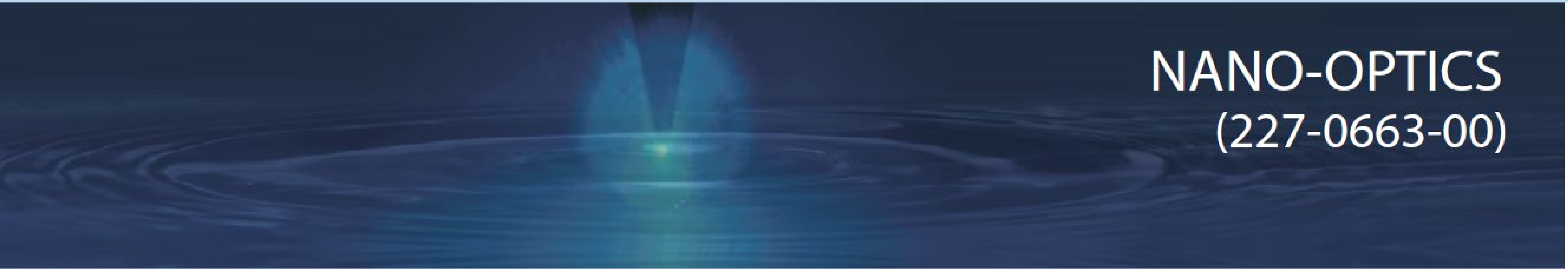
Welcome again!

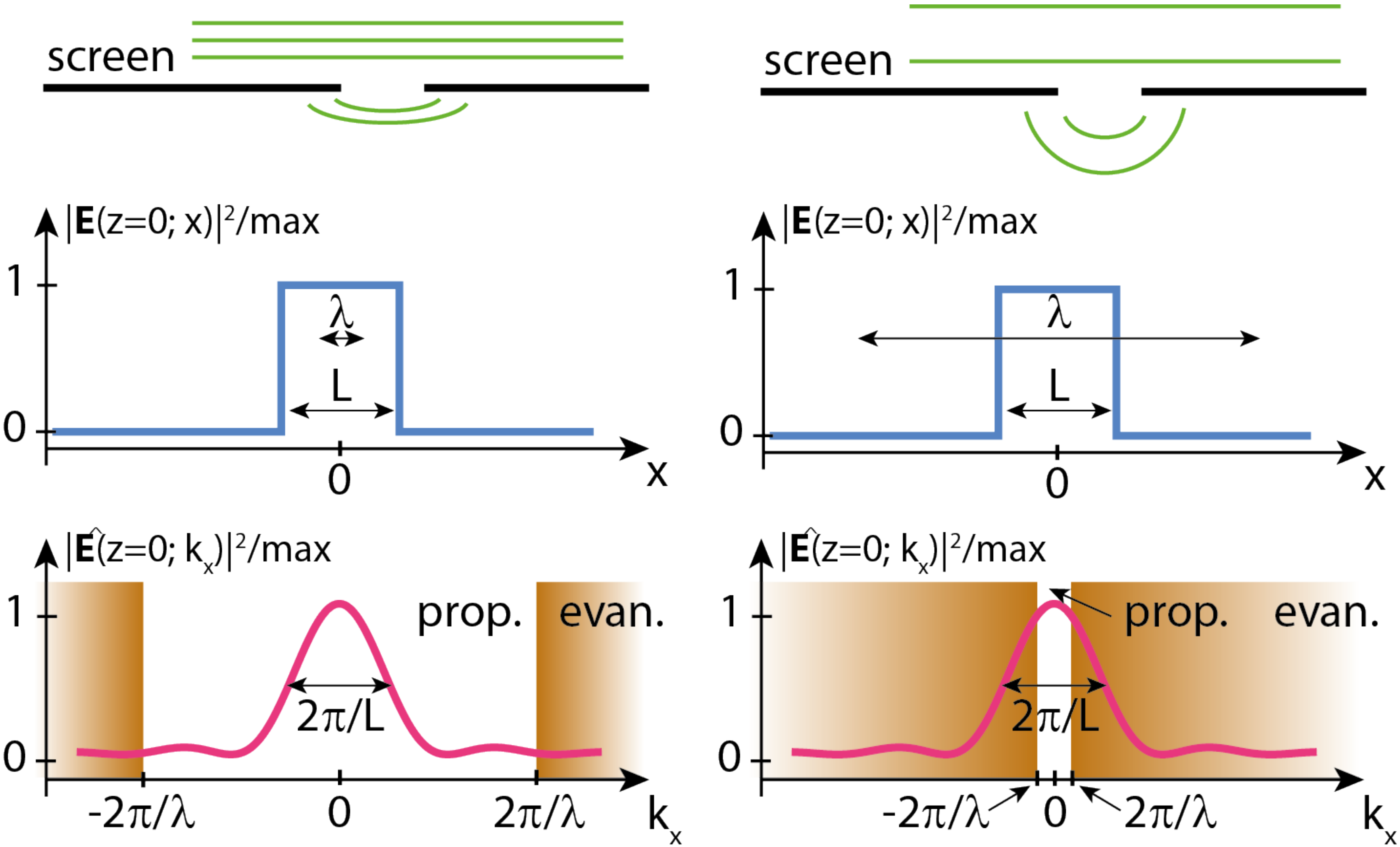


Martin Frimmer (mfrimmer@ethz.ch)
Photonics Laboratory
HPP M24

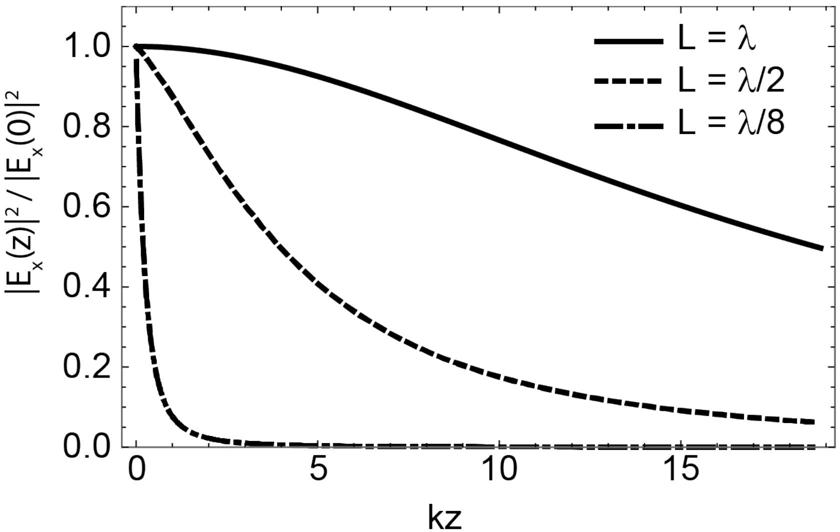
Where do we stand?

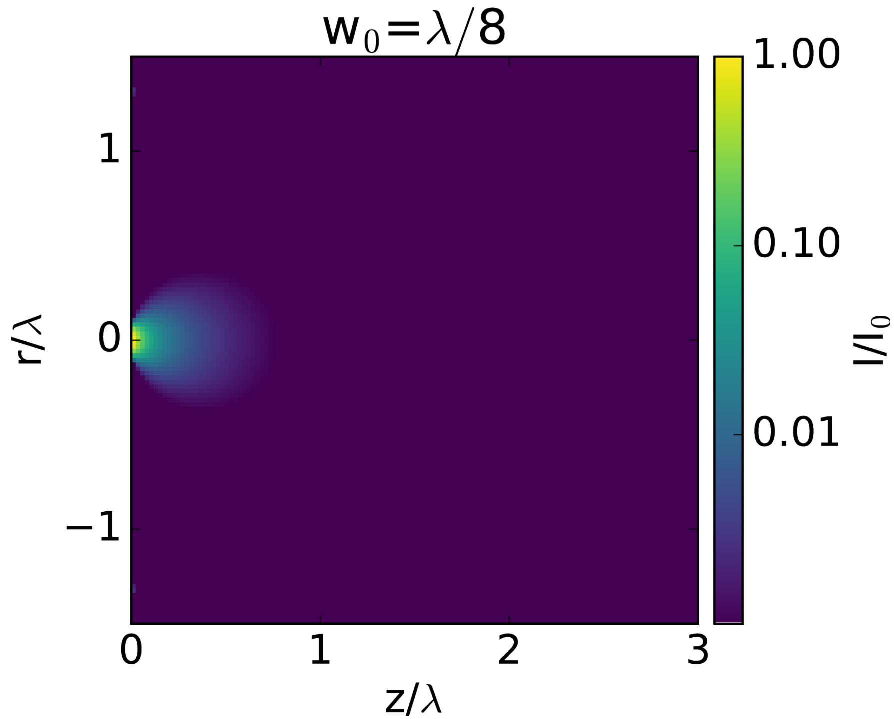
- Optical imaging:
  - Focusing by a lens
    - Angular spectrum
    - Paraxial approximation
    - Gaussian beams
    - Method of stationary phase
  - The diffraction limit: How well can we focus light?
  - Optical microscopy
    - Optical imaging systems
    - Real-world (dipolar) sources: Fluorophores and scatterers
    - Example: Fluorescence microscopy
    - Example: STED microscopy
    - Example: Localization microscopy
    - Example: Scanning probe microscopy

