is a single emitter
- Single emitter can only emit one photon at a time
- If there is a photon on D1 there cannot be a photon on D2 → antibunching
- Photon antibunching is at odds with classical electromagnetism
- $g^{(2)}(\tau=0) = 0$ is the signature of a single photon source

Intensity correlation – counting single photons



Beveratos, PhD thesis, Univ. Paris Sud (2002)



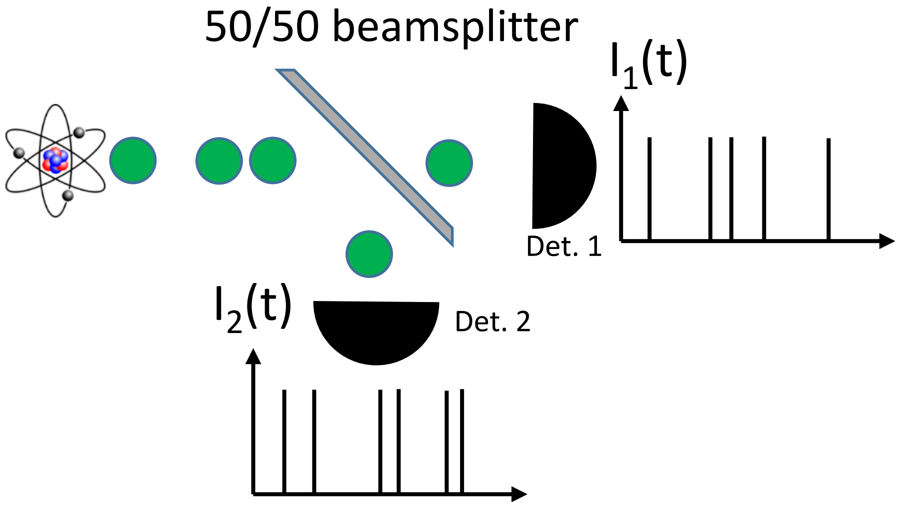
- How do you know your emitter is a single photon source?

For n emitters:

$$g^2(0) = 1 - \frac{1}{n}$$

- How does the lifetime show up in the correlation function?

Intensity correlation – counting single photons



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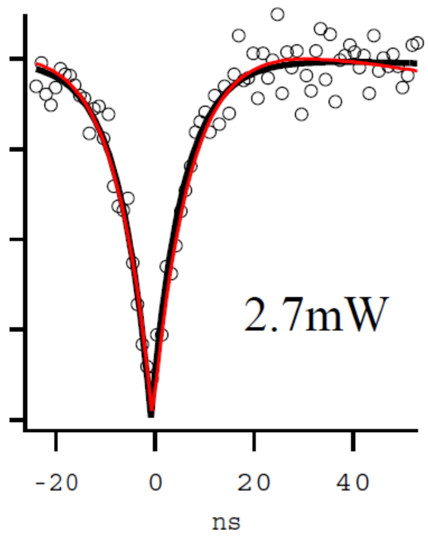


- How do you know your emitter is a single photon source?

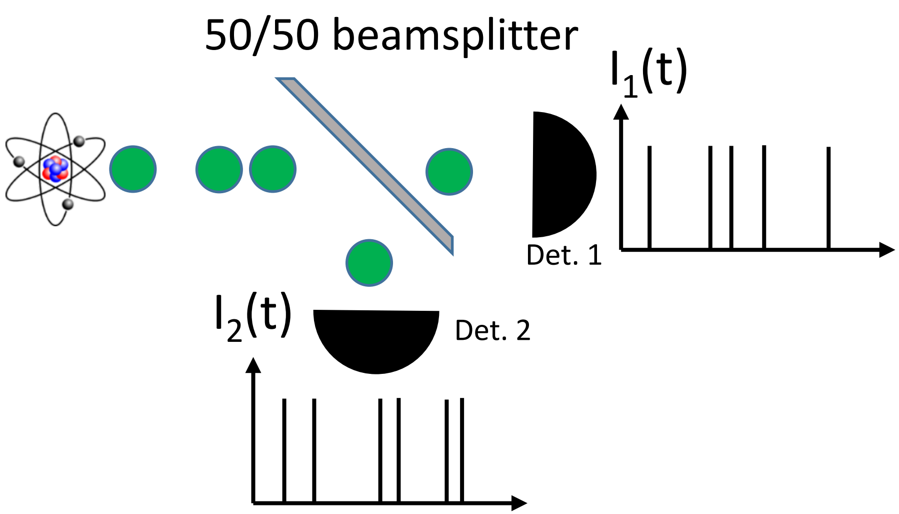
  For n emitters:

$$g^2(0) = 1 - \frac{1}{n}$$

- How does the lifetime show up in the correlation function?

  Rise time is lifetime in the case of weak pumping.

Single photon sources



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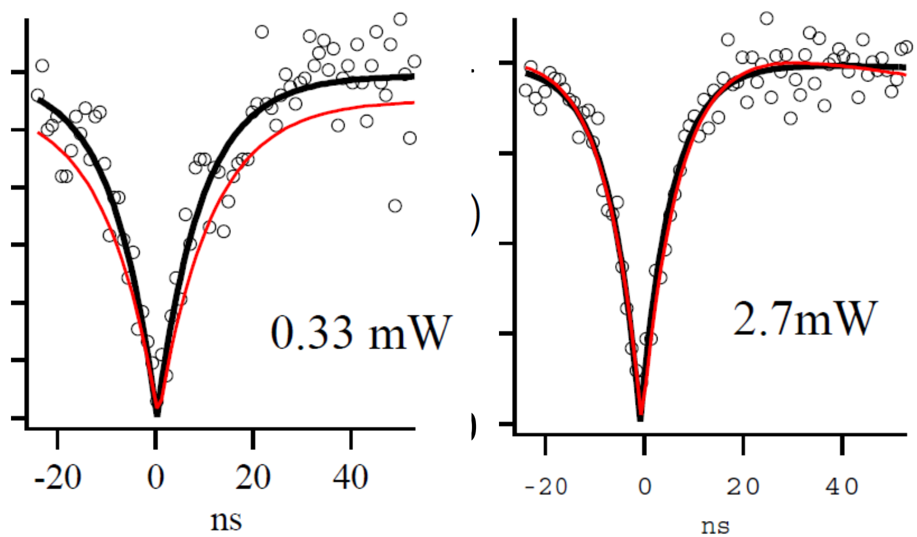

