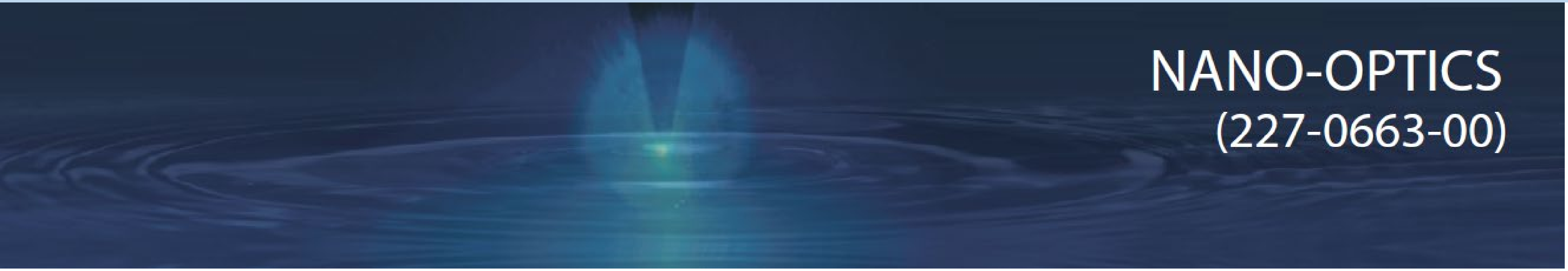
Welcome again!



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Photonics Laboratory
HPP M24

- Check out the “Orientation sheet” on Moodle

Administrative details

- 3 homework problems: find on Moodle, hand in on Moodle
- Dates for paper presentations fixed (please be available)
- Exams will take place in last week of semester (precise schedule to be published)

- You should have all necessary information to decide whether you want to follow this course. If you don't want to get credit/take the exam, please sign out of the course by Tuesday, 29 Sep. (You are welcome to keep following the lectures and I can add you as a guest in case you want to access the Moodle.)

Administrative details: Paper Presentations

- I will assign the students I haven't had feedback from
- I'll share the schedule as soon as possible

Today's question

Why do I not see the atoms that make up your skin when I look at you?

Maxwell's equations for complex fields

$$\nabla \cdot \mathbf{D}(\mathbf{r}) = \rho(\mathbf{r})$$

$$\nabla \times \mathbf{E}(\mathbf{r}) = i\omega \mathbf{B}(\mathbf{r})$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = -i\omega \mathbf{D}(\mathbf{r}) + \mathbf{j}(\mathbf{r})$$

$$\nabla \cdot \mathbf{B}(\mathbf{r}) = 0$$

Note that I dropped the underscore for complex fields!

$$\mathrm{D}(\mathbf{r}) = \varepsilon_0 \varepsilon(\omega) \mathrm{E}(\mathbf{r}) \qquad \mathrm{B}(\mathbf{r}) = \mu_0 \mu(\omega) \mathrm{H}(\mathbf{r})$$

Spectral representation:

$$\hat{\mathrm{E}}(\mathbf{r}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathbf{E}(\mathbf{r}, t)\, \mathrm{e}^{i\omega t}\, \mathrm{d}t$$

For monochromatic fields:

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}\left\{\mathbf{E}(r)\mathrm{e}^{-\mathrm{i}\omega t}\right\}$$

The Helmholtz equation, plane and evanescent waves

$$\left[\qquad \qquad \right] \boldsymbol{E}(\boldsymbol{r}) = 0$$

1. Plane waves: $\mathbf{E} = \mathbf{E}_0 \mathrm{e}^{\pm \mathrm{i}\mathbf{kr}}$

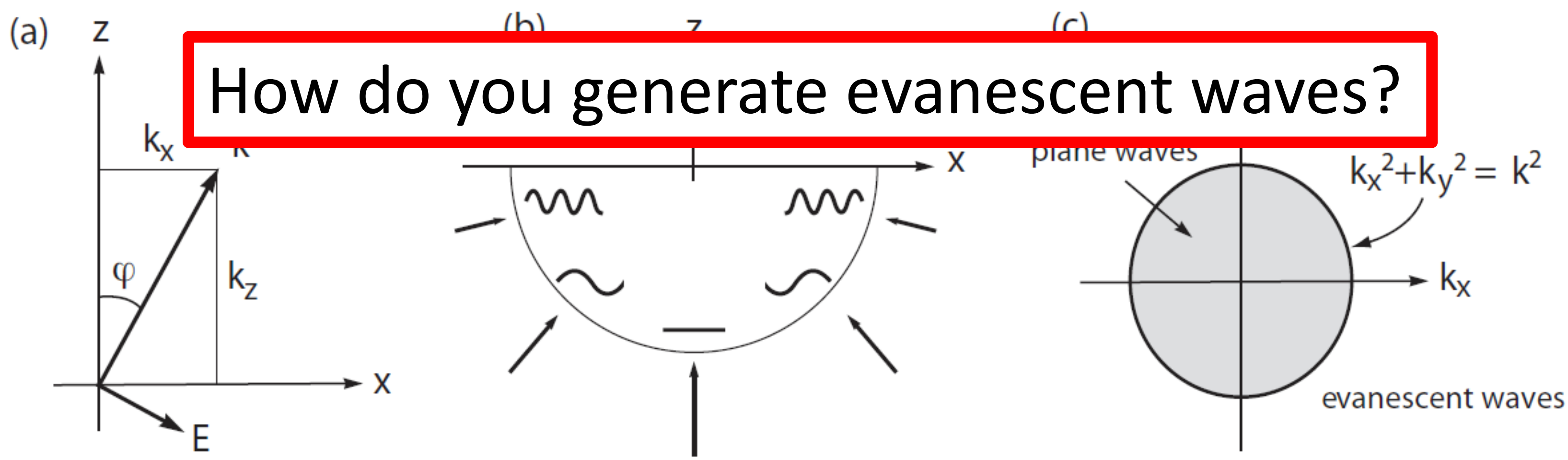
2. Dispersion relation: $k_x^2 + k_y^2 + k_z^2 = n^2 \frac{\omega^2}{c^2}$

3. Speed of light: $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

4. Refractive index: $n = \sqrt{\varepsilon \mu}$

for $k_x^2 + k_y^2 > n^2 \frac{\omega^2}{c^2}$

$$\mathbf{E}(\mathbf{r}, t) \; = \; \mathrm{Re}\{\mathbf{E}_0 \, \mathrm{e}^{\pm i(k_x x + k_y y) - i\omega t}\} \; \mathrm{e}^{\mp |k_z| z}$$



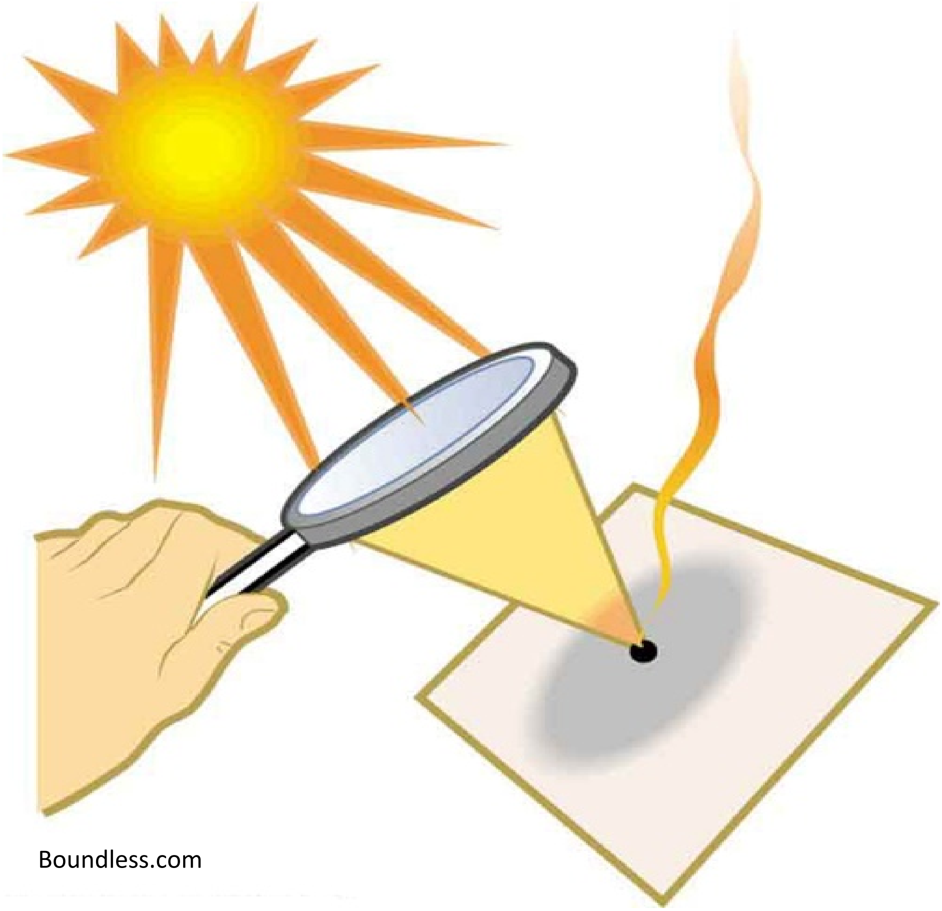
How do you generate evanescent waves?