Fields behind an aperture

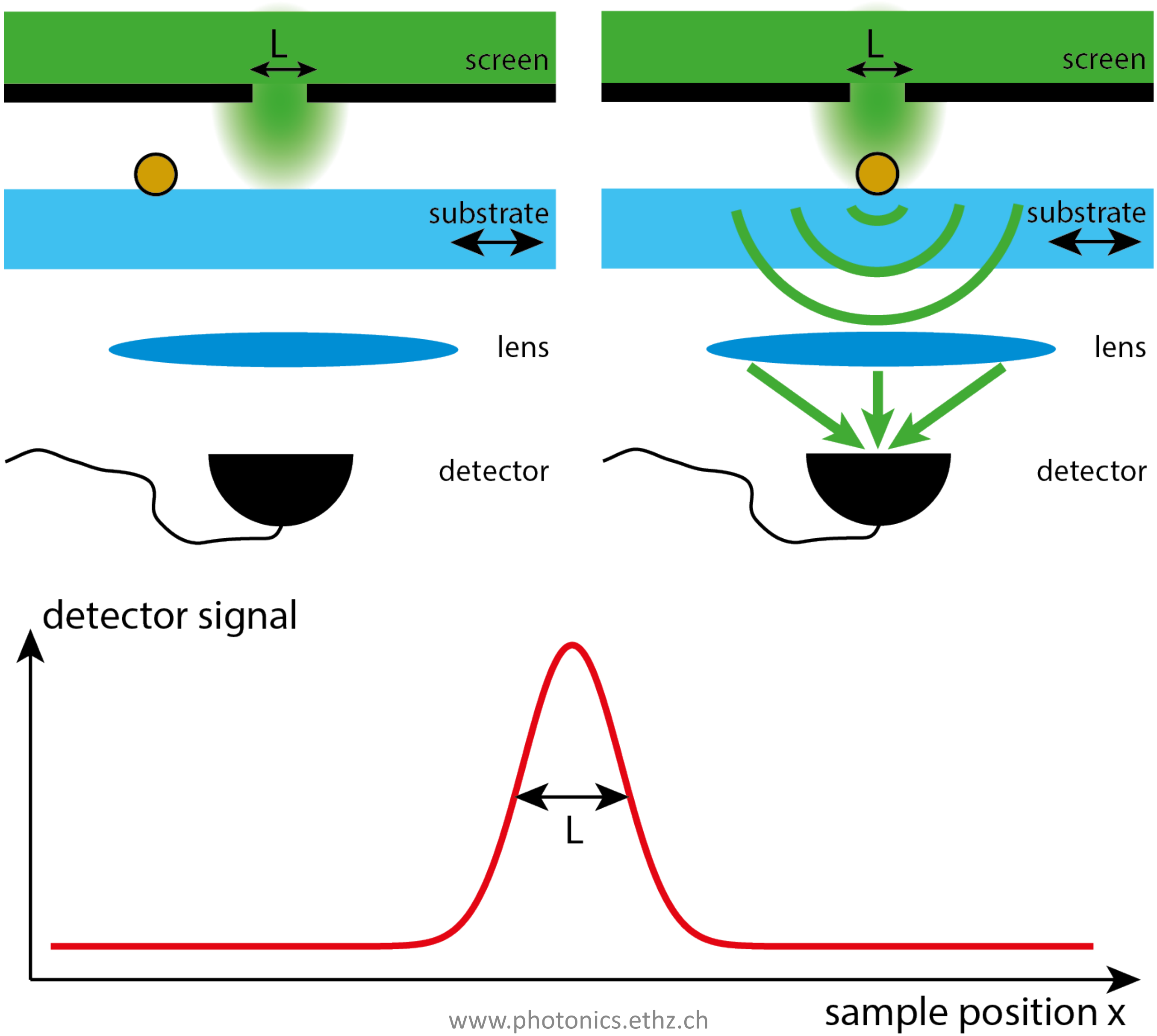


Fields behind a (Gaussian) aperture





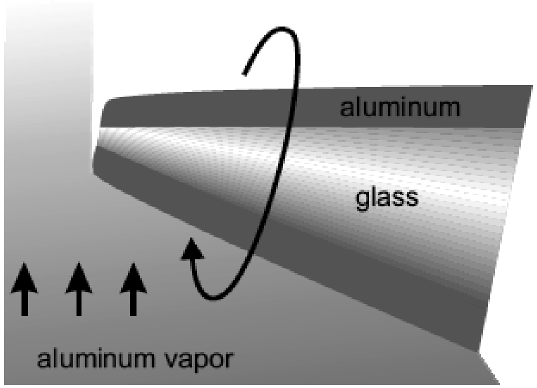
The principle of NSOM

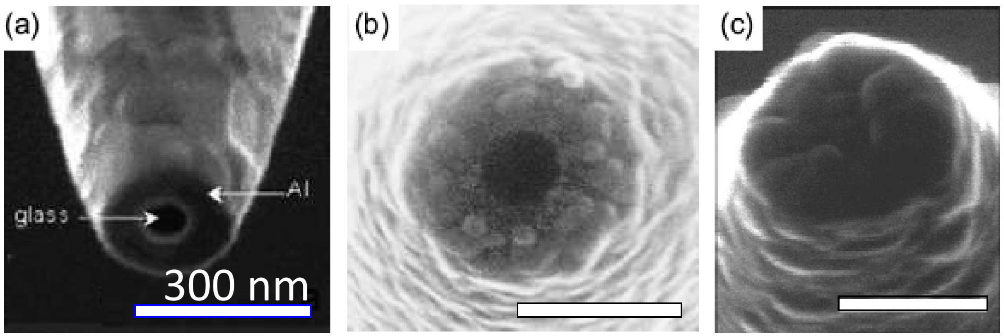


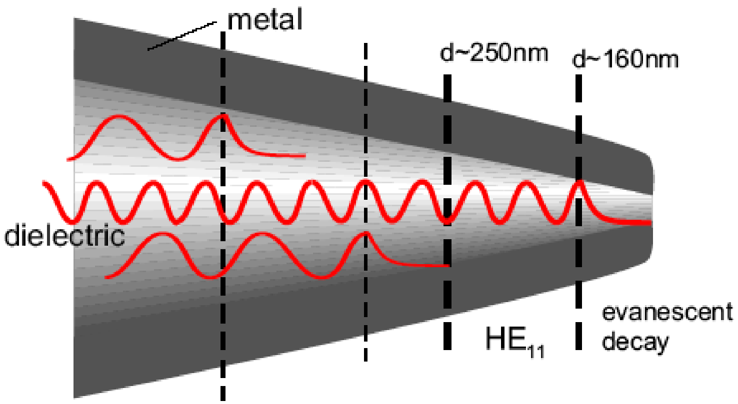
NSOM – how it's really done

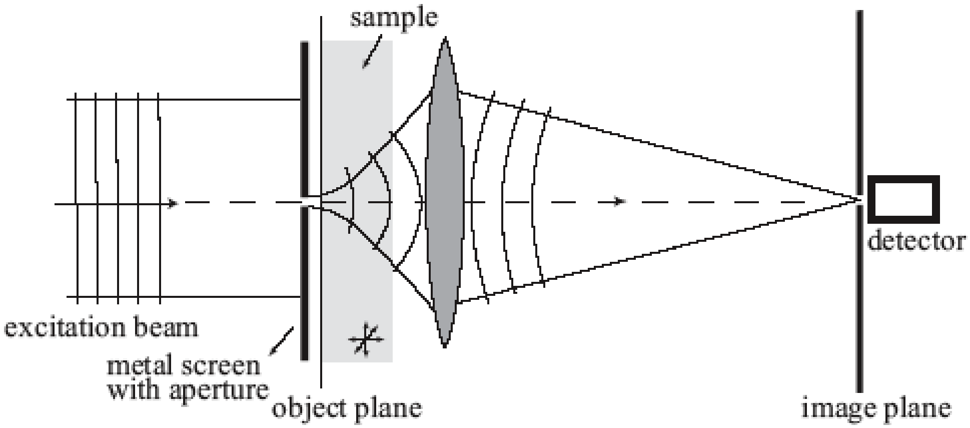
- Metal coated fiber tip

Hecht et al., J Chem. Phys. 112, 7761

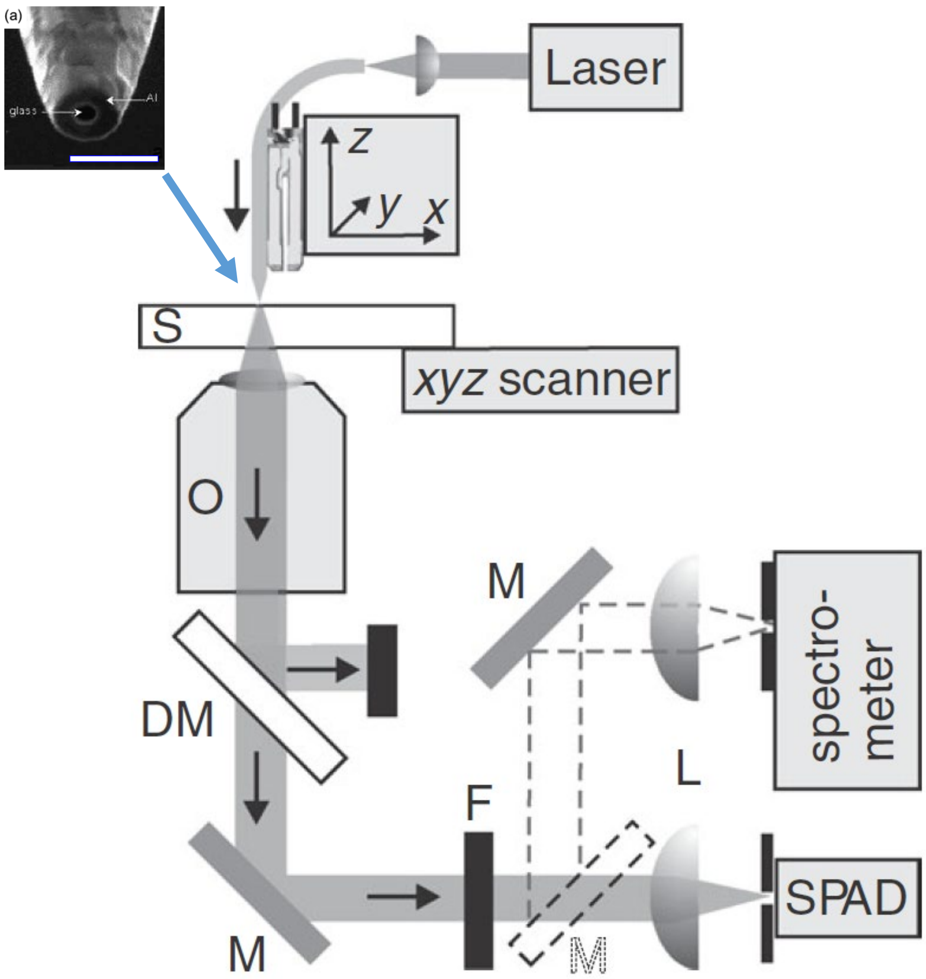








NSOM – operation modes



Localized excitation

- Create subdiffraction-sized illumination spot with aperture probe
- Collect scattered field/fluorescence with conventional far-field optics