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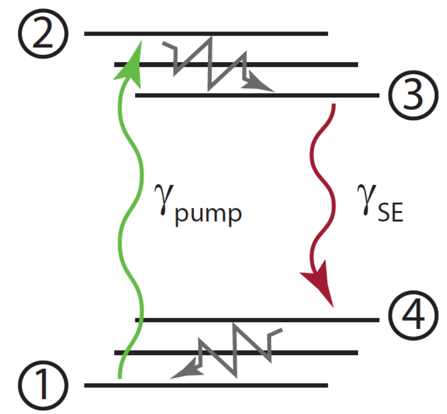
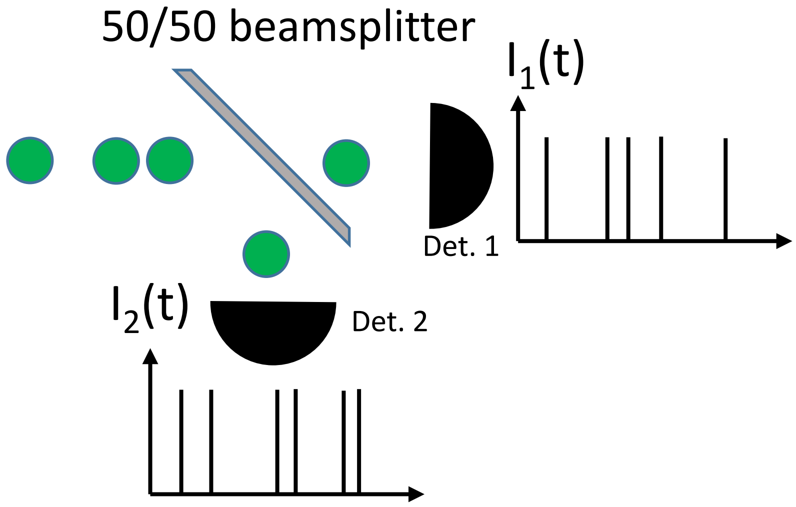
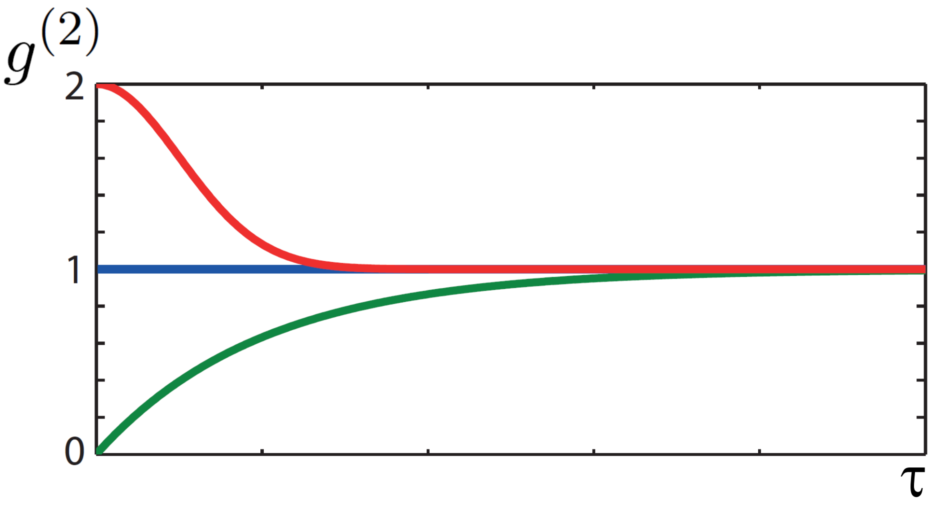