Simple imaging systems

On the menu today

- Motivation: Why nano-optics?
- Repetition: electromagnetism
- Optical imaging:
  - Focusing by a lens
    - Angular spectrum
    - Paraxial approximation
    - Gaussian beams
    - Method of stationary phase
  - The diffraction limit
  - Fluorophores
  - Example: Fluorescence microscopy
  - Example: STED microscopy
  - Example: Localization microscopy
  - Example: Scanning probe microscopy

How does focusing by a lens work?

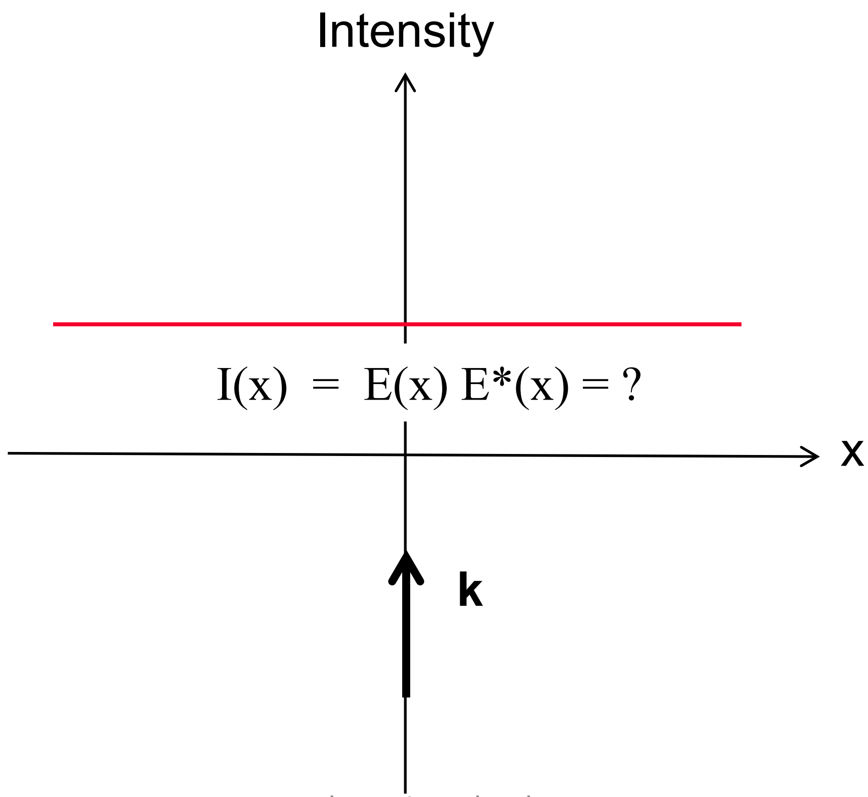


Boundless.com

How does focusing by a lens work?

$$\mathbf{E}(x,\, z=0) \;=\; E_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} e^{\imath k x \sin[\theta_1]}$$

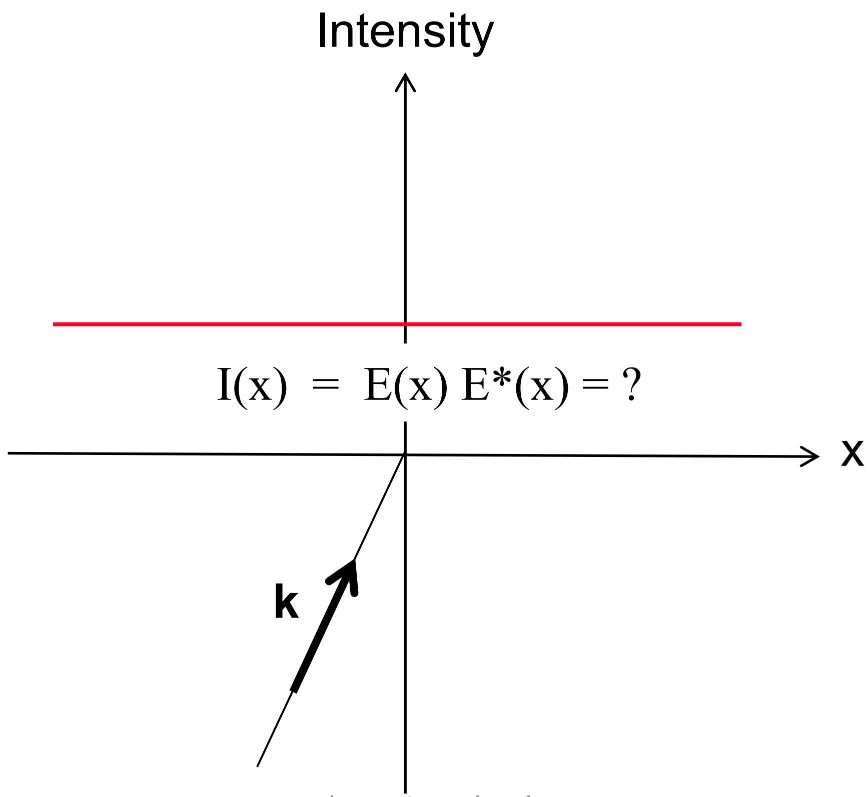
$\theta_1 = 0^\circ$



How does focusing by a lens work?

$$\mathbf{E}(x,\, z=0) \;=\; E_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} e^{\imath k x \sin[\theta_1]}$$

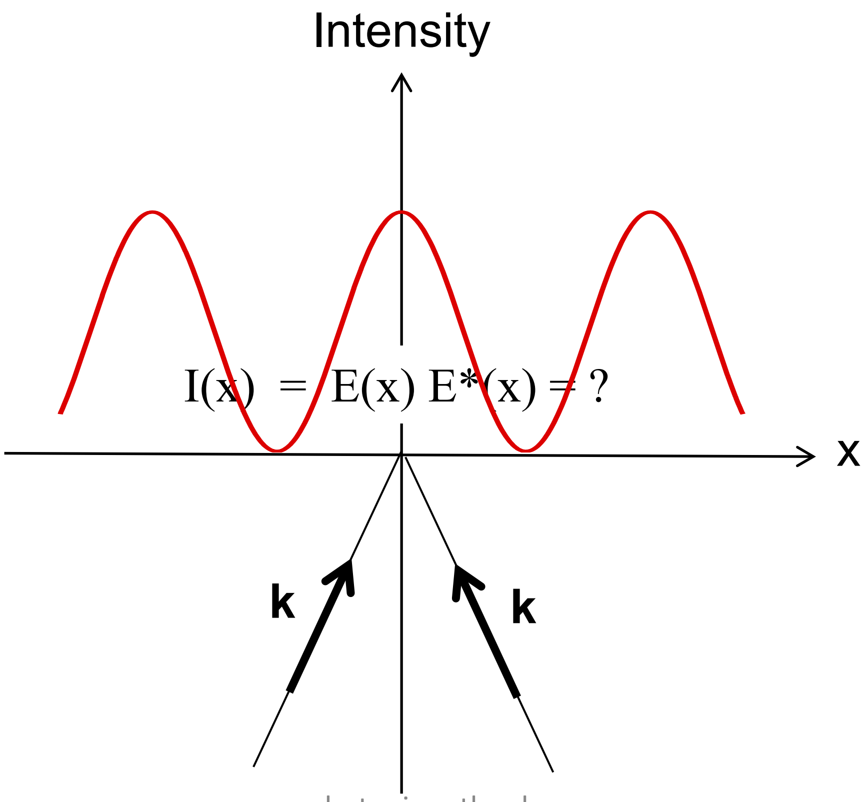
$\theta_1 = 20^\circ$



How does focusing by a lens work?