Localized detection

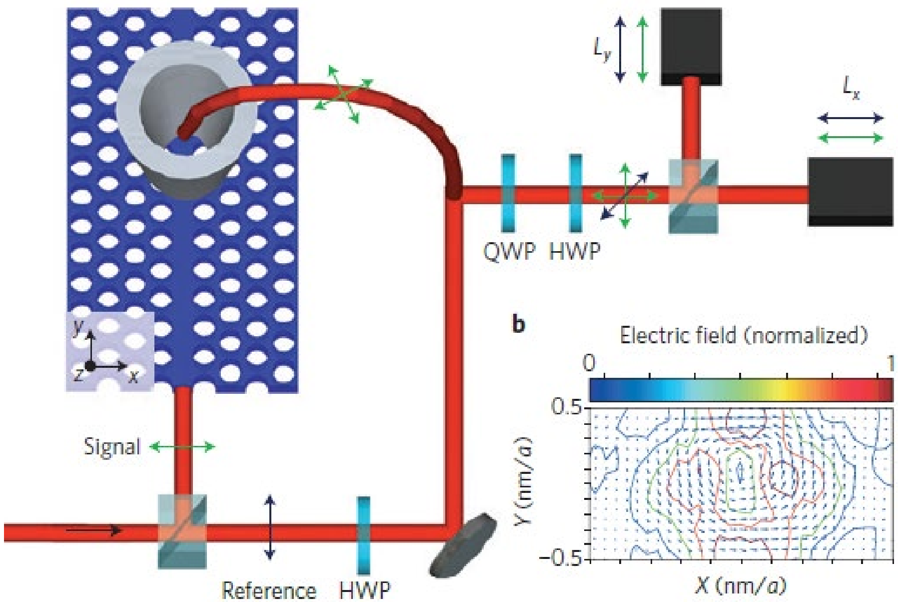
- Excite with conventional far-field optics
- Collect scattered field/fluorescence with aperture probe

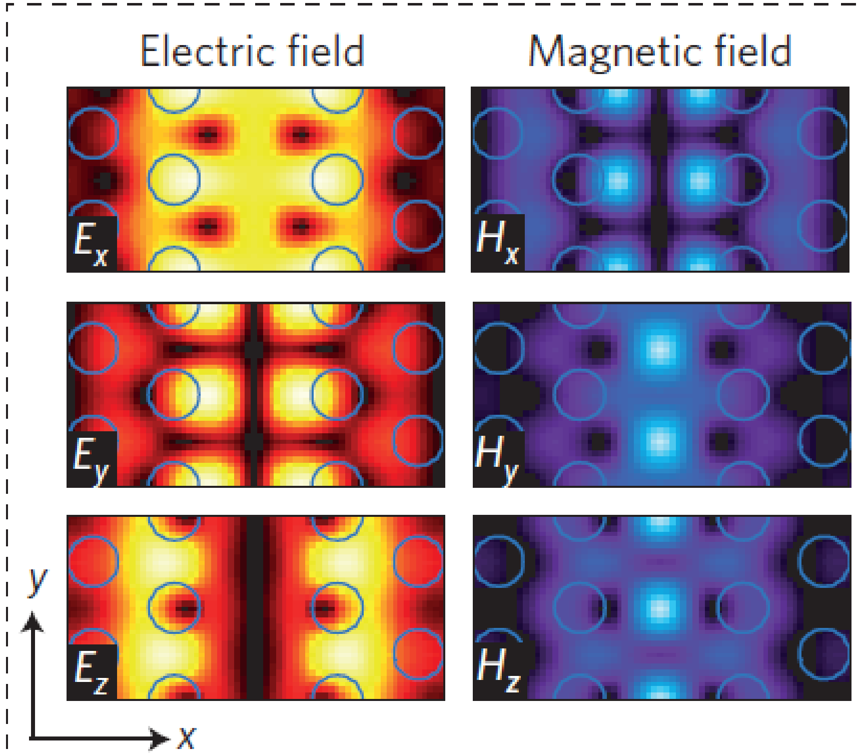
Localized excitation and detection

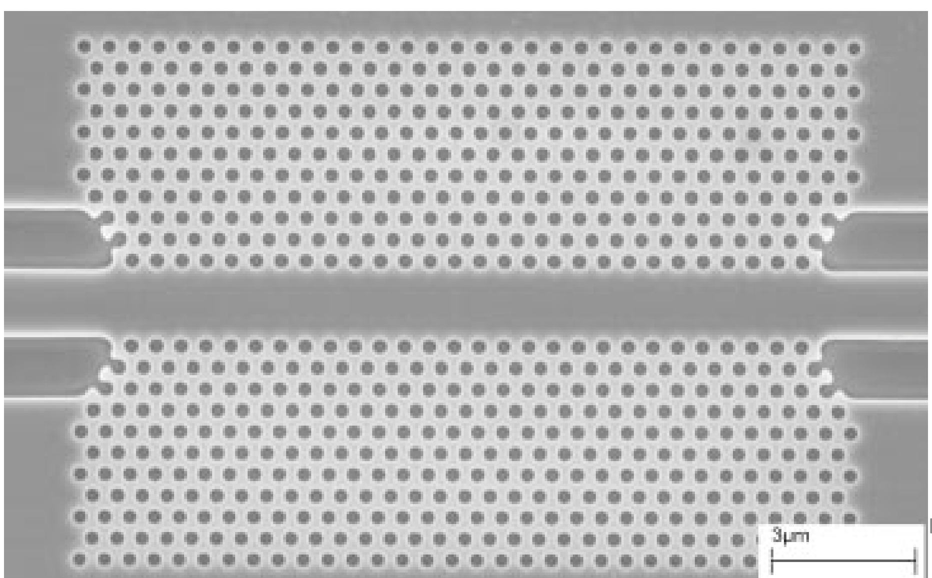
- …

NSOM – localized detection

Gersen et al., Phys. Rev. Lett. 94, 123901
Rothenberg and Kuipers, Nat. Phot. 8, 919



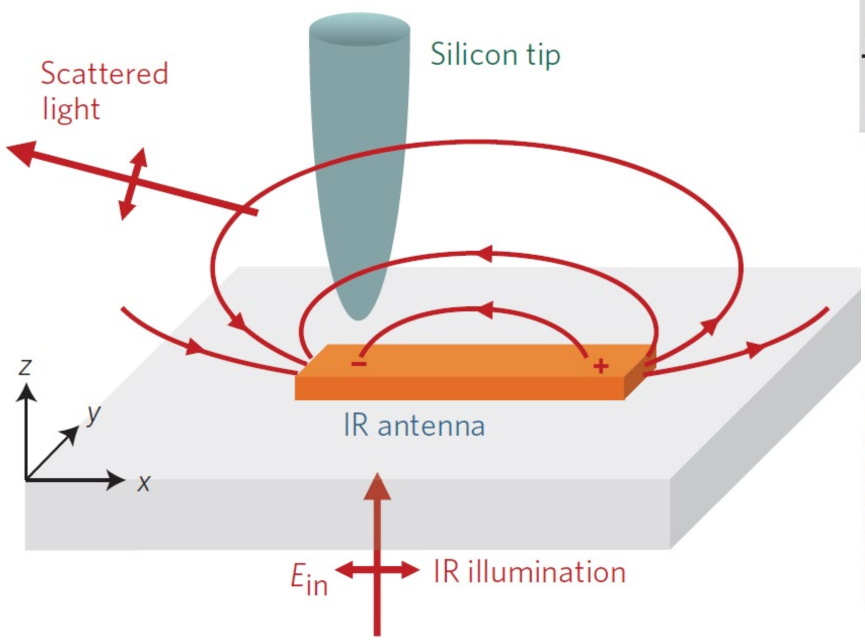


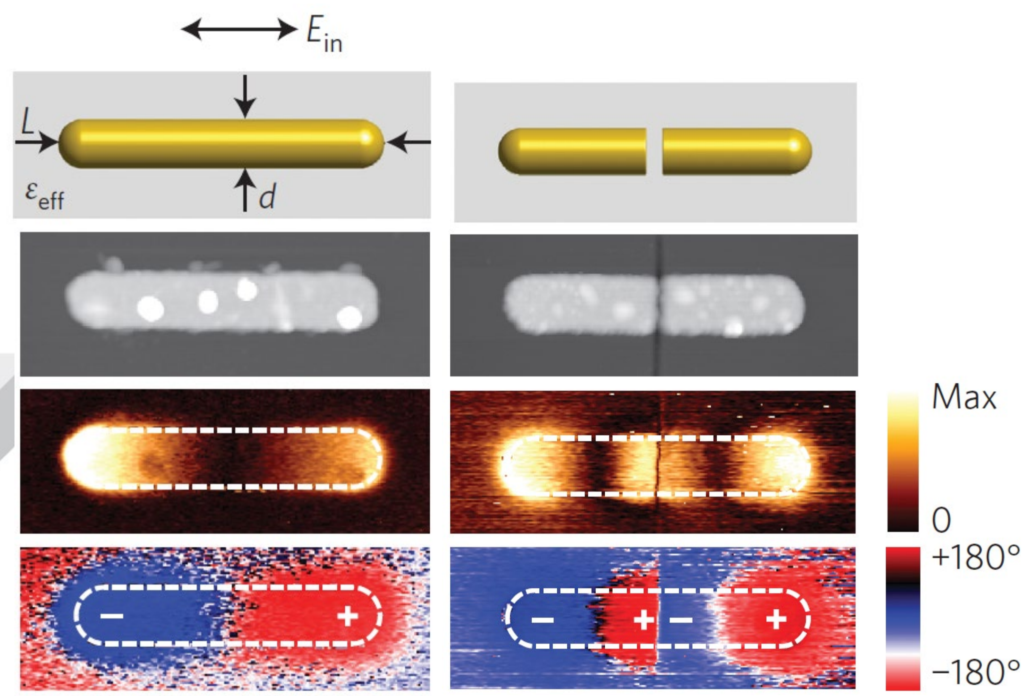


- Field distribution in photonic crystal waveguide
- Interferometric technique allows phase sensitive mapping of field

Scattering NSOM

Schnell et al., Nature Photonics 3, 287 - 291 (2009)