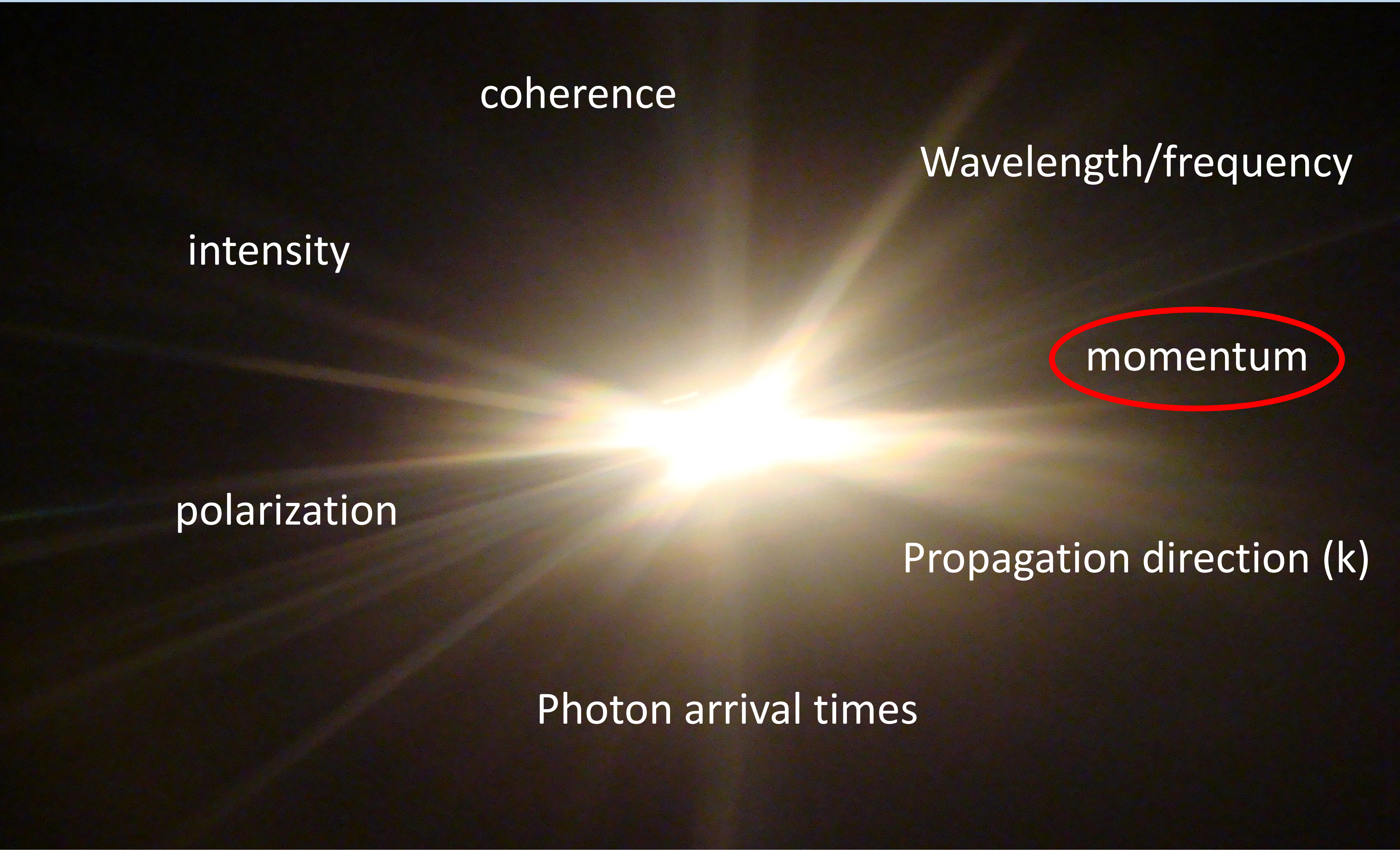
Intensity correlation – summary



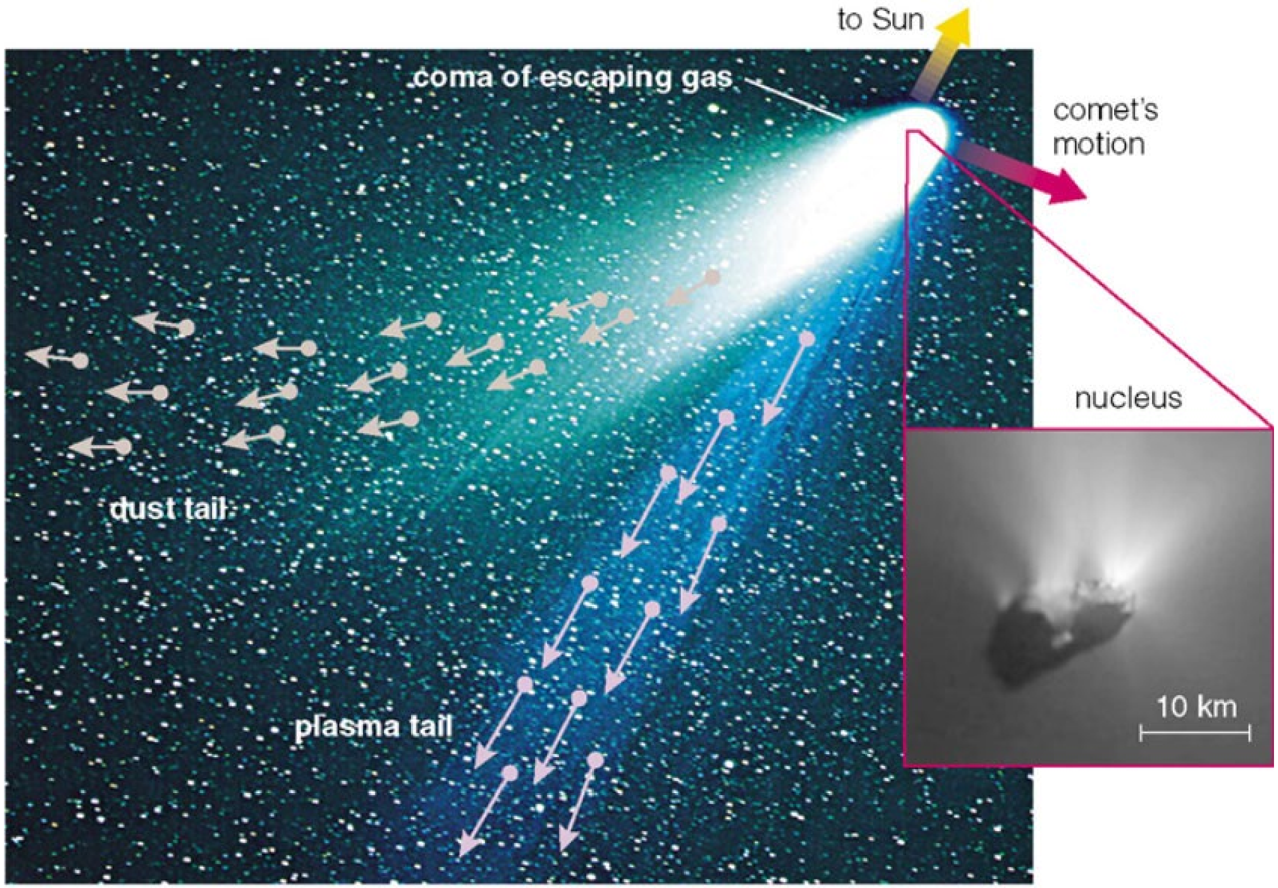


- Second-order correlation function measures temporal intensity correlation
- Bunching: photons tend to “arrive together”, classically allowed/expected
- Antibunching: photons tend to “arrive alone”, classically forbidden

Properties of “light”

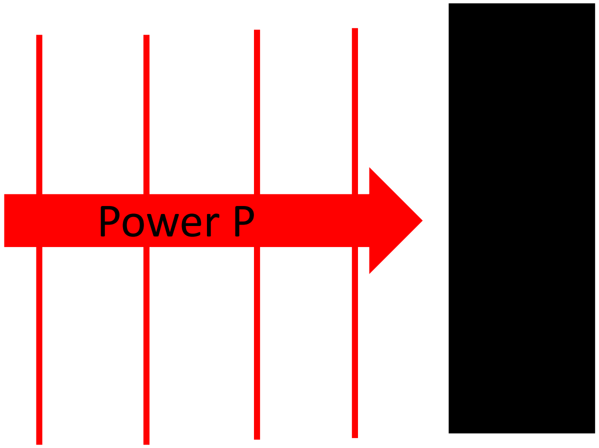


The forces exerted by light



Radiation pressure for plane wave

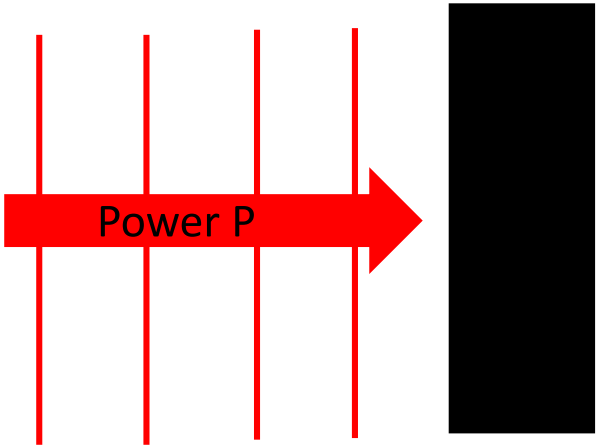
Radiation pressure force:



- Consider stream of photons hitting a black object (perfect absorber)

Radiation pressure for plane wave

Radiation pressure force: $$\mathbf{F} = \frac{P}{c}\mathbf{n}_k$$

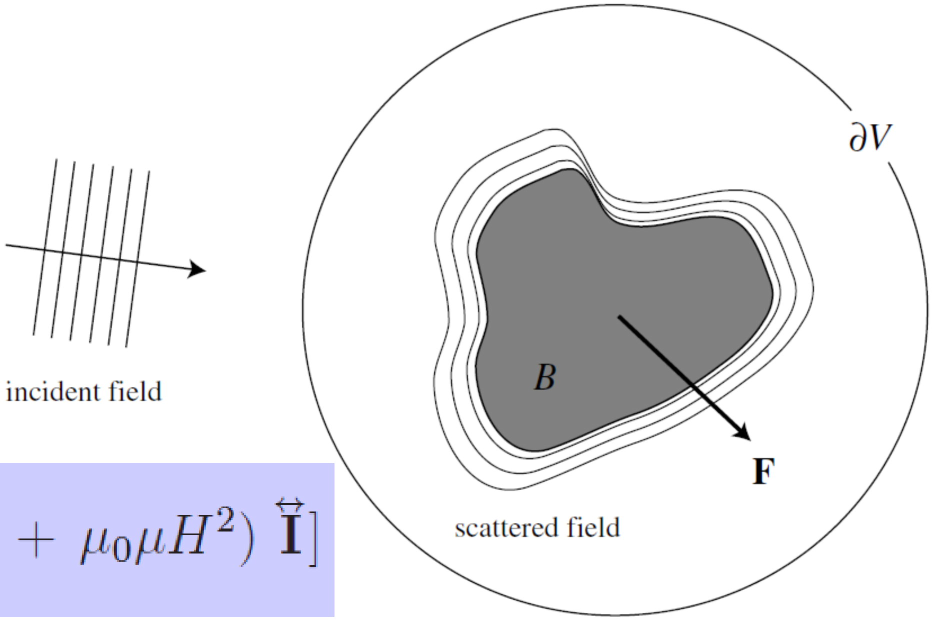


- Consider stream of photons hitting a black object (perfect absorber)
- Each photon carries momentum h/λ
- Radiation pressure is a purely classical effect (Planck's constant h drops out)

Forces of light – Maxwell stress tensor

Read E&M lecture notes for details

$$\langle \mathbf{F} \rangle = \int_{\partial V} \langle \overset{\leftrightarrow}{\mathbf{T}}(\mathbf{r},t) \rangle \cdot \mathbf{n}(\mathbf{r}) \, da$$