$$\mathbf{E}(x,\, z=0) \;=\; E_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mathrm{e}^{\imath k x \sin[\theta_1]}$$

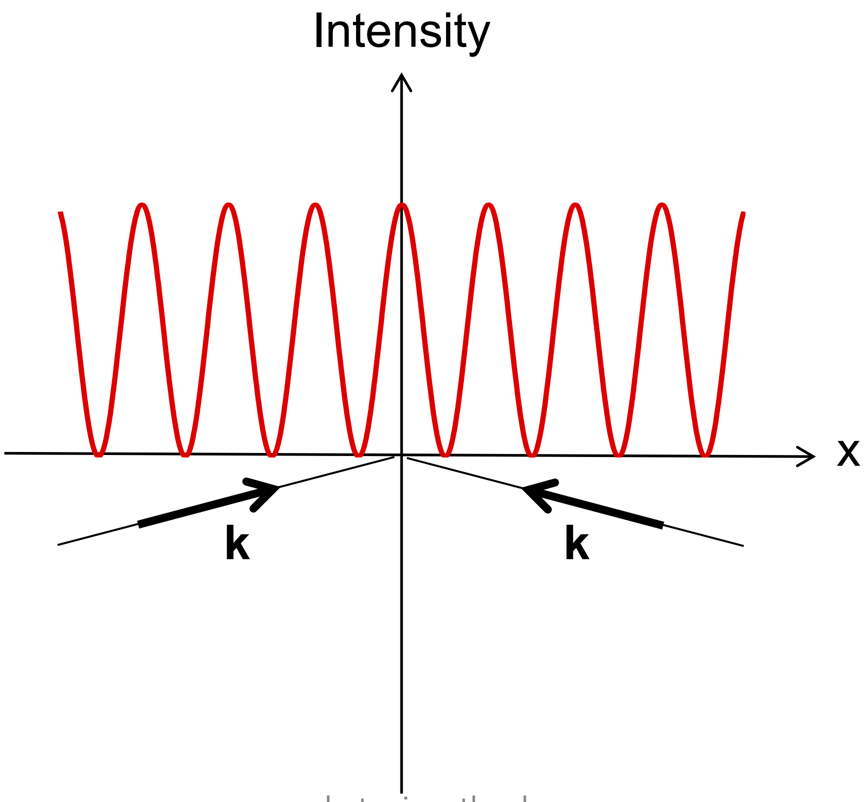
$\theta_1 = \pm\, 20^\circ$



How does focusing by a lens work?

$$\mathbf{E}(x,\, z=0) \;=\; E_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mathrm{e}^{\imath k x \sin[\theta_1]}$$

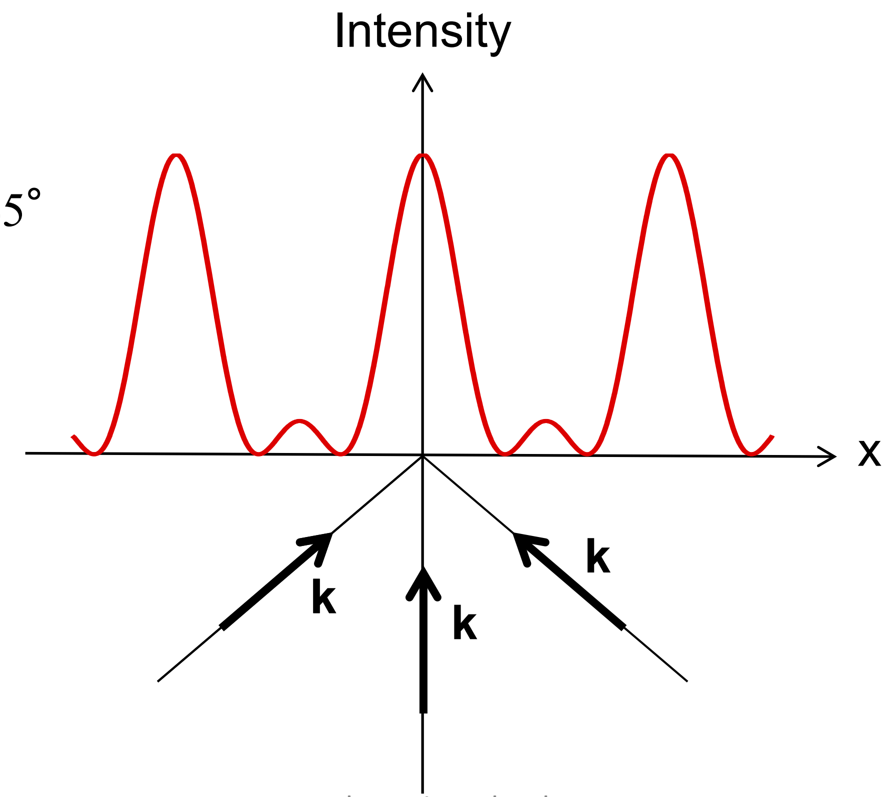
$\theta_1 = \pm\, 80^\circ$



How does focusing by a lens work?

$$\mathbf{E}(x, z=0) = E_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} e^{\imath kx \sin[\theta_1]}$$

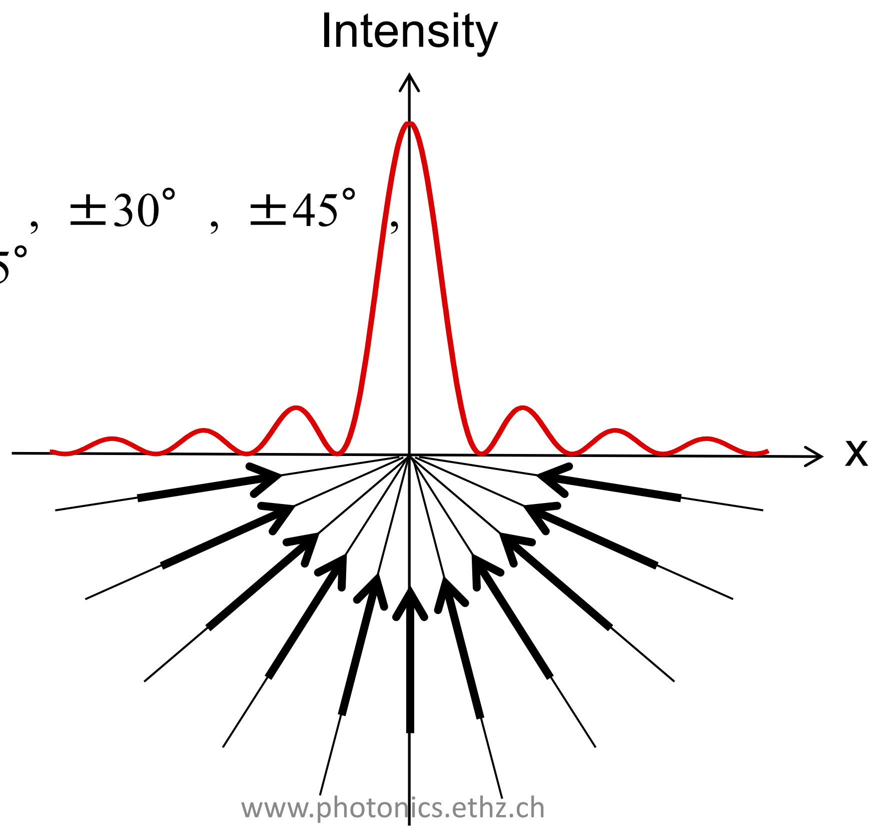
$\theta 1 = 0^\circ$, $\pm 45^\circ$



How does focusing by a lens work?

$$\mathbf{E}(x,\, z=0) \;=\; E_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mathrm{e}^{\imath k x \sin[\theta_1]}$$