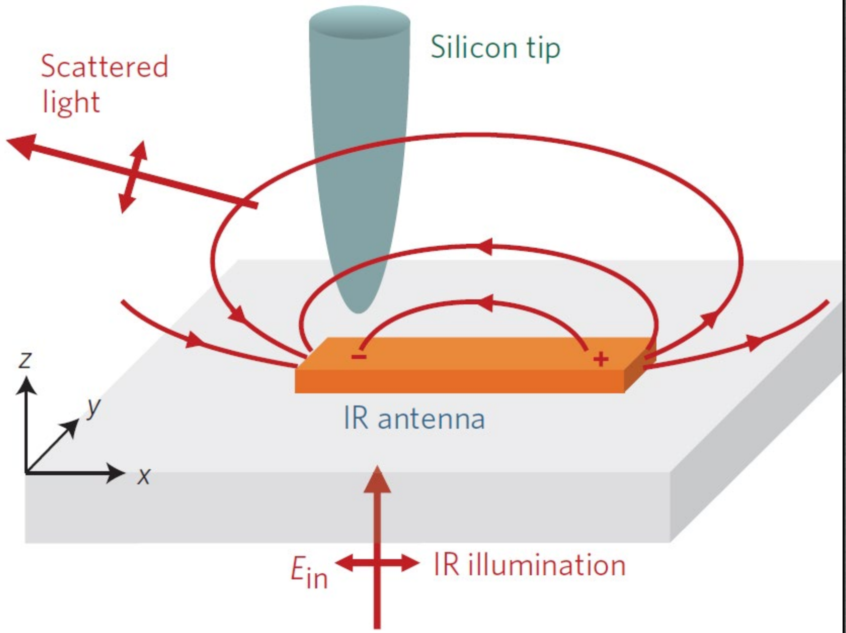


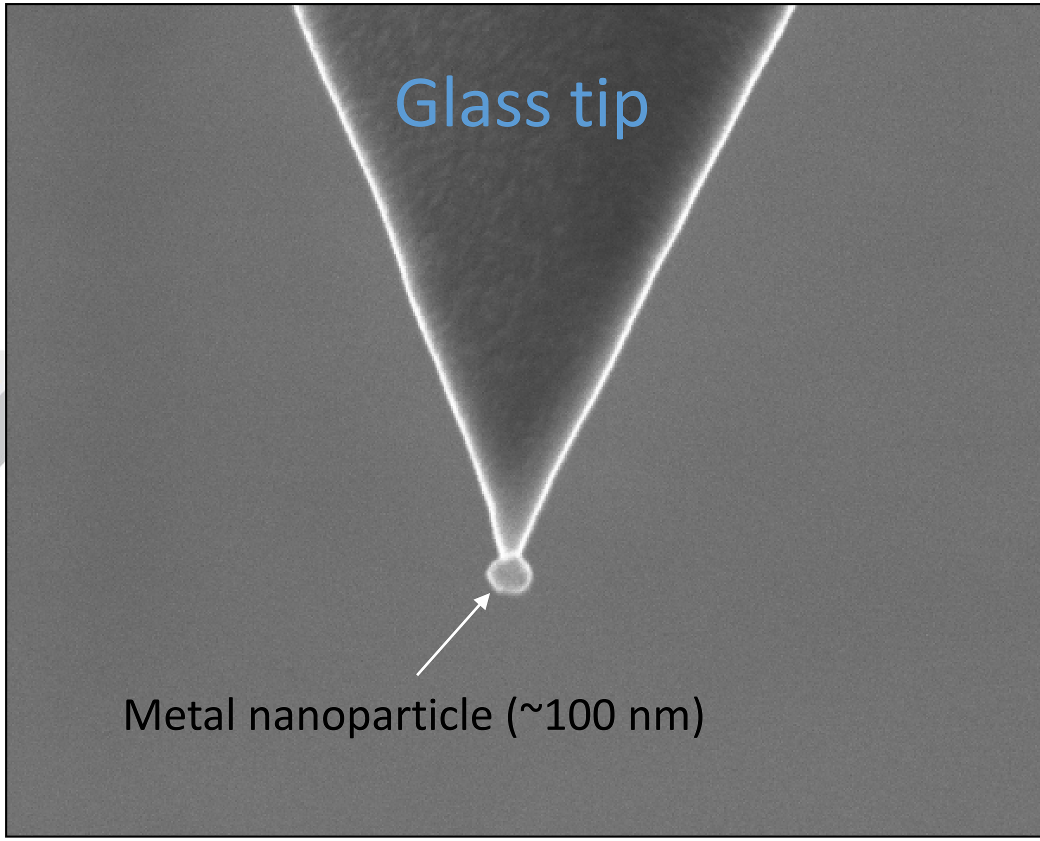


- L<<λ
- Illuminate with far field
- insert tip to <u>scatter out near-field components into far-field detector</u>

Scattering NSOM

Schnell et al., Nature Photonics 3, 287 - 291 (2009)

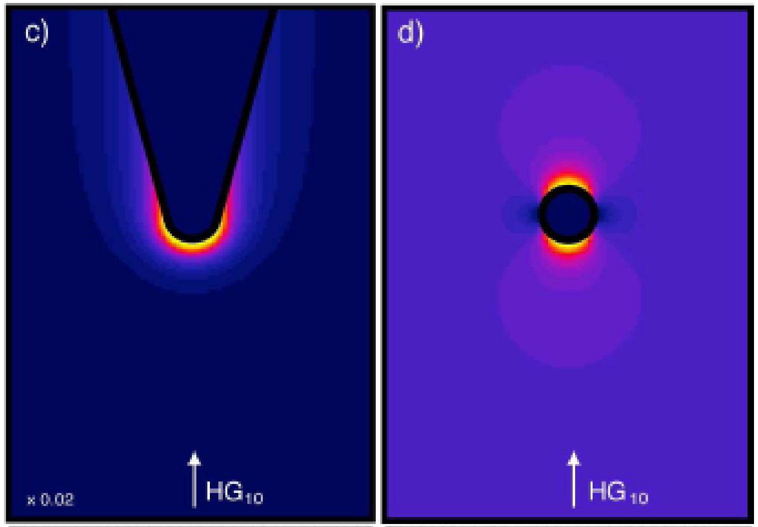


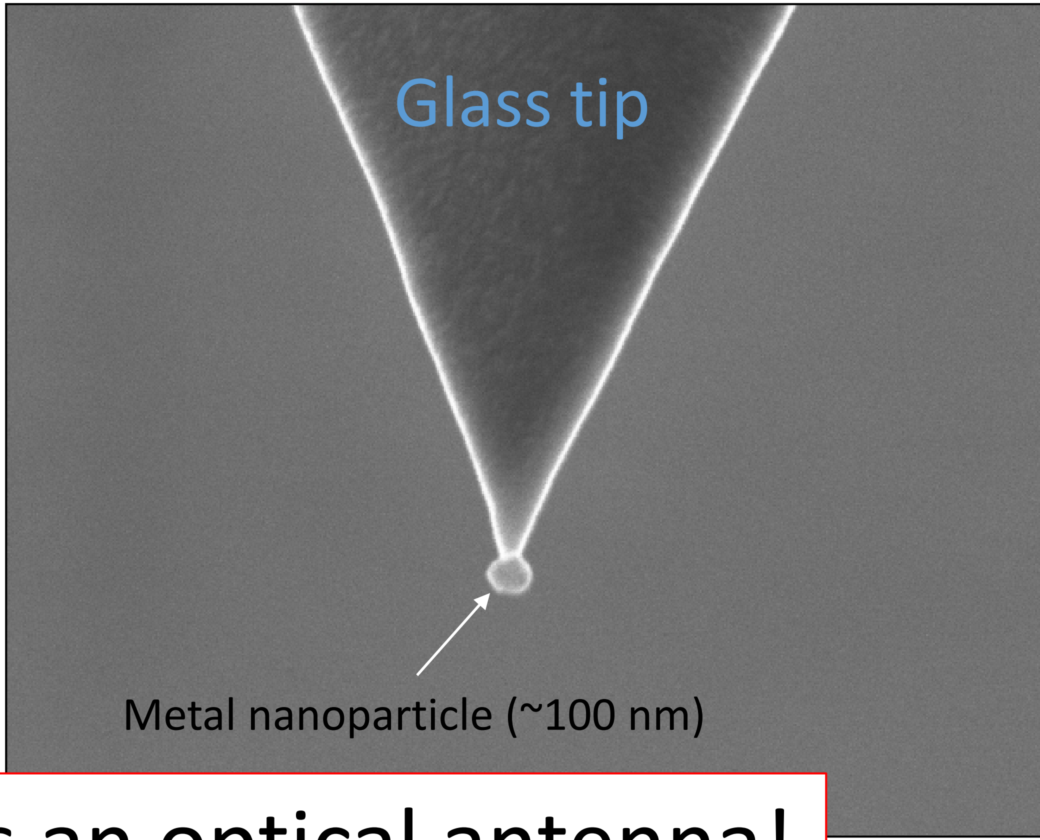


- L<<λ
- Illuminate with far field
- insert tip to scatter out near-field components into far-field detector
- Implementation of Synge’s idea: metal nano-particle at end of glass tip

Scattering NSOM

Schnell et al., Nature Photonics 3, 287 - 291 (2009)



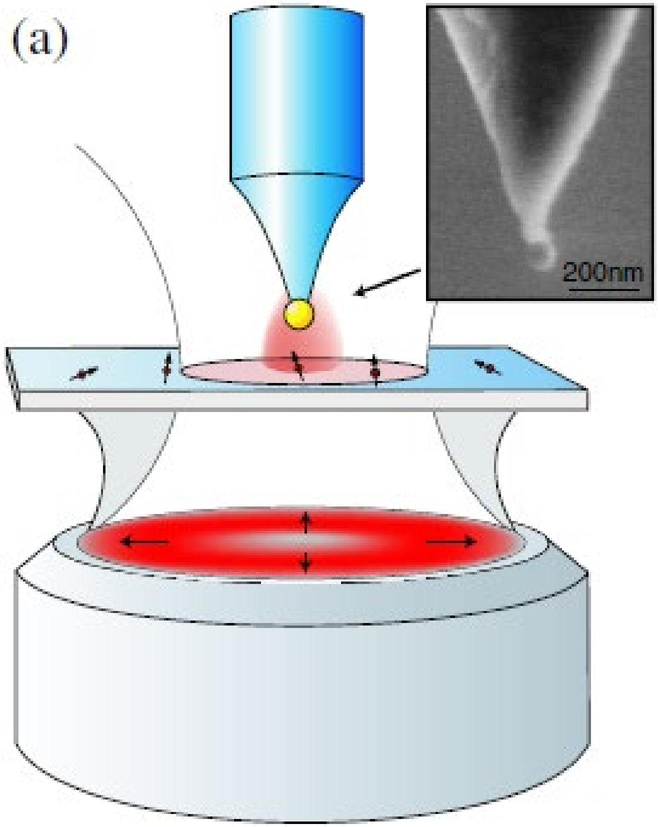


Particle acts as an optical antenna!