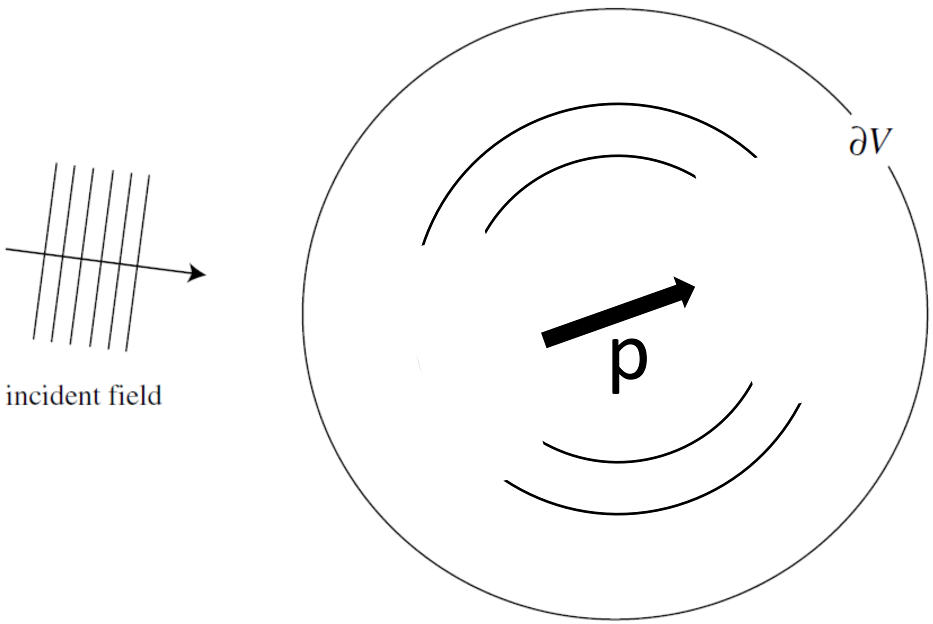
$$\overset{\leftrightarrow}{\mathbf{T}} = \left[\varepsilon_0 \varepsilon \mathbf{E}\mathbf{E} + \mu_0 \mu \mathbf{H}\mathbf{H} - \frac{1}{2} (\varepsilon_0 \varepsilon E^2 + \mu_0 \mu H^2) \overset{\leftrightarrow}{\mathbf{I}} \right]$$

- Calculate force acting on a volume by integrating MW stress tensor over enclosing surface
- Fields required are total (real) fields (incoming and scattered)
- Which force is at work here?

Force on a dipole in an electromagnetic field

- Dipole oscillating at frequency ω in field at frequency ω
- How can we get the force on the dipole?
  - Integrate Stress tensor

    or
  - Calculate Lorentz force

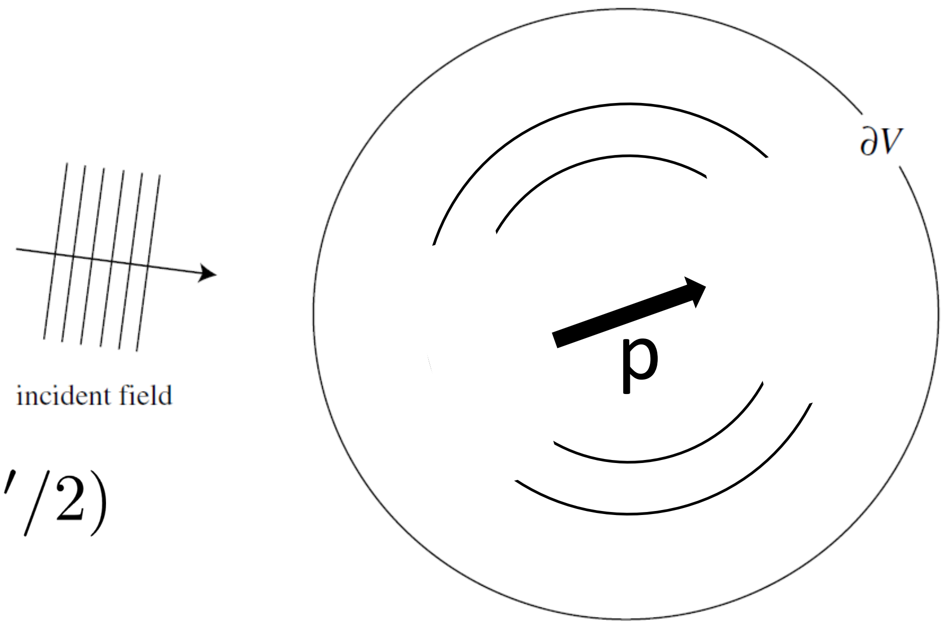


Force on a dipole in an electromagnetic field

- Dipole oscillating at frequency ω in field at frequency ω
- What is the force on the dipole?

$$\frac{\mathrm{d}\mathbf{F}}{\mathrm{d}V} = \rho\mathbf{E} + \mathbf{j} \times \mathbf{B}(\mathbf{r}, t)$$

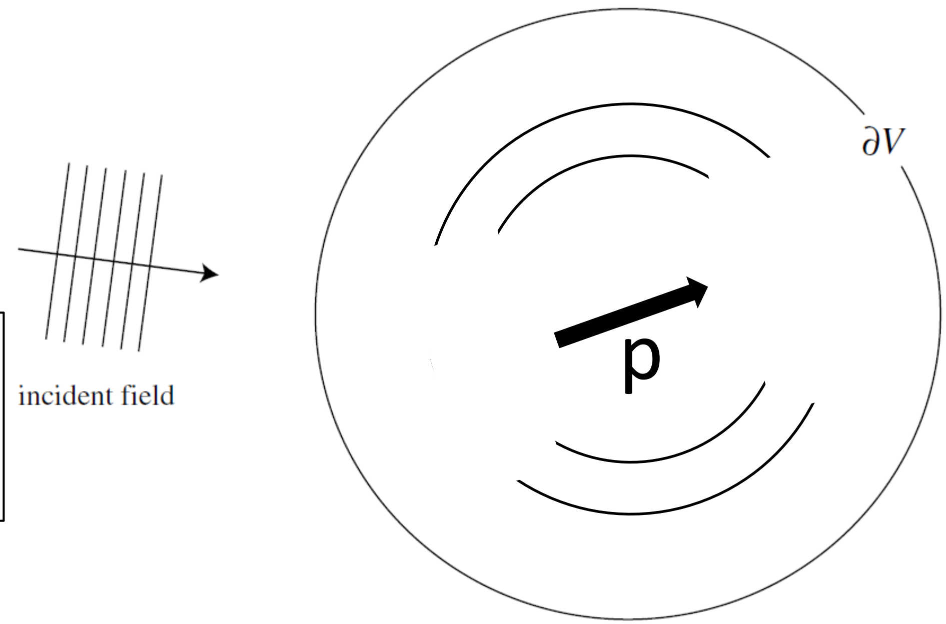
$$\rho_{\mathrm{dip}} = q[\delta(\mathbf{r} - \mathbf{r}'/2) - \delta(\mathbf{r} + \mathbf{r}'/2)$$



Force on a dipole in an electromagnetic field

- Dipole oscillating at frequency ω in field at frequency ω
- What is the force on the dipole?

$$\mathbf{F}(\mathbf{r}) = \sum_i \frac{1}{2}\mathrm{Re}\left\{p_i^*\,\nabla E_i(\mathbf{r})\right\}$$



Force on a dipole in an electromagnetic field

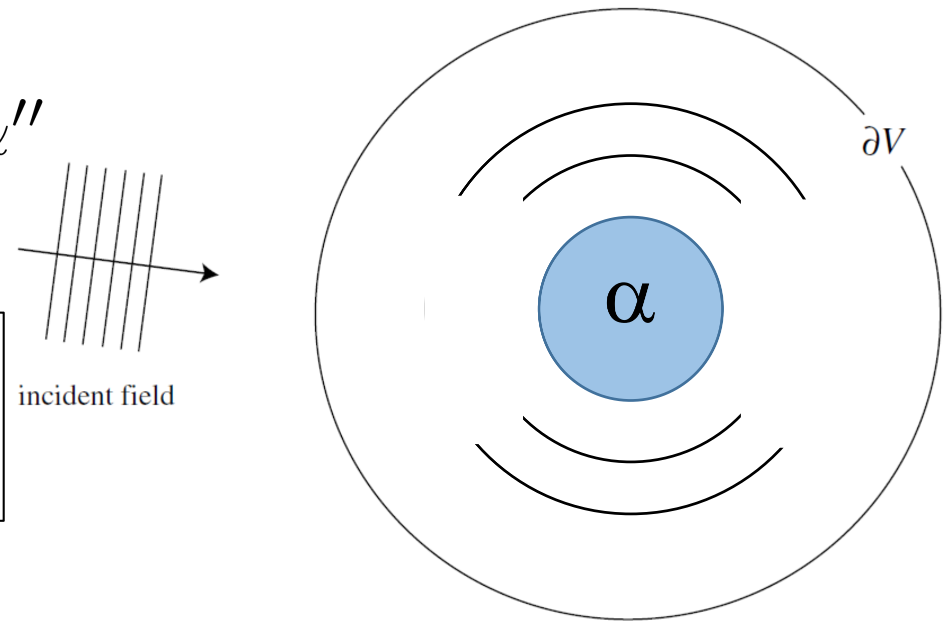
Force on dipolar scatterer

Induced dipole moment $$\mathbf{p} = \alpha \mathbf{E}$$

$$\alpha = \alpha' + \mathrm{i}\alpha''$$

Force on dipole in field:

$$\mathbf{F}(\mathbf{r}) = \sum_i \frac{1}{2} \mathrm{Re}\left\{ p_i^* \nabla E_i(\mathbf{r}) \right\}$$