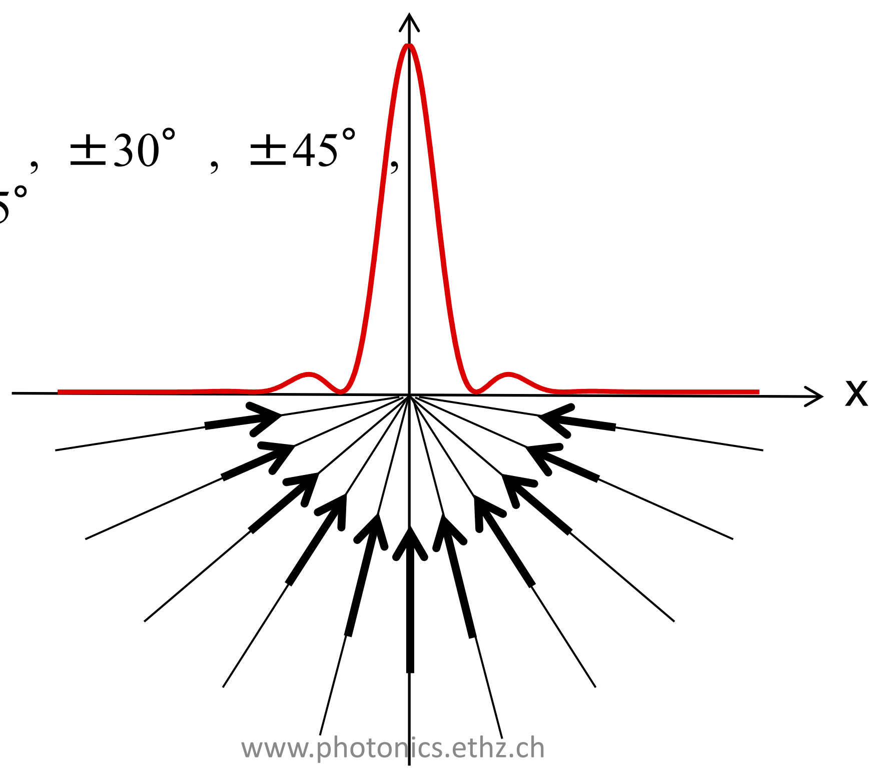
$\theta 1 = 0^\circ$, $\pm 15^\circ$, $\pm 30^\circ$, $\pm 45^\circ$,
$\pm 60^\circ$, $\pm 75^\circ$



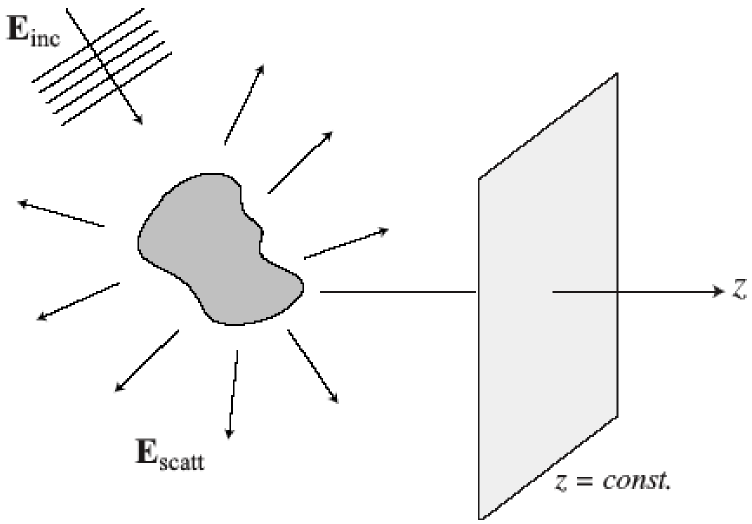
How does focusing by a lens work?

$$\mathbf{E}(x,\, z=0) \;=\; E_0 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mathrm{e}^{\imath k x \sin[\theta_1]}$$

$\theta 1 = 0^\circ$, $\pm 15^\circ$, $\pm 30^\circ$, $\pm 45^\circ$,
$\pm 60^\circ$, $\pm 75^\circ$
+apodization



Angular spectrum

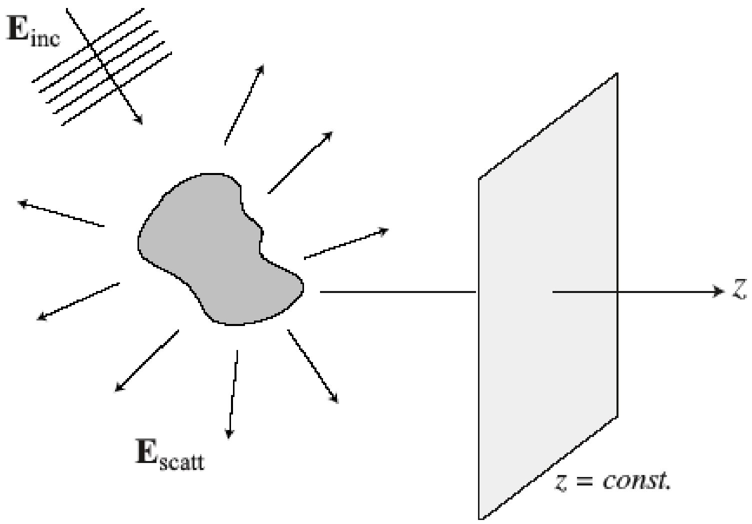


MATH :

$$\hat{\mathbf{E}}(k_x, k_y; z) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \mathbf{E}(x, y, z)\, \mathrm{e}^{-i\,[k_x x + k_y y]}\, dx\, dy$$

$$\mathbf{E}(x, y, z) = \iint_{-\infty}^{\infty} \hat{\mathbf{E}}(k_x, k_y; z)\, \mathrm{e}^{i\,[k_x x + k_y y]}\, dk_x\, dk_y$$

Angular spectrum



MATH :

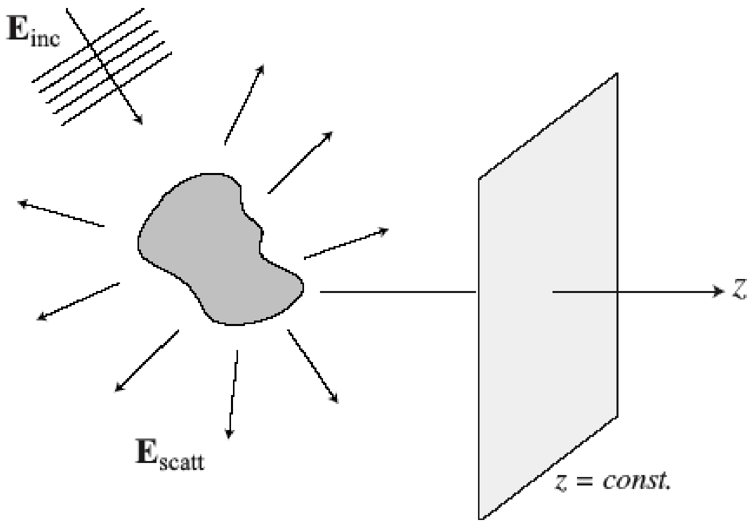
$$\hat{\mathbf{E}}(k_x, k_y; z) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \mathbf{E}(x, y, z)\, e^{-i[k_x x + k_y y]}\, dx\, dy$$

$$\mathbf{E}(x, y, z) = \iint_{-\infty}^{\infty} \hat{\mathbf{E}}(k_x, k_y; z)\, e^{i[k_x x + k_y y]}\, dk_x\, dk_y$$

PHYS :

$$(\nabla^2 + k^2)\, \mathbf{E}(\mathbf{r}) = 0 \longrightarrow \hat{\mathbf{E}}(k_x, k_y; z) = \hat{\mathbf{E}}(k_x, k_y; 0)\, e^{\pm i k_z z}$$

Angular spectrum



MATH :

$$\hat{\mathbf{E}}(k_x, k_y; z) \;=\; \frac{1}{4\pi^2} \iint\limits_{-\infty}^{\infty} \mathbf{E}(x, y, z)\, \mathrm{e}^{-i\,[k_x x + k_y y]}\, dx\, dy$$

$$\mathbf{E}(x, y, z) \;=\; \iint\limits_{-\infty}^{\infty} \hat{\mathbf{E}}(k_x, k_y; z)\, \mathrm{e}^{i\,[k_x x + k_y y]}\, dk_x\, dk_y$$