- L<<λ
- Illuminate with far field
- insert tip to scatter out near-field components into far-field detector
- Implementation of Synge's idea: metal nano-particle at end of glass tip

A metal nanoparticle as an optical antenna

Anger et al., PRL **96**, 113002 (2006)

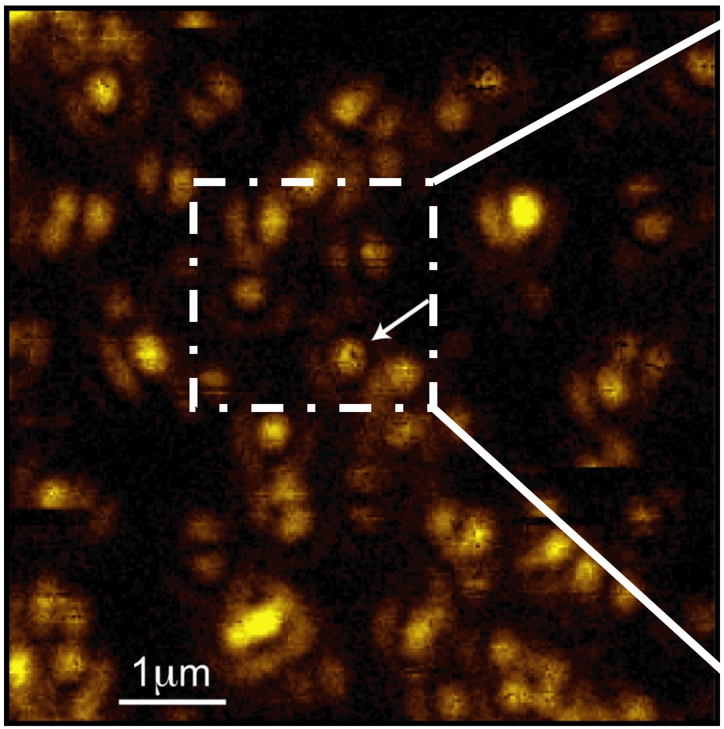


- Particle gets polarized by pump field and generates large local (dipolar) field
- Scan tip over sample with single fluorescing molecules

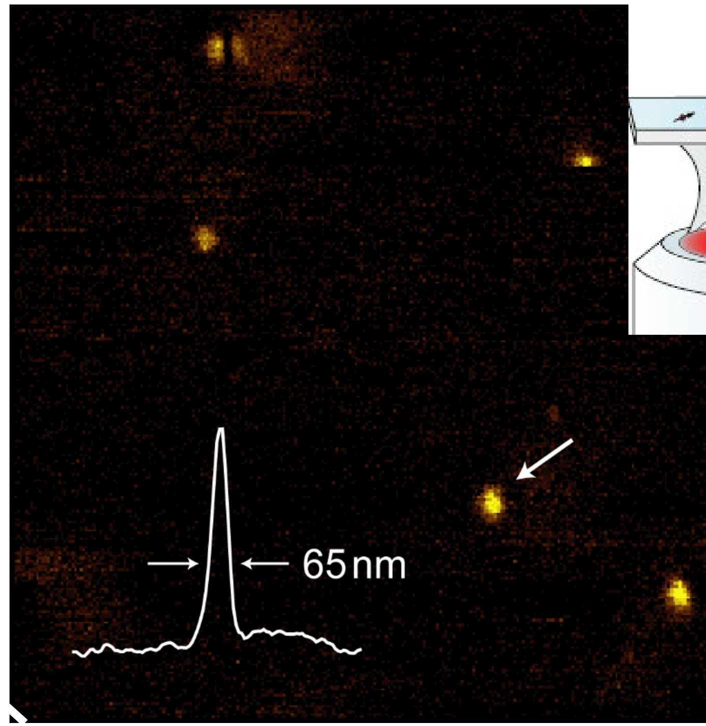
A metal nanoparticle as an optical antenna

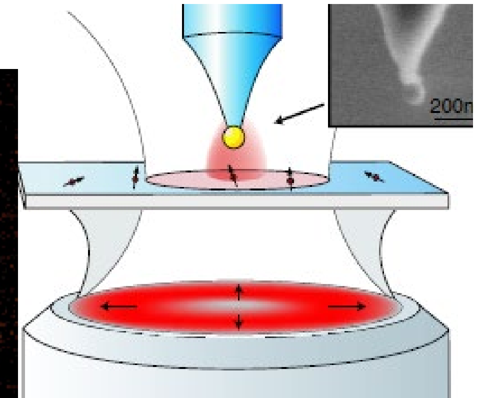
Anger et al., PRL **96**, 113002 (2006)

without Au particle :



with Au particle antenna:





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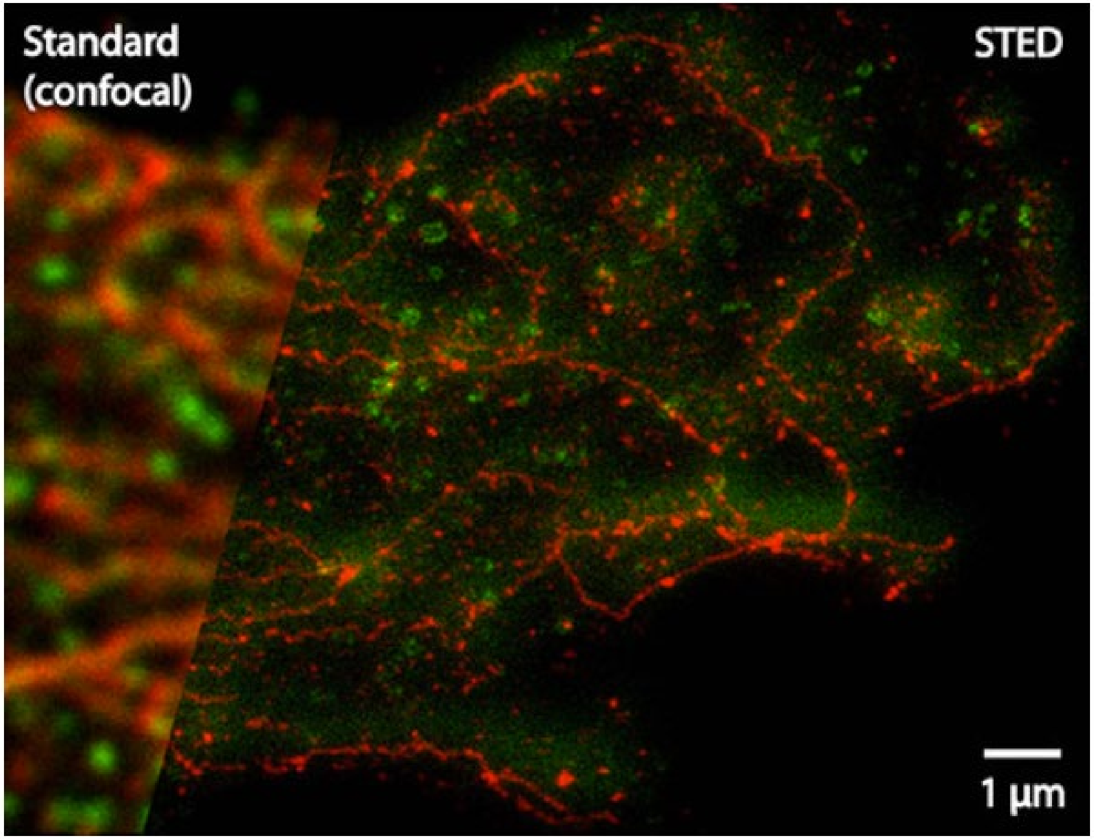
So far…

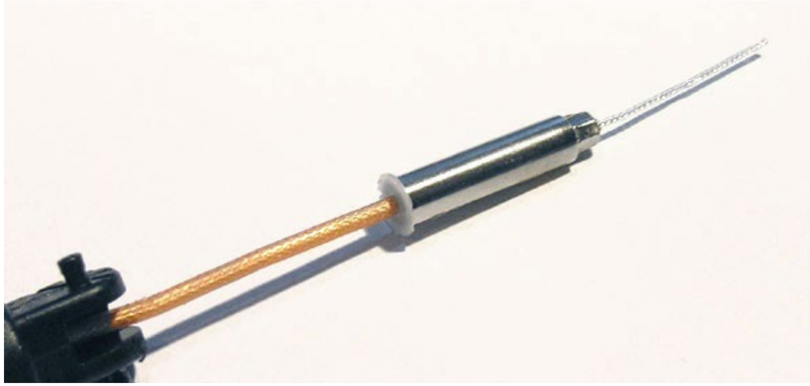


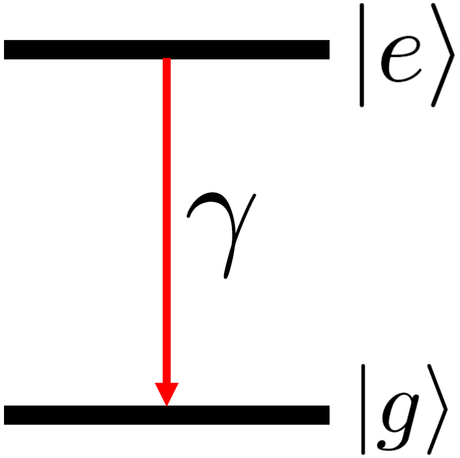
- So far, light emitters just reported their position
- But there is more: light emitters probe their local environment

On the menu

Radiation sources

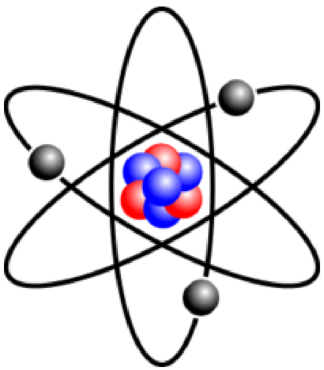
- The electric dipole
- Green function
- Fields of electric dipole
- Power dissipated by an oscillating dipole
- The local density of optical states (LDOS)
- Decay rate of quantum emitters
- Decay rate engineering



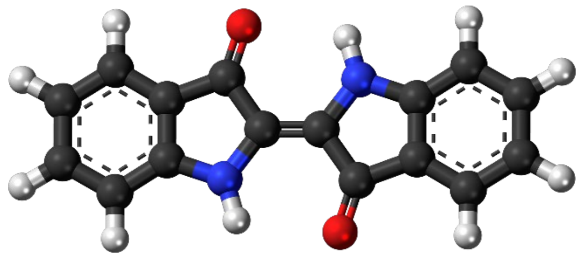


Radiation sources

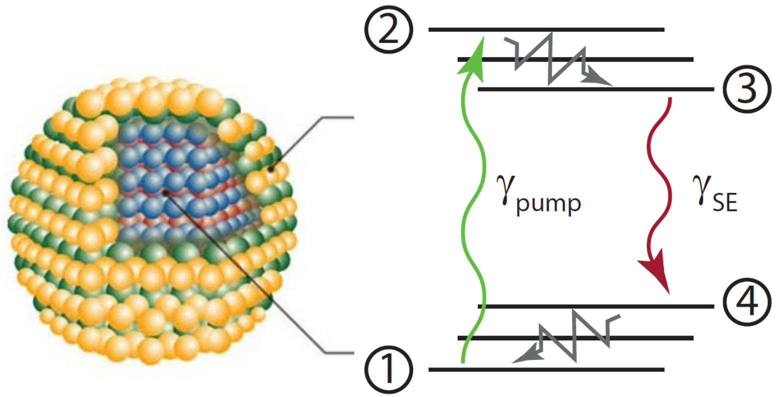
Radiating sources at 1000 THz (visible):