Force on dipolar scatterer

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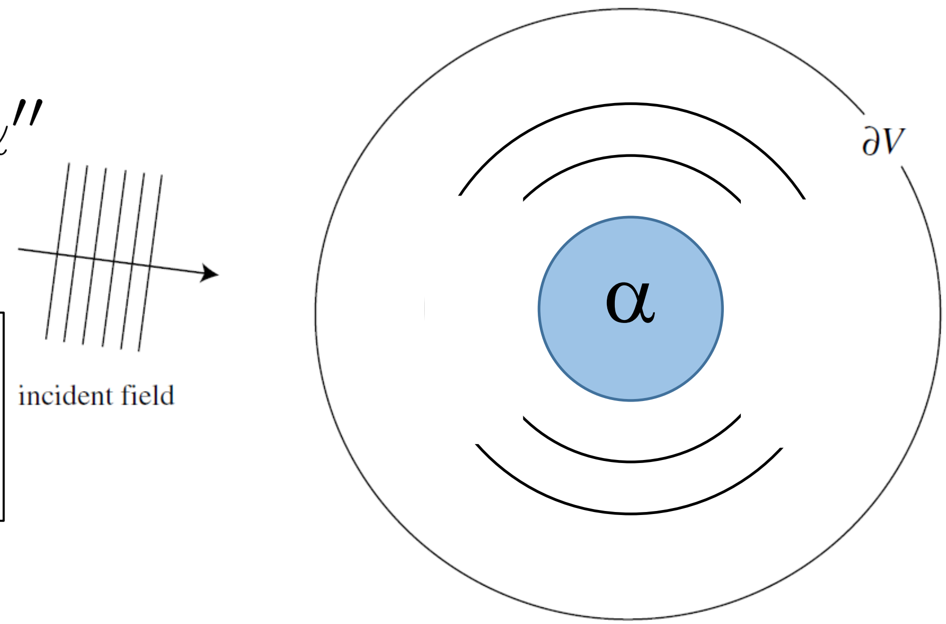

Force on dipole in field:

$$\mathbf{F}(\mathbf{r}) = \sum_i \frac{1}{2} \mathrm{Re}\left\{ p_i^* \, \nabla E_i(\mathbf{r}) \right\}$$

$$\mathbf{F}(\mathbf{r}) = \frac{\alpha'}{2} \sum_i \mathrm{Re}\left\{ E_i^*(\mathbf{r}) \, \nabla E_i(\mathbf{r}) \right\} + \frac{\alpha''}{2} \sum_i \mathrm{Im}\left\{ E_i^*(\mathbf{r}) \, \nabla E_i(\mathbf{r}) \right\}$$

Force on dipolar scatterer

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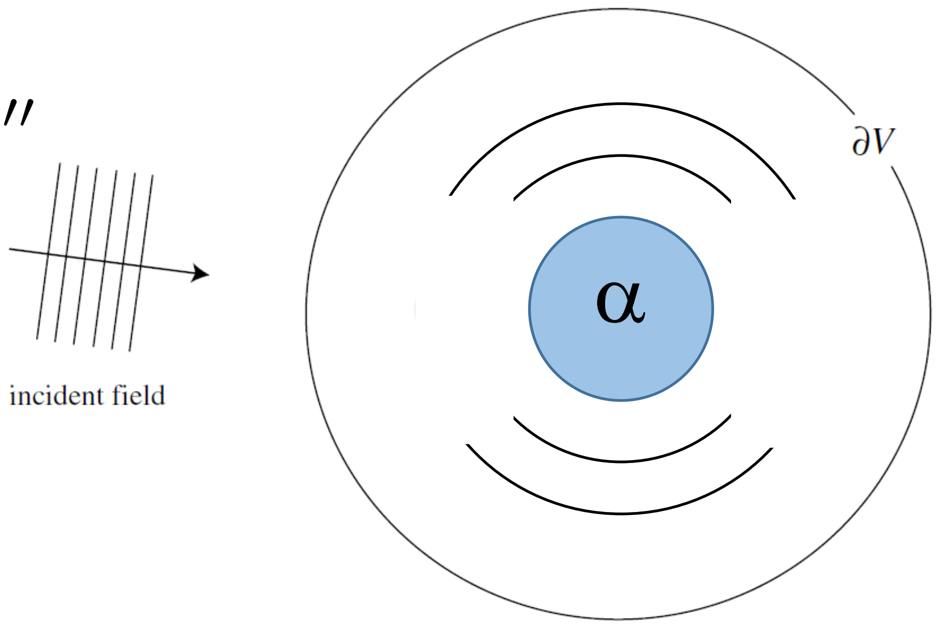


Force on dipole in field:

$$\mathbf{F}(\mathbf{r}) \; = \; \sum_i \frac{1}{2} \mathrm{Re}\left\{ p_i^* \, \nabla E_i(\mathbf{r}) \right\}$$

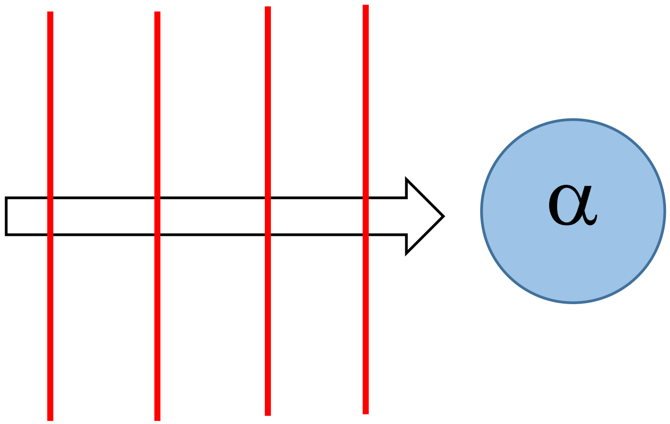
$$\mathbf{F}(\mathbf{r}) \; = \; \underbrace{\frac{\alpha'}{2} \sum_i \mathrm{Re}\left\{ E_i^*(\mathbf{r}) \, \nabla E_i(\mathbf{r}) \right\}}_{\text{Gradient force}} + \underbrace{\frac{\alpha''}{2} \sum_i \mathrm{Im}\left\{ E_i^*(\mathbf{r}) \, \nabla E_i(\mathbf{r}) \right\}}_{\text{Scattering force}}$$

$$\mathbf{F}_{\mathrm{grad}} = (\alpha'/4) \nabla (\mathbf{E}^* \cdot \mathbf{E})$$