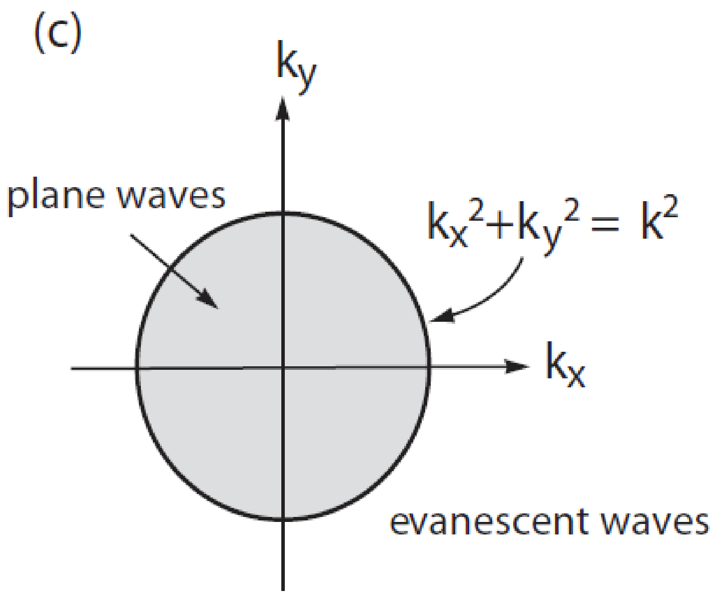
PHYS :

$$(\nabla^2 + k^2)\,\mathbf{E}(\mathbf{r}) \;=\; 0 \quad\longrightarrow\quad \hat{\mathbf{E}}(k_x, k_y; z) \;=\; \hat{\mathbf{E}}(k_x, k_y; 0)\, \mathrm{e}^{\pm i\, k_z z}$$

Together:

$$\mathbf{E}(x, y, z) \;=\; \iint\limits_{-\infty}^{\infty} \hat{\mathbf{E}}(k_x, k_y; 0)\, \mathrm{e}^{i\,[k_x x + k_y y \pm k_z z]}\; \mathrm{d}k_x\, \mathrm{d}k_y$$

Angular spectrum



PHYS : $$(\nabla^2 + k^2)\,\mathbf{E}(\mathbf{r}) = 0 \longrightarrow \hat{\mathbf{E}}(k_x, k_y; z) = \hat{\mathbf{E}}(k_x, k_y; 0)\, \mathrm{e}^{\pm i\, k_z\, z}$$

Together: $$\mathbf{E}(x, y, z) = \iint_{-\infty}^{\infty} \hat{\mathbf{E}}(k_x, k_y; 0)\, \mathrm{e}^{i\,[k_x x + k_y y \pm k_z z]}\, \mathrm{d}k_x\, \mathrm{d}k_y$$

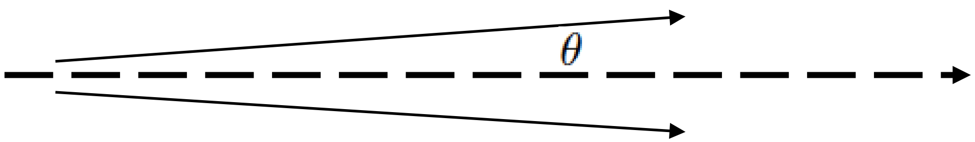
Paraxial approximation

$$\mathbf{E}(x, y, z) = \iint_{-\infty}^{\infty} \hat{\mathrm{E}}(k_x, k_y; 0)\, \mathrm{e}^{i[k_x x + k_y y \pm k_z z]}\, \mathrm{d}k_x\, \mathrm{d}k_y$$

mit $$k_z = k\sqrt{1 - (k_x^2 + k_y^2)/k^2} \approx k - \frac{(k_x^2 + k_y^2)}{2\,k}$$

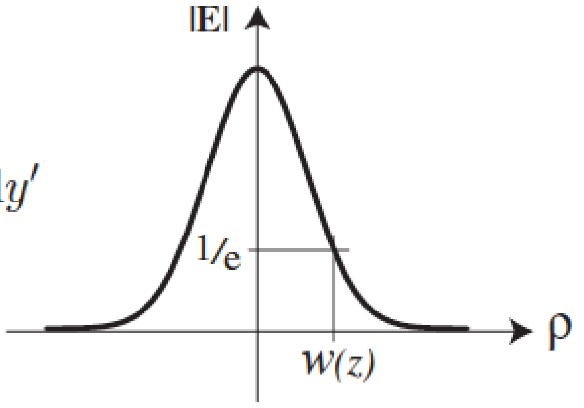
$$k_z = k\cos\theta = k\,[1 - \theta^2/2 + ..]$$

→ Fields propagate predominantly in z-direction !



Gaussian beams

$$\mathbf{E}(x', y', 0) = \mathbf{E}_o \, e^{-\frac{x'^2+y'^2}{w_o^2}}$$



$$\longrightarrow \quad \hat{\mathrm{E}}(k_x, k_y; 0) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \mathrm{E}_0 \, e^{-\frac{x'^2+y'^2}{w_0^2}} \, e^{-\mathrm{i}[k_x x' + k_y y']} \, \mathrm{d}x' \, \mathrm{d}y'$$

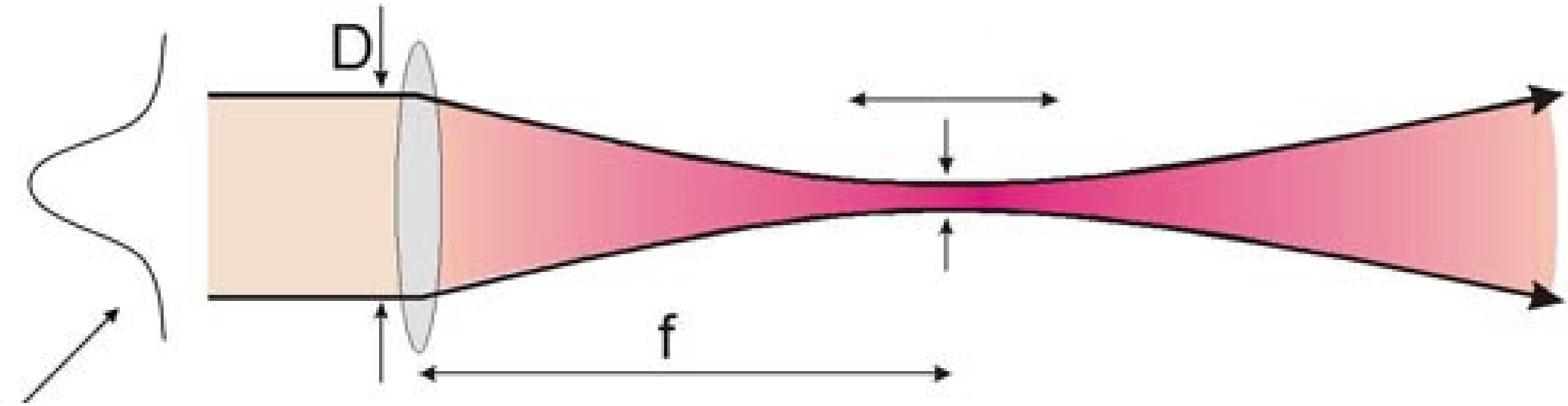
$$= \mathbf{E}_o \frac{w_o^2}{4\pi} e^{-(k_x^2+k_y^2)\frac{w_o^2}{4}}$$

$$\longrightarrow \quad \mathbf{E}(x, y, z) = \mathbf{E}_o \frac{w_o^2}{4\pi} e^{ikz} \iint_{-\infty}^{\infty} e^{-(k_x^2+k_y^2)\left(\frac{w_o^2}{4}+\frac{iz}{2k}\right)} \, e^{i[k_x x + k_y y]} \, dk_x \, dk_y \; ;$$

$$\int_{-\infty}^{\infty} \exp(-ax^2 + \mathrm{i}bx) \, \mathrm{d}x = \sqrt{\pi/a} \, \exp(-b^2/4a)$$

$$\int_{-\infty}^{\infty} x \exp(-ax^2 + \mathrm{i}bx) \, \mathrm{d}x = \mathrm{i}b\sqrt{\pi} \exp(-b^2/4a) / \, (2a^{3/2})$$