Atoms



Dye molecules

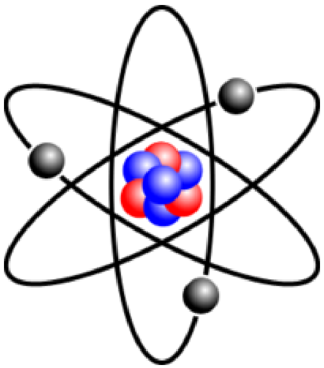


Quantum dots

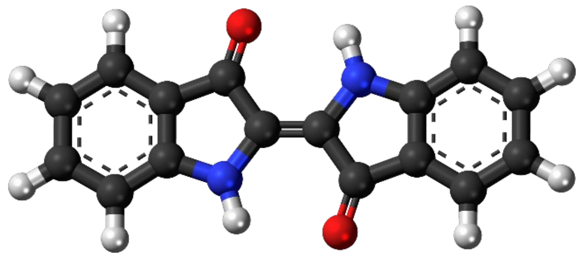
Wikimedia; Emory.edu

Radiation sources

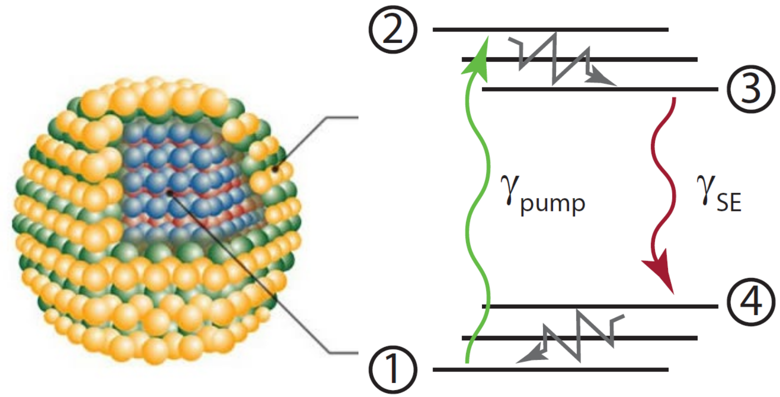
Radiating sources at 1000 THz (visible):



Atoms

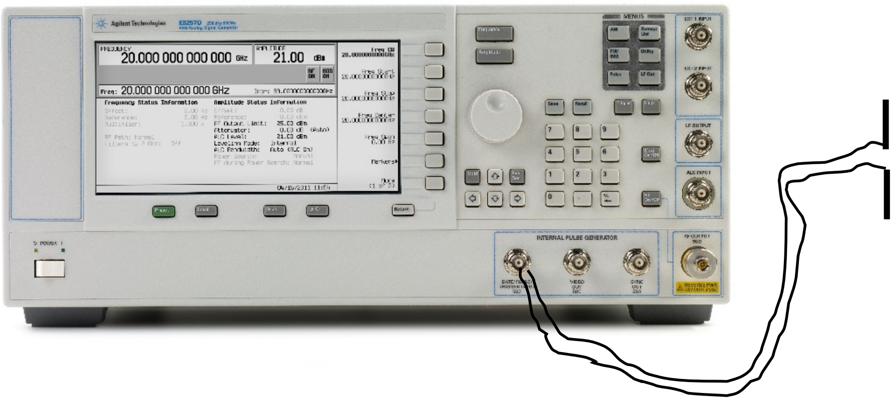


Dye molecules



Quantum dots

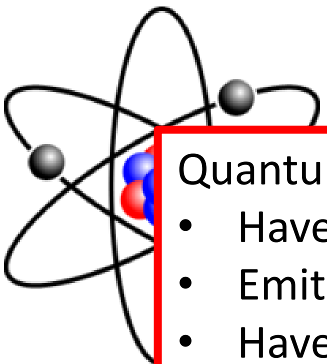
Radiating source up to GHz:

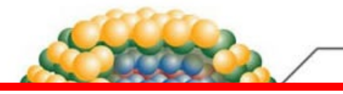


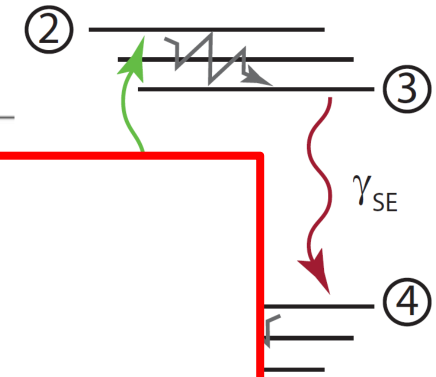
Wikimedia; Emory.edu

Radiation sources

Radiating sources at 1000 THz (visible):







At

Quantum emitters:

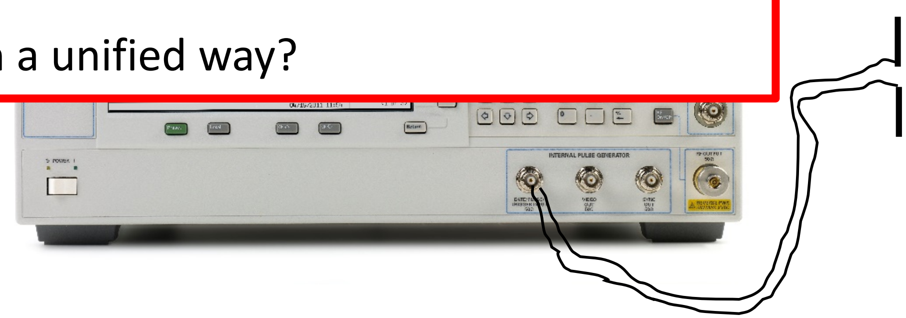
- Have a discrete level scheme
- Emit one photon at a time
- Have a fluorescence lifetime (= inverse of decay rate γ_{SE})

Classical radiation sources:

- Are driven by a voltage/current source with tunable frequency
- May have a resonance given by their geometry

Can we think about these sources in a unified way?

Radi



Wikimedia; Emory.edu

Where does radiation come from?

- From the source terms in the inhomogeneous wave equation

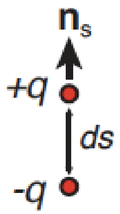
$$\nabla \times \nabla \times \mathbf{E} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = -\mu_0 \frac{\partial}{\partial t}\left(\mathbf{j} + \frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M}\right)$$

- In the monochromatic case (remember HW1)

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) - k^2 \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0\mu(\omega)\boldsymbol{j}_0(\boldsymbol{r})$$

For which source current distribution **j**(r) should we solve this equation?

The oscillating dipole



$$\mathbf{p}(t) = q(t)\, ds \quad \longrightarrow \quad \mathbf{j}(\mathbf{r},t) = \frac{\partial}{\partial t}\mathbf{p}(t)\,\delta(\mathbf{r}-\mathbf{r}_0)$$

Harmonic time dependence:

$$\mathbf{p}(t) = \mathrm{Re}\{\mathbf{p}\,\exp[-i\omega t]\} \quad \longrightarrow \quad \mathbf{j}(\mathbf{r}) = -i\omega\,\mathbf{p}\,\delta(\mathbf{r}-\mathbf{r}_0)$$

An oscillating dipole is a point-like time-harmonic current source.

The Green function of the wave equation

$$\nabla \times \nabla \times \overset{\leftrightarrow}{\boldsymbol{G}}(\boldsymbol{r}) - k^2 \overset{\leftrightarrow}{\boldsymbol{G}}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0\mu(\omega)\underbrace{(-\mathrm{i}\omega p)\, \mathbb{1}\, \delta(\boldsymbol{r} - \boldsymbol{r}')}_{\mathbf{j}(\mathbf{r}) \;=\; -i\omega\, \mathbf{p}\, \delta(\mathbf{r} - \mathbf{r}_0)}$$

With **G** we can calculate the field distribution E of any current distribution **j**!

$$\mathbf{E}(\mathbf{r}) \;=\; i\,\omega\mu_0\mu \int_V \overset{\leftrightarrow}{\mathbf{G}}_0(\mathbf{r}, \mathbf{r}')\, \mathbf{j}(\mathbf{r}')\, dV'$$

The Green function of the wave equation

$$\mathbf{E}(\mathbf{r}) \;=\; i\,\omega\mu_0\mu \int_V \overleftrightarrow{\mathbf{G}}_0\,(\mathbf{r},\mathbf{r}')\,\mathbf{j}(\mathbf{r}')\,dV' \qquad \mathbf{r} \notin V$$

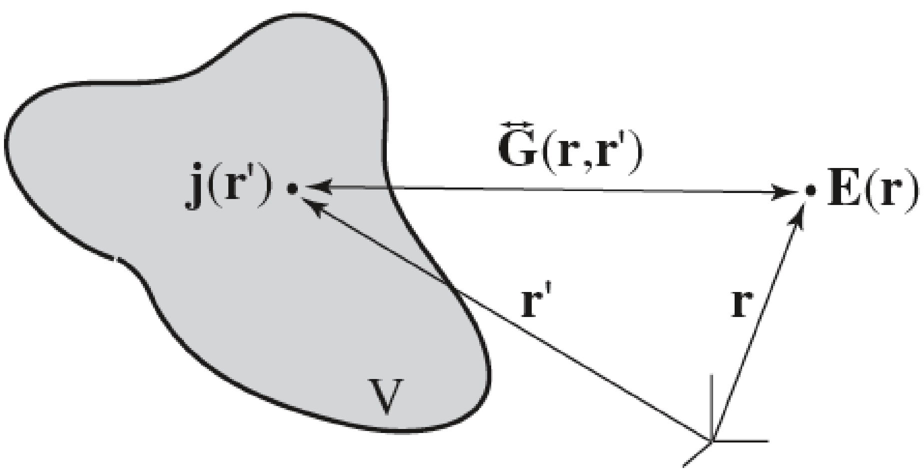
For dipole: $\mathbf{j}(\mathbf{r}) \;=\; -i\omega\,\mathbf{p}\,\delta(\mathbf{r}-\mathbf{r}_0)$

[redacted]

The Green function of the wave equation

$$\mathbf{E}(\mathbf{r}) \;=\; i\,\omega\mu_0\mu \int_V \overleftrightarrow{\mathbf{G}}_0\,(\mathbf{r},\mathbf{r}')\;\mathbf{j}(\mathbf{r}')\,dV' \qquad \mathbf{r} \notin V$$