Example: Scatterer illuminated by plane wave

$$\mathbf{F}(\mathbf{r}) = \frac{\alpha'}{4}\nabla(\mathbf{E}^*\mathbf{E}) + \frac{\alpha''}{2}\sum_i \mathrm{Im}\left\{E_i^*\nabla E_i\right\}$$

$$\mathbf{E}(\mathbf{r}) = E_0 \mathrm{e}^{\mathrm{i}k_z z}\mathbf{n}_x$$

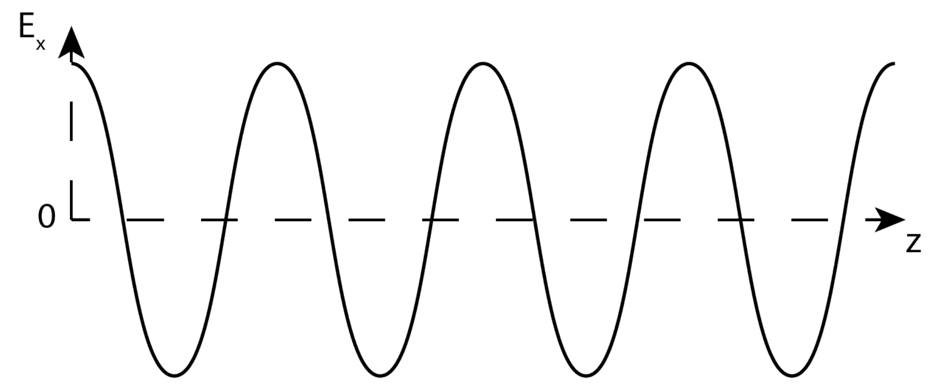


Example: Dipolar scatterer in standing wave

$$\mathbf{F}(\mathbf{r}) = \frac{\alpha'}{4}\nabla(\mathbf{E}^*\mathbf{E}) + \frac{\alpha''}{2}\sum_i \mathrm{Im}\left\{E_i^*\nabla E_i\right\}$$

Field of standing wave:

$$\mathbf{E} = E_0\cos(kz)\mathbf{n}_x$$



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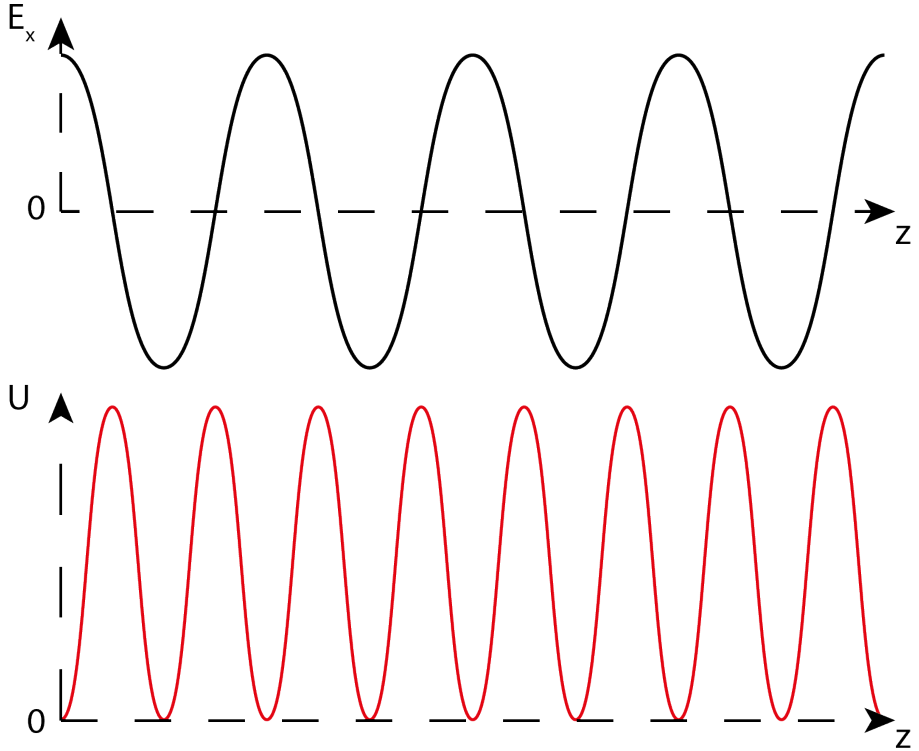
Field of standing wave:

$$\mathbf{E} = E_0\cos(kz)\mathbf{n}_x$$

$$\mathbf{F}_{\mathrm{grad}} = -\frac{\alpha' k}{4}|E_0|^2\sin(2kz)$$



- No scattering force
- Field intensity forms an “optical potential”

General force on dipolar scatterer

<u>Gradient force</u> proportional to

- Real part of polarizability $\mathrm{Re}[\alpha]$
- Intensity gradient

$$\mathbf{F}_{\mathrm{grad}} = (\alpha'/4)\nabla(\mathbf{E}^{*}\cdot\mathbf{E})$$

<u>Scattering force</u> proportional to

- Imaginary part of polarizability

$$\mathbf{F}_{\mathrm{scatt}} = \frac{\sigma_{\mathrm{ext}}}{c}\mathbf{S} + c\sigma_{\mathrm{ext}}[\nabla \times \mathbf{L}]$$

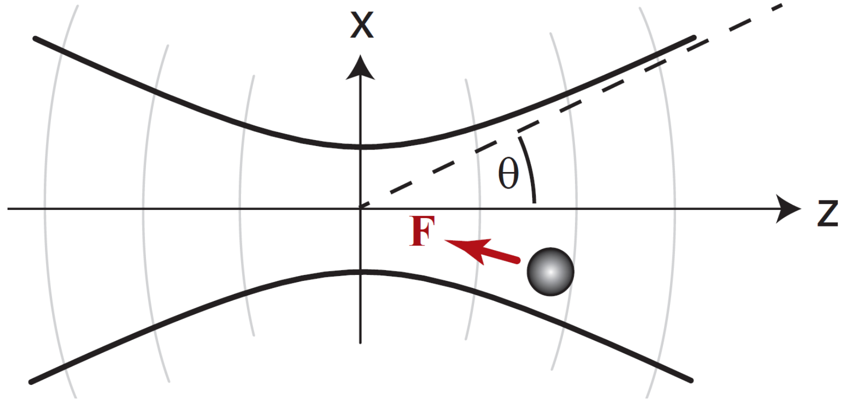
S = Poynting vector,
L = spin density of field (exists in certain circularly polarized fields)

$$\mathbf{F}(\mathbf{r}) = \underbrace{\frac{\alpha'}{2}\sum_i \mathrm{Re}\Big\{E_i^*(\mathbf{r})\,\nabla E_i(\mathbf{r})\Big\}}_{\text{Gradient force}} + \underbrace{\frac{\alpha''}{2}\sum_i \mathrm{Im}\Big\{E_i^*(\mathbf{r})\,\nabla E_i(\mathbf{r})\Big\}}_{\text{Scattering force}}$$

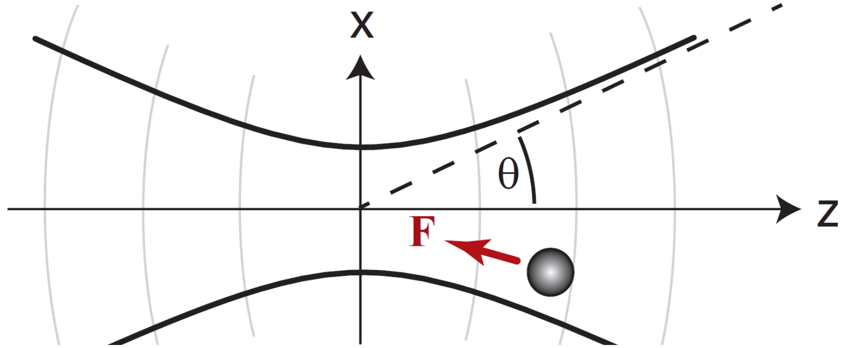
Gradient force: $\mathbf{F}_{\mathrm{grad}} = (\alpha'/4)\nabla(\mathbf{E}^{*}\cdot\mathbf{E})$