The Gaussian Beam



Field in focal plane z=0: $\mathbf{E}(x', y', 0) = \mathbf{E}_o \, \mathrm{e}^{-(x'^2+y'^2)/w_0^2}$

$$\mathbf{E}(\rho, z) = \mathbf{E}_o \frac{w_o}{w(z)} \, \mathrm{e}^{-\frac{\rho^2}{w^2(z)}} \, \mathrm{e}^{i\,[kz - \eta(z) + k\rho^2/2R(z)]}$$

$w(z) = w_o\,(1 + z^2/z_o^2)^{1/2}$ Beam waist

$R(z) = z\,(1 + z_o^2/z^2)$ Wavefront radius

$\eta(z) = \arctan z/z_o$ Phase correction (Gouy phase)

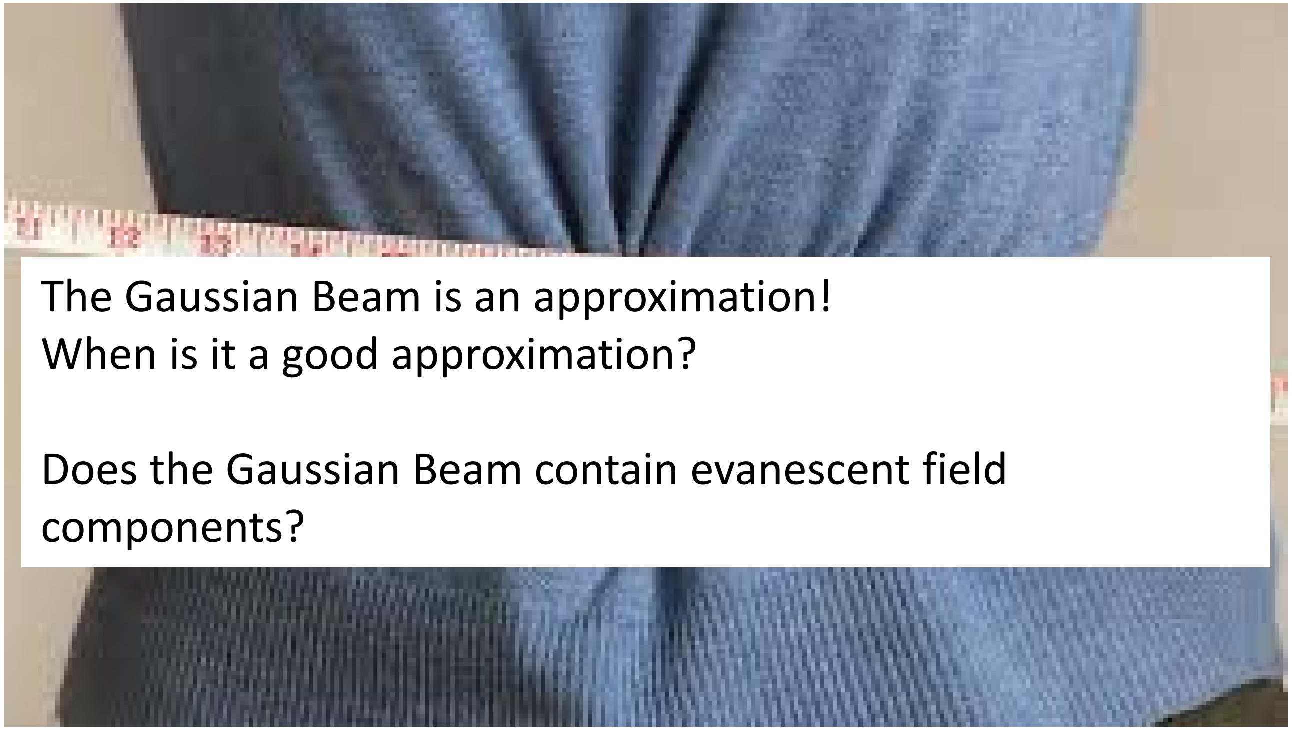
$z_o = \frac{k\,w_o^2}{2}$ Rayleigh length

The Gaussian Beam

$$z_o = \frac{k\, w_o^2}{2} \qquad \theta = \frac{2}{k\, w_o}$$

The Gaussian Beam has one free parameter. Which one?

The Gaussian Beam



The Gaussian Beam is an approximation!
When is it a good approximation?

Does the Gaussian Beam contain evanescent field components?

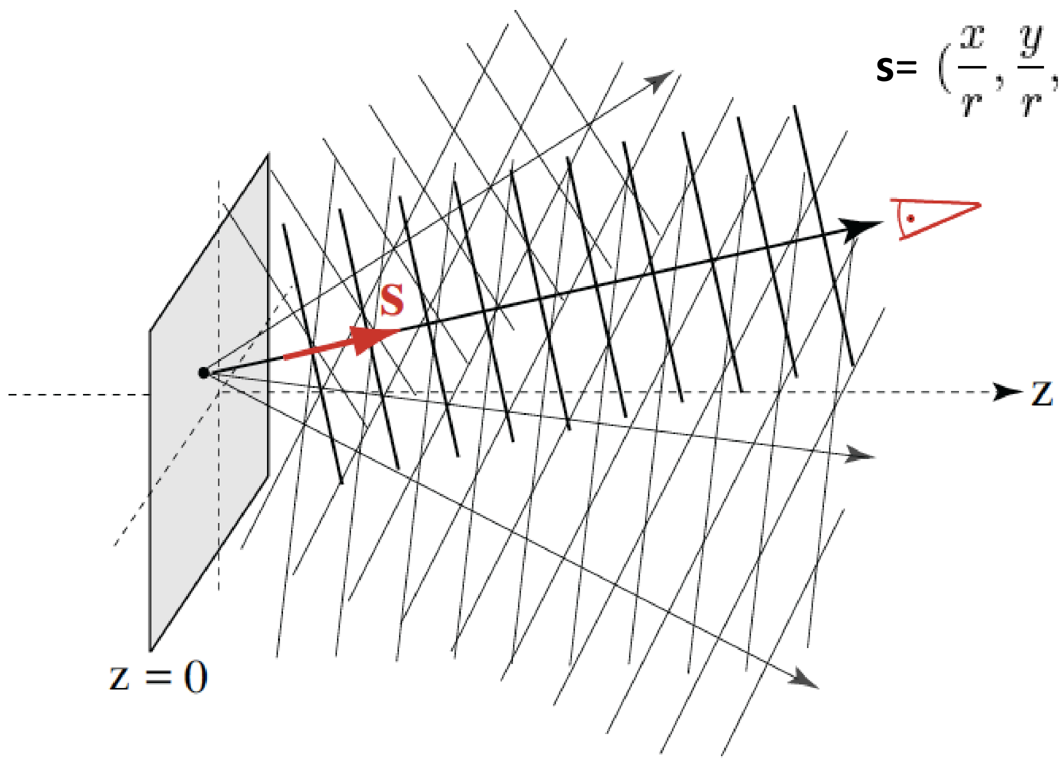
$$z_o = \frac{k\, w_o^2}{2} \qquad \theta = \frac{2}{k\, w_o}$$

The Gaussian Beam has one free parameter. Which one?