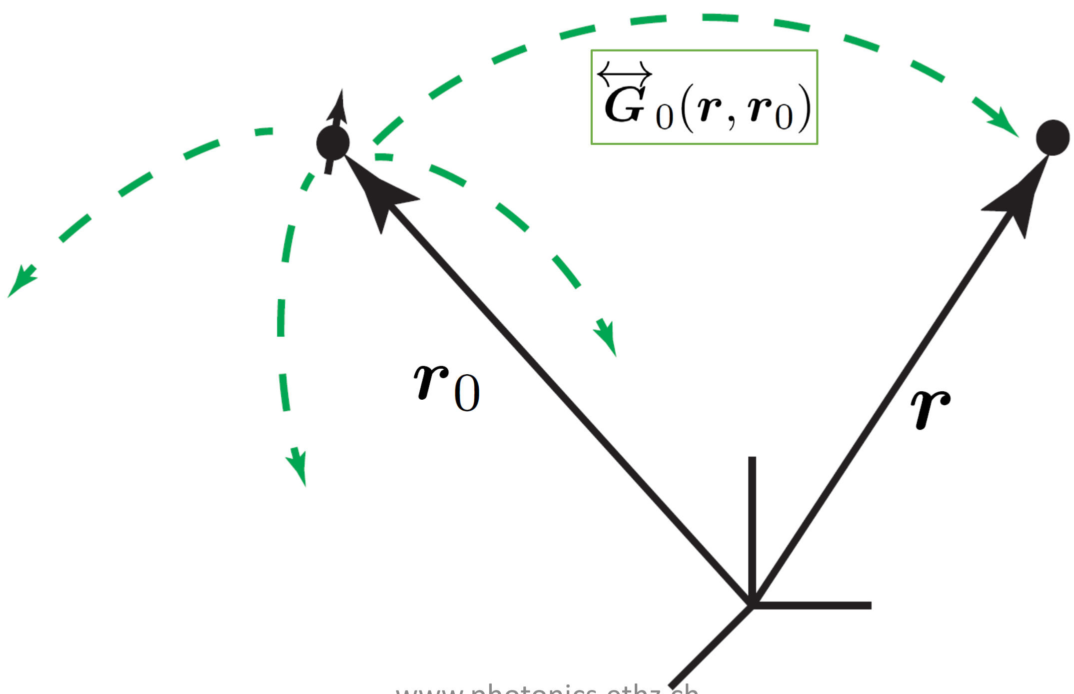
For dipole: $\mathbf{j}(\mathbf{r}) \;=\; -i\omega\,\mathbf{p}\,\delta(\mathbf{r}-\mathbf{r}_0)$ $\longrightarrow$ $\mathbf{E}(\mathbf{r}) \;=\; \omega^2\mu_0\mu\,\overleftrightarrow{\mathbf{G}}\,(\mathbf{r},\mathbf{r}_0)\;\mathbf{p}$



The Green function

$$\mathbf{E}(\mathbf{r}) \;=\; \omega^2 \mu_0 \mu \, \overleftrightarrow{\mathbf{G}}(\mathbf{r}, \mathbf{r}_0) \, \mathbf{p}$$

Field at **r** generated by dipole at $\mathbf{r}_0$



The Green function of free space

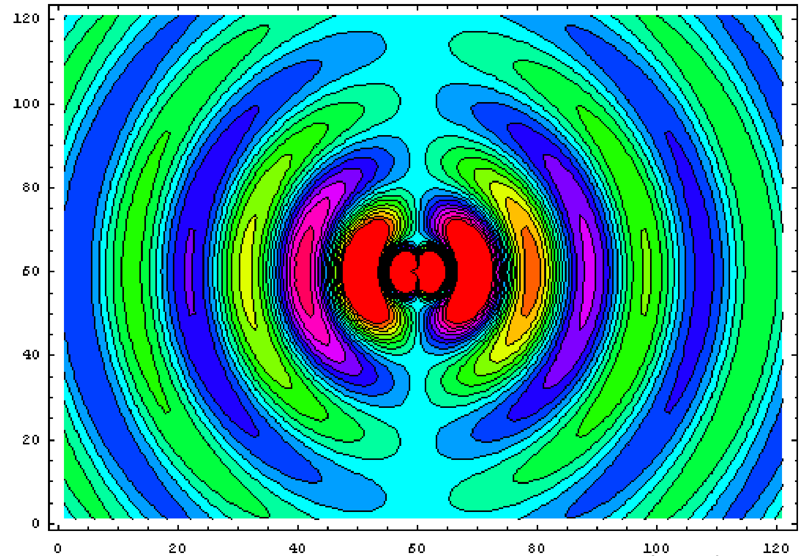
$$\nabla \times \nabla \times \overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}) - k^2 \overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0\mu(\omega)(-\mathrm{i}\omega p)\,\mathbb{1}\,\delta(\boldsymbol{r}-\boldsymbol{r}')$$

In cartesian coordinates and in a linear, homogeneous and isotropic medium (see EM notes for derivation):

$$\overleftrightarrow{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = \frac{\exp[\mathrm{i}kR]}{4\pi R}\left[\left(1+\frac{\mathrm{i}kR-1}{k^2R^2}\right)\overleftrightarrow{\boldsymbol{I}} + \frac{3-3\mathrm{i}kR-k^2R^2}{k^2R^2}\frac{\boldsymbol{R}\boldsymbol{R}}{R^2}\right]$$

with $R = |\mathbf{r}-\mathbf{r}'|$

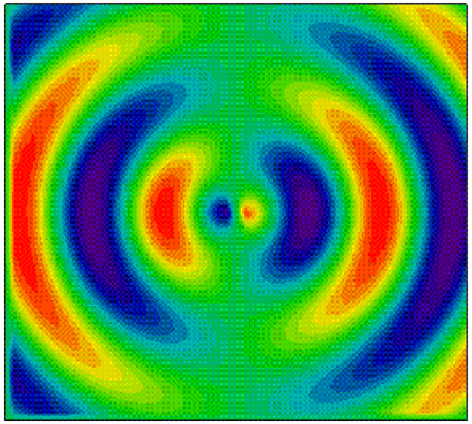
Dipole fields



Wikipedia.org

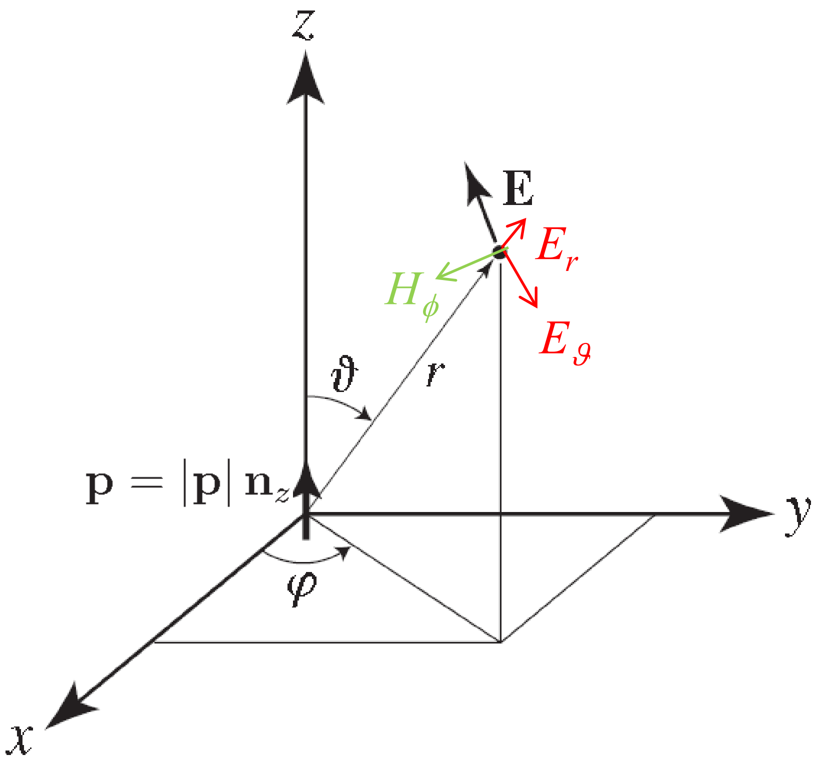
- Polarization
- Radiation pattern
- Near-field vs. far-field

Dipole fields for z-oriented dipole



$$E_r = \frac{|\mathbf{p}|\cos\vartheta}{4\pi\varepsilon_0\varepsilon}\,\frac{\exp(\mathrm{i}kr)}{r}\,k^2\left[\frac{2}{k^2r^2} - \frac{2\mathrm{i}}{kr}\right],$$

NF IF



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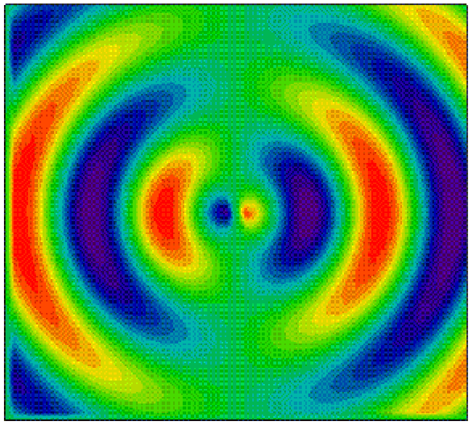


NF IF

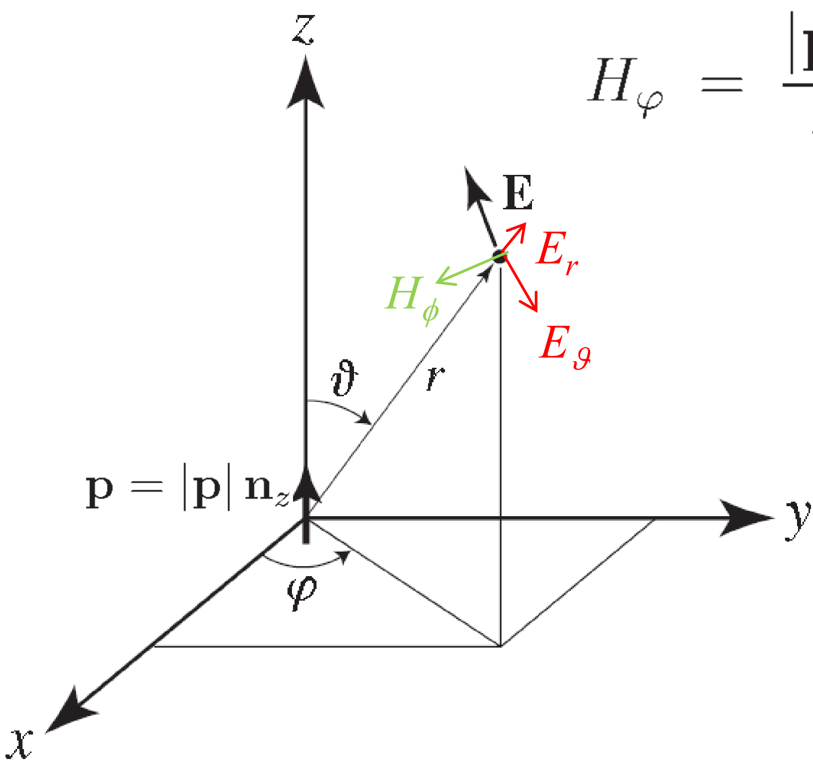
$$E_r = \frac{|\mathbf{p}|\cos\vartheta}{4\pi\varepsilon_0\varepsilon}\,\frac{\exp(ikr)}{r}\,k^2\left[\frac{2}{k^2r^2} - \frac{2i}{kr}\right],$$