Scattering force

Dipolar scatterer in focused field



Dipolar scatterer in focused field



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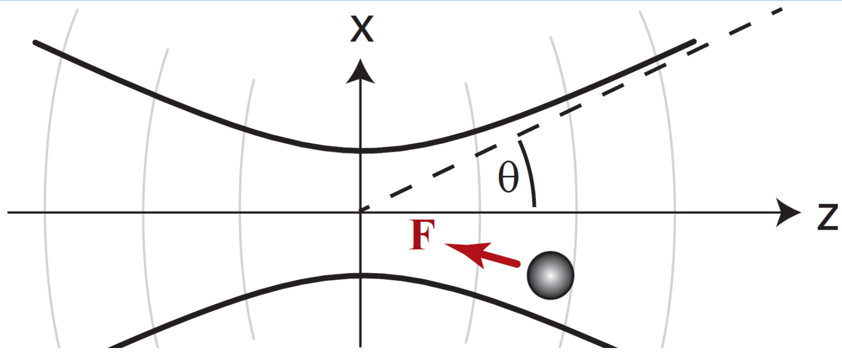
Gradient force: $\mathbf{F}_{\mathrm{grad}} = (\alpha'/4)\nabla(\mathbf{E}^*\cdot\,\mathbf{E})$

Scattering force

- Gradient force pulls scatterer to region of largest field intensity
- Scattering force pushes scatterer along propagation direction

Dipolar scatterer in focused field

$$\alpha = \alpha' + i\alpha''$$



For small particles:

$$\alpha'' = \frac{k^3}{6\pi\varepsilon_0}\alpha'^2$$

$$\mathbf{F}(\mathbf{r}) = \underbrace{\frac{\alpha'}{2}\sum_i \mathrm{Re}\left\{E_i^*(\mathbf{r})\,\nabla E_i(\mathbf{r})\right\}}_{\text{Gradient force}} + \underbrace{\frac{\alpha''}{2}\sum_i \mathrm{Im}\left\{E_i^*(\mathbf{r})\,\nabla E_i(\mathbf{r})\right\}}_{\text{Scattering force}}$$

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Scattering force

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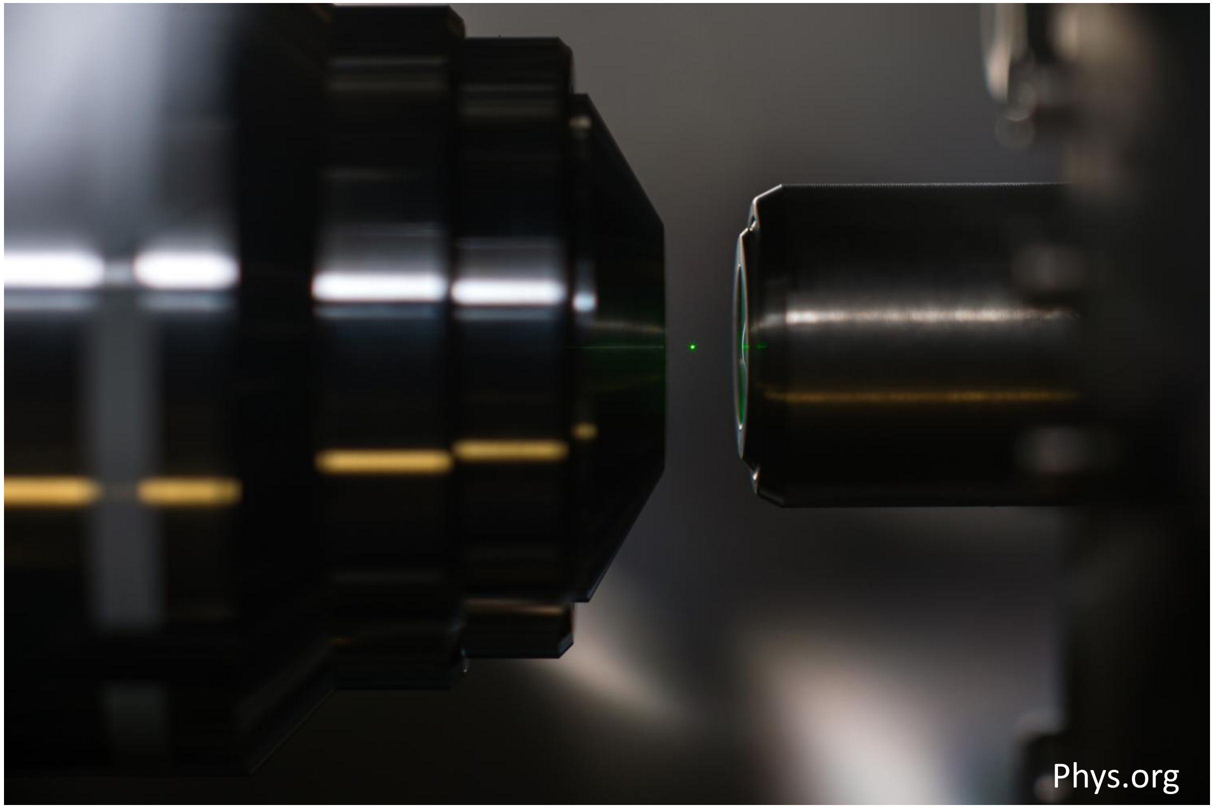
Example: Optical trapping



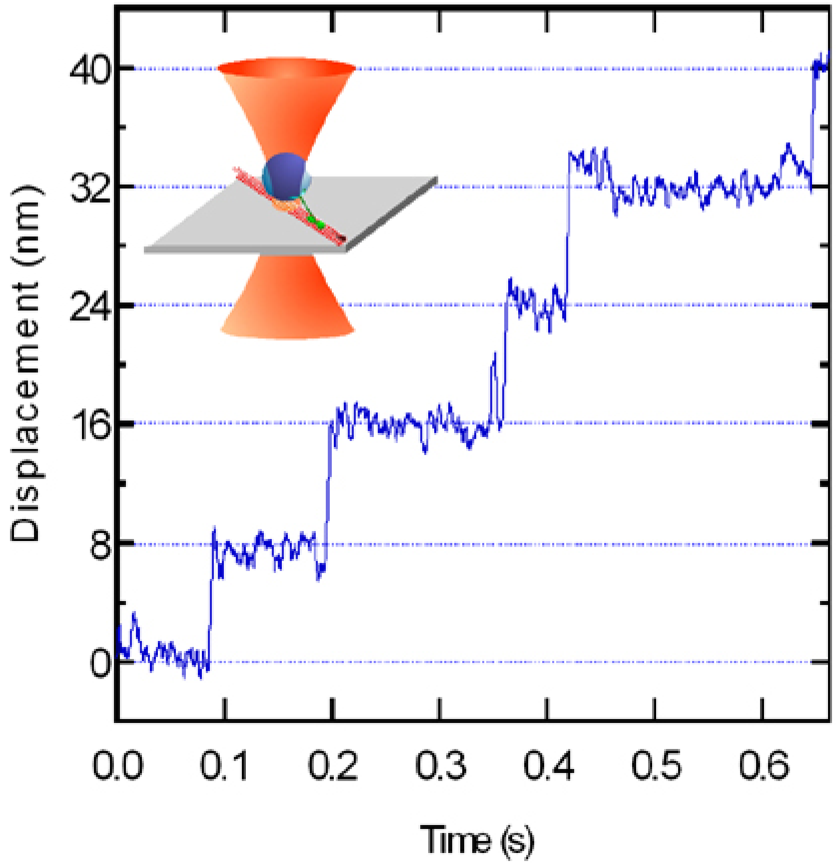
- Optical forces allow trapping and levitation of nano- and micro-particles in vacuum, gas and liquid