Far-field

$$\mathbf{E}_\infty(s_x, s_y) \;=\; -2\pi \mathrm{i} k\, s_z\, \hat{\mathbf{E}}(ks_x, ks_y; 0)\, \frac{\mathrm{e}^{\mathrm{i}kr}}{r}$$

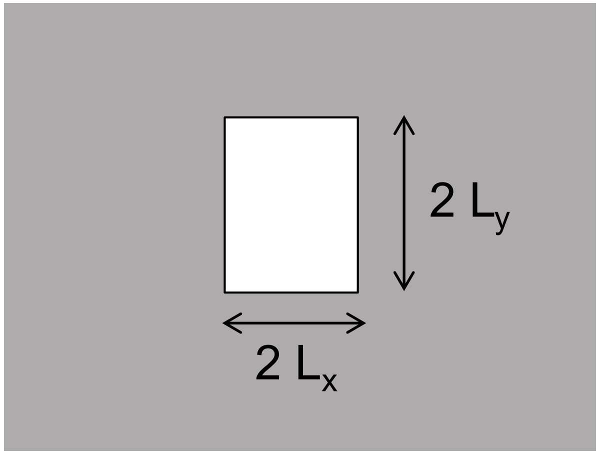
s= $\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right) = \left(\frac{k_x}{k}, \frac{k_y}{k}, \frac{k_z}{k}\right)$



Example

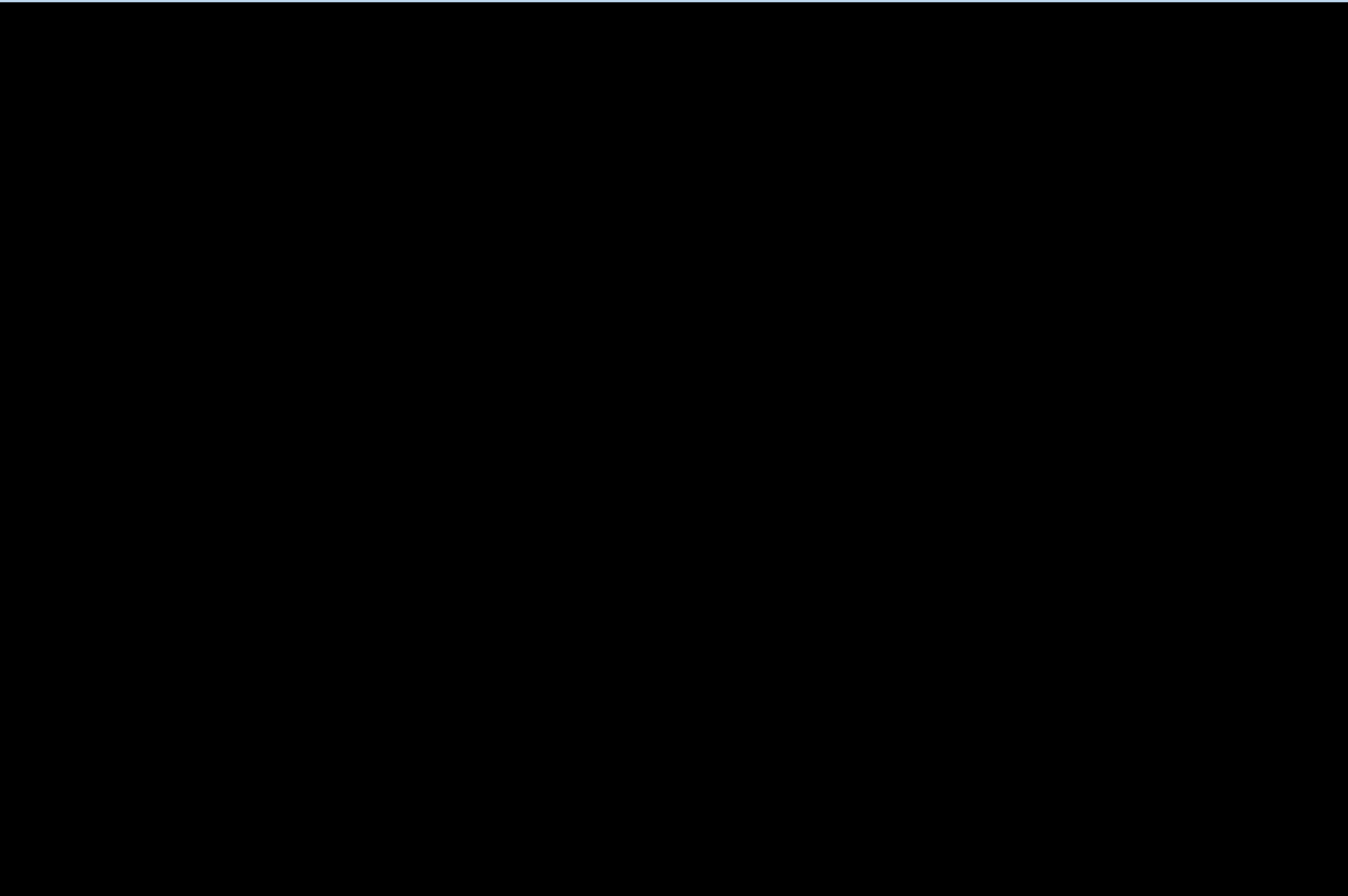
$$\hat{\mathbf{E}}(k_x, k_y; 0) = \frac{\mathbf{E}_0}{4\pi^2} \int_{-L_y}^{+L_y} \int_{-L_x}^{+L_x} \mathrm{e}^{-\mathrm{i}\,[k_x x' + k_y y']} \, \mathrm{d}x' \, \mathrm{d}y'$$

$$= \mathbf{E}_0 \frac{L_x L_y}{\pi^2} \frac{\sin(k_x L_x)}{k_x L_x} \frac{\sin(k_y L_y)}{k_y L_y}$$

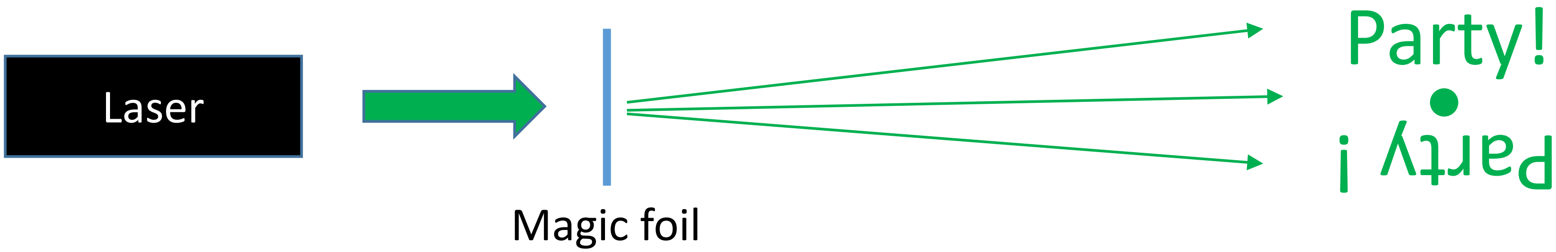


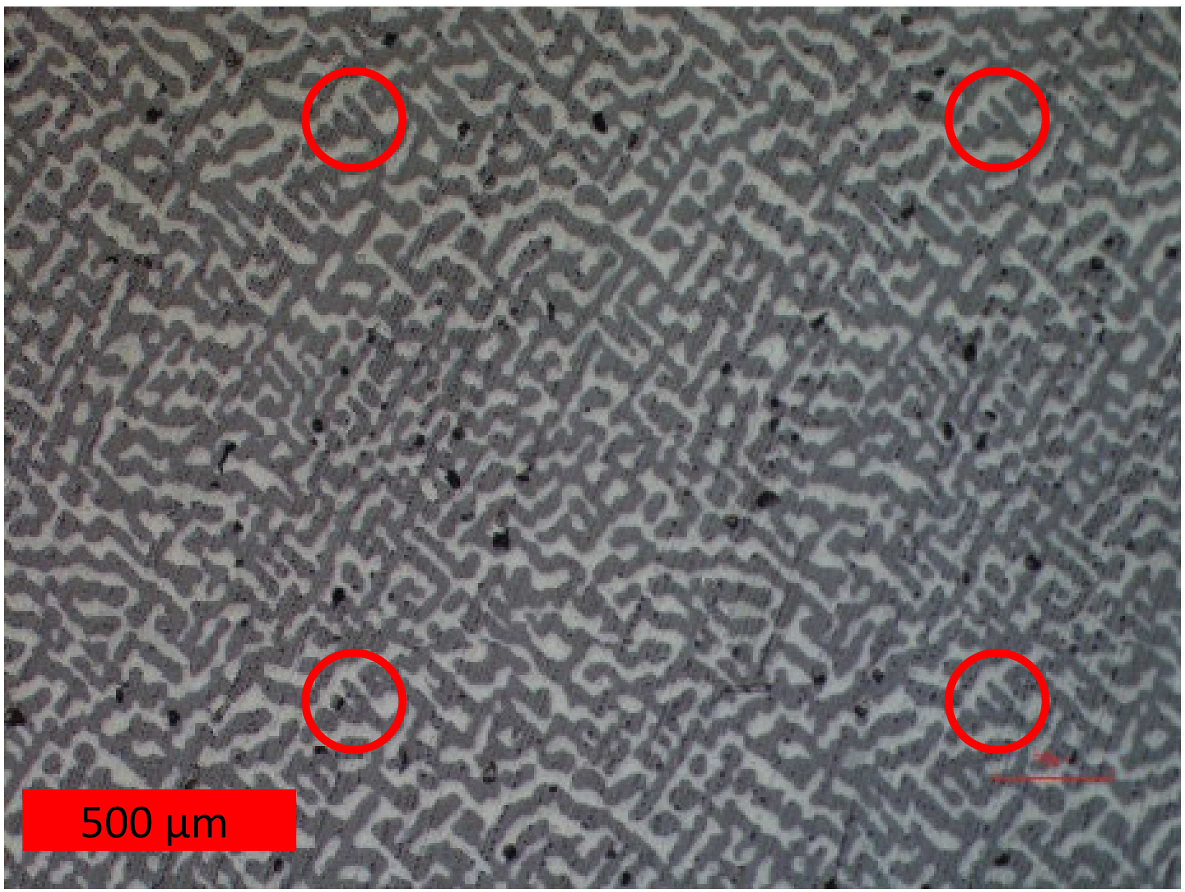
$$\mathbf{E}_\infty(s_x, s_y) = -\mathrm{i} k s_z \mathbf{E}_0 \frac{2 L_x L_y}{\pi} \frac{\sin(k s_x L_x)}{k s_x L_x} \frac{\sin(k s_y L_y)}{k s_y L_y} \frac{\mathrm{e}^{\mathrm{i}kr}}{r}$$

Example: Party-Party Goggles



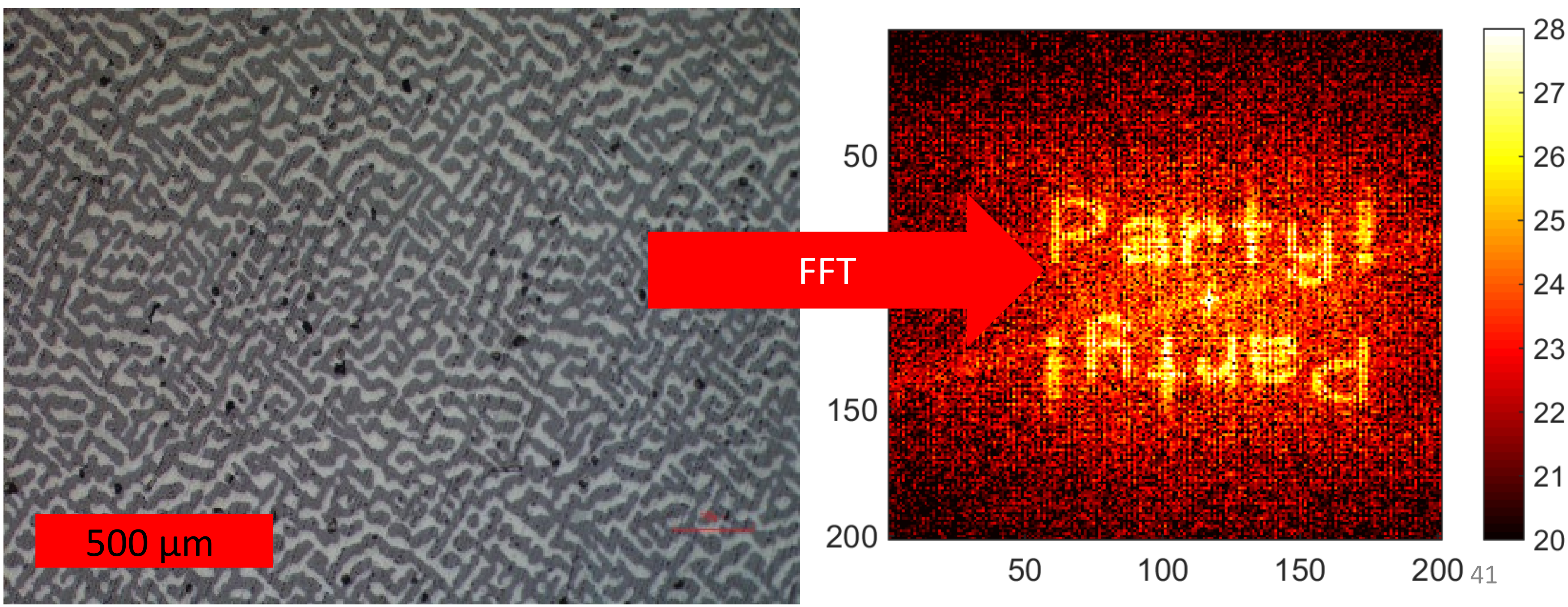
Party-Party Goggles





Party-Party

Laser
Magic foil
Party!



500 µm
FFT
50
150
200
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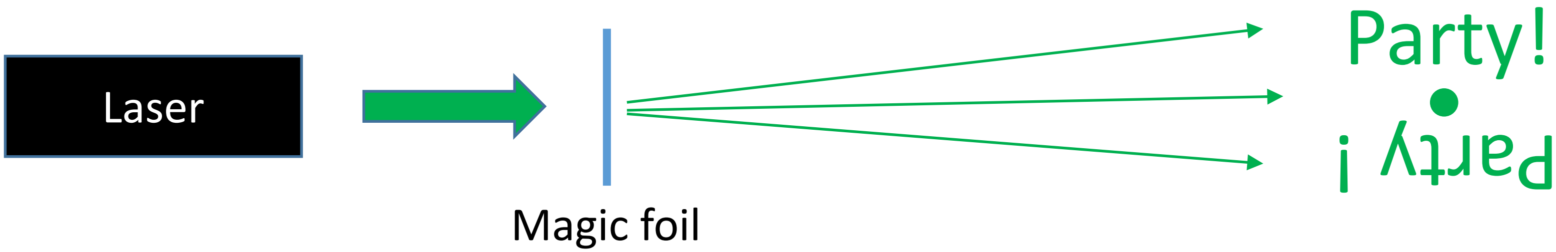
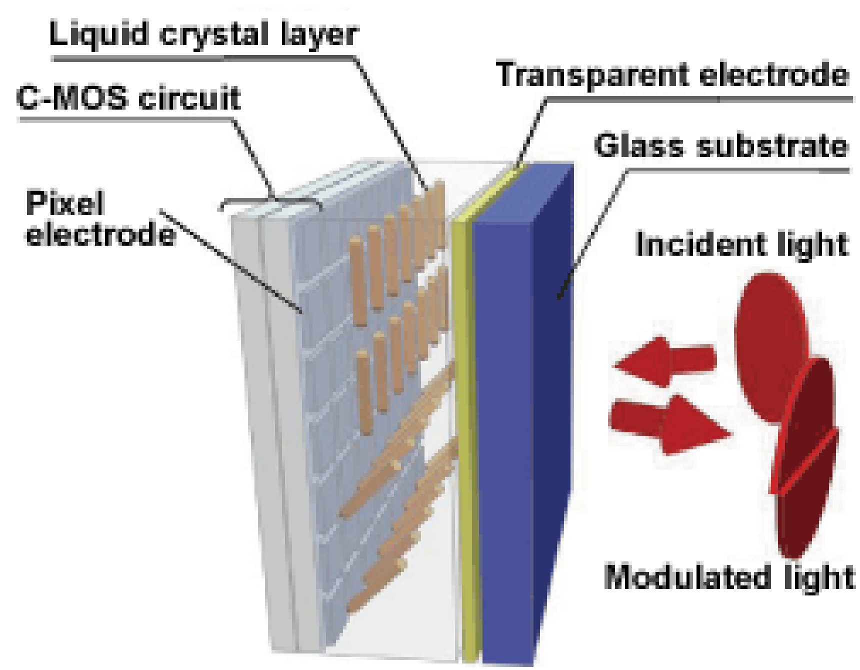
20

SLM technology uses Fourier optics



Adaptive Version: <u>Spatial light modulator (SLM)</u>



A better description of focused fields

Better than Gaussian beams, but not much to do analytically.