Nobel prize in Physics 2018

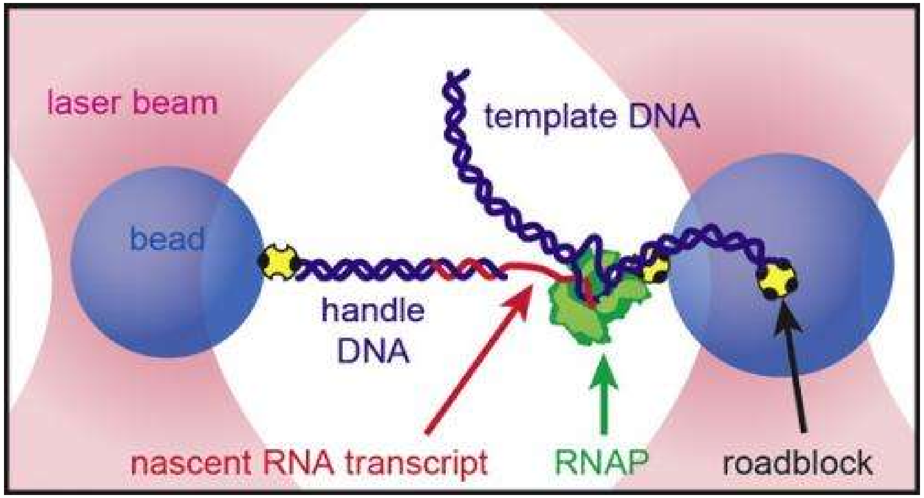
Arthur Ashkin

Prize motivation: "for the optical tweezers and their application to biological systems."

Applications of optical trapping in biology



Block lab, Stanford

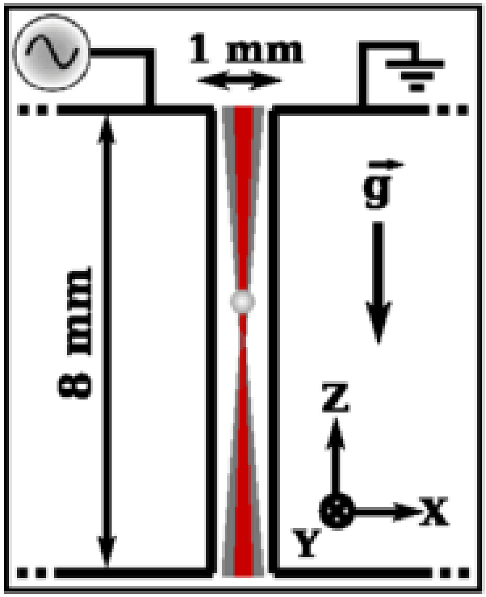


- Molecular motor taking steps against a pN force
- Absolute measurement of motor force and step size

"By keeping the tension between the two beads constant and measuring the distance between them as they moved apart, Block was able to gauge the changing length of the new RNA strand.
"What we got was a blow-by-blow readout of how RNA folds as it is processed by RNA polymerase," said Block."
(from phys.org)

Read more at: https://phys.org/news/2012-10-optical-tweezers-sub-nanoscale-precision-processand.html#jCp

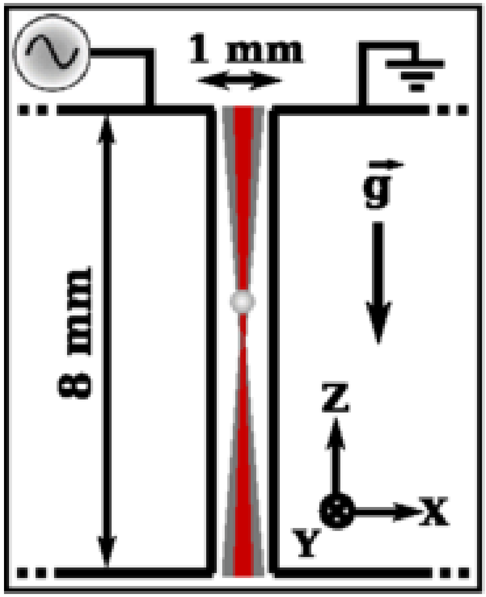
Applications of optical trapping in physics



Moore et al., PRL 113, 251801

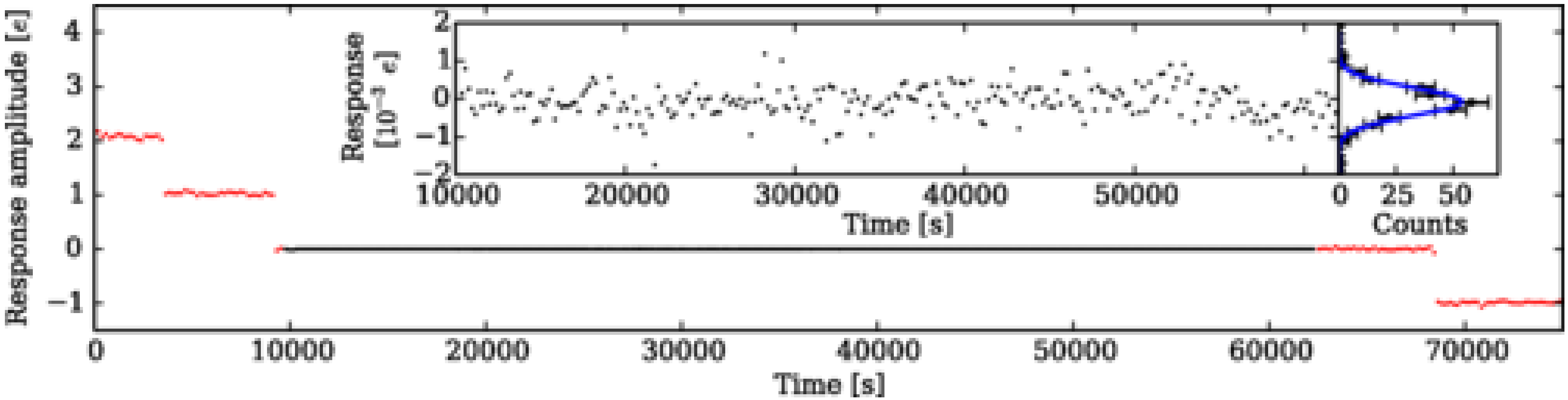
- Search for millicharged particles
- Charged optically levitated nanoparticle is an ultrasensitive force sensor
- Here: Coulomb force
- Are there charges with q<<e?

Applications of optical trapping in physics

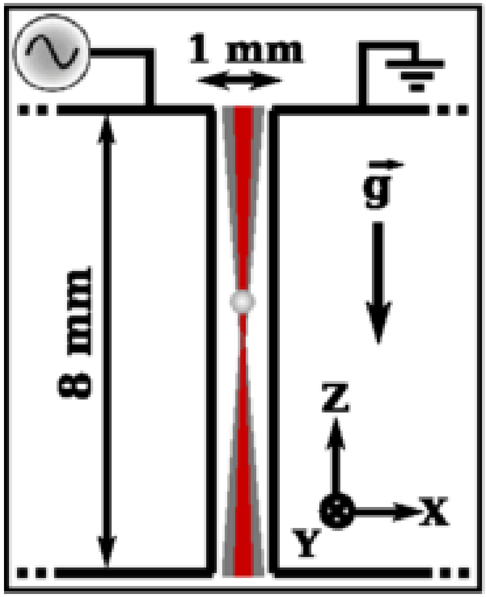


Moore et al., PRL 113, 251801

- Search for millicharged particles
- Charged optically levitated nanoparticle is an ultrasensitive force sensor
- Here: Coulomb force
- Are there charges with q<<e?



Applications of optical trapping in physics



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An optically levitated particle can serve as a Heisenberg microscope (measurement at the Heisenberg uncertainty limit)!