NF IF FF

$$E_\vartheta = \frac{|\mathbf{p}|\sin\vartheta}{4\pi\varepsilon_0\varepsilon}\,\frac{\exp(ikr)}{r}\,k^2\left[\frac{1}{k^2r^2} - \frac{i}{kr} - 1\right],$$

IF FF

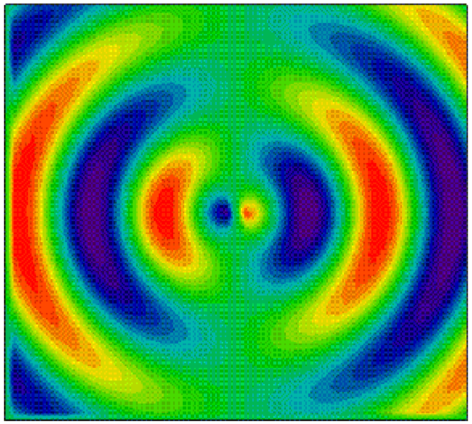
$$H_\varphi = \frac{|\mathbf{p}|\sin\vartheta}{4\pi\varepsilon_0\varepsilon}\,\frac{\exp(ikr)}{r}\,k^2\left[-\frac{i}{kr} - 1\right]\sqrt{\frac{\varepsilon_0\varepsilon}{\mu_0\mu}}$$



NB:

- There is no magnetic near-field
- Far-fields are transverse
- Intermediate field is 90° out of phase with near- and far-field

Distance dependence of dipole fields



NF   IF

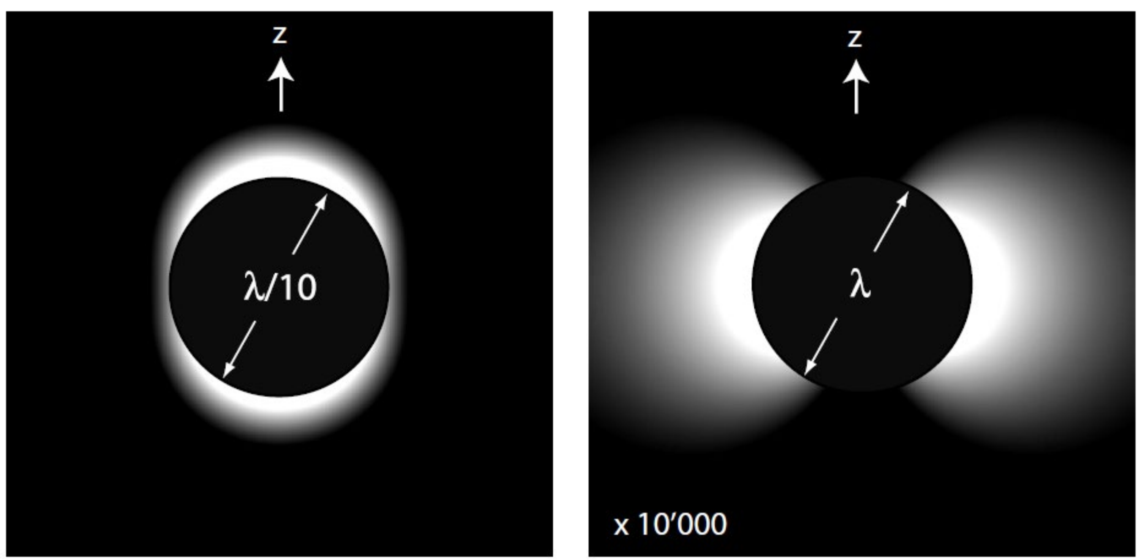
$$E_r = \frac{|\mathbf{p}|\cos\vartheta}{4\pi\varepsilon_0\varepsilon}\frac{\exp(ikr)}{r}k^2\left[\frac{2}{k^2r^2} - \frac{2i}{kr}\right],$$

NF   IF   FF

$$E_\vartheta = \frac{|\mathbf{p}|\sin\vartheta}{4\pi\varepsilon_0\varepsilon}\frac{\exp(ikr)}{r}k^2\left[\frac{1}{k^2r^2} - \frac{i}{kr} - 1\right],$$

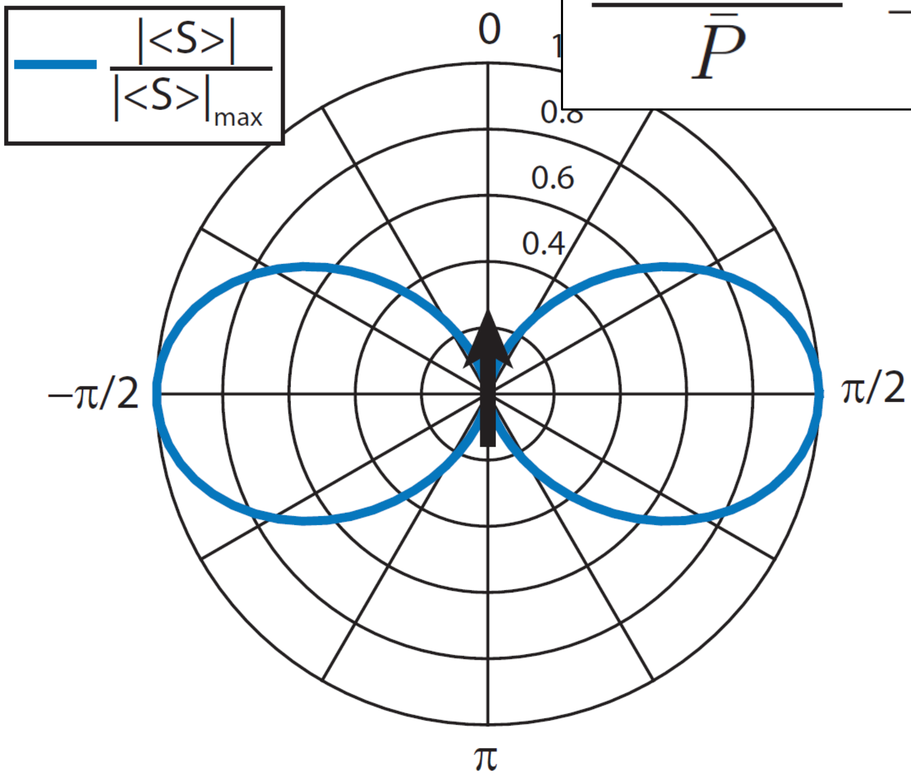
Caution: only far-field shown here!

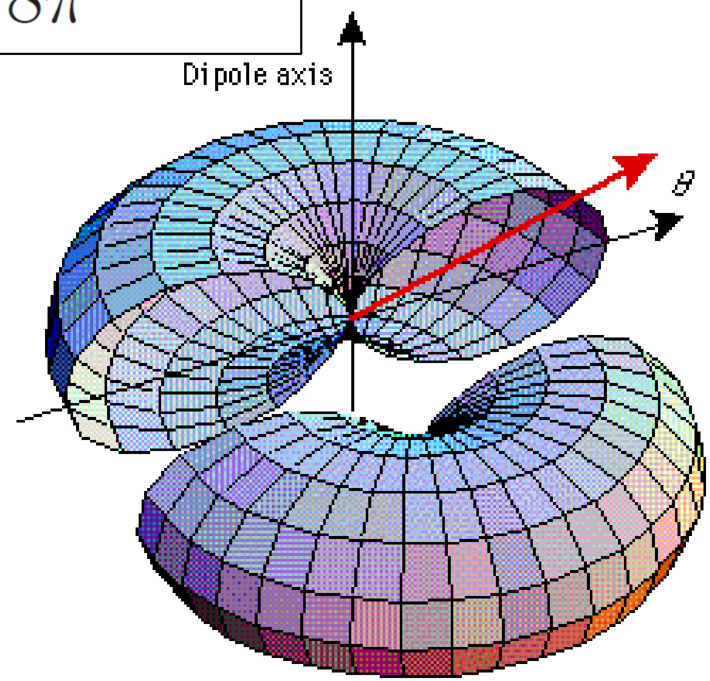
Time averaged energy density:



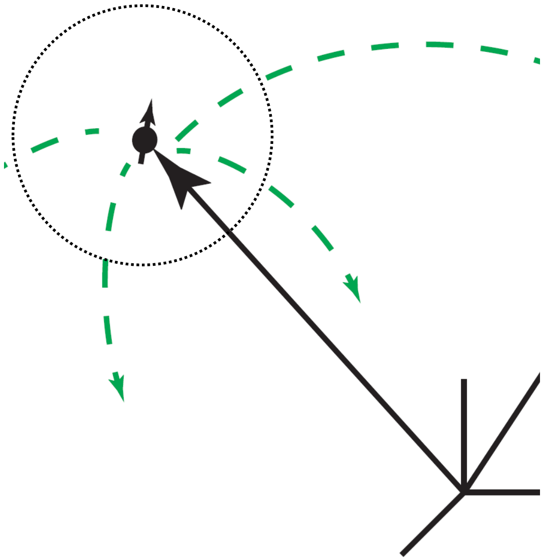
Dipole radiation pattern

$$\frac{\bar{P}(\vartheta,\varphi)}{\bar{P}} = \frac{3}{8\pi}\sin^2\vartheta$$





Power radiated by dipole in homogeneous medium



We calculated the power radiated by a dipole in free space by integrating the Poynting vector flux through a large sphere

$$\langle \boldsymbol{S}(\boldsymbol{r}) \rangle = \frac{1}{2} \mathrm{Re} \left[\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}^*(\boldsymbol{r}) \right]$$

$$P = \int_{\partial S} r^2 \sin\theta \mathrm{d}\theta \mathrm{d}\phi \, \boldsymbol{n}_r \langle \boldsymbol{S}(\boldsymbol{r}) \rangle = \frac{|p|^2 \quad}{12\pi\varepsilon_0\varepsilon}$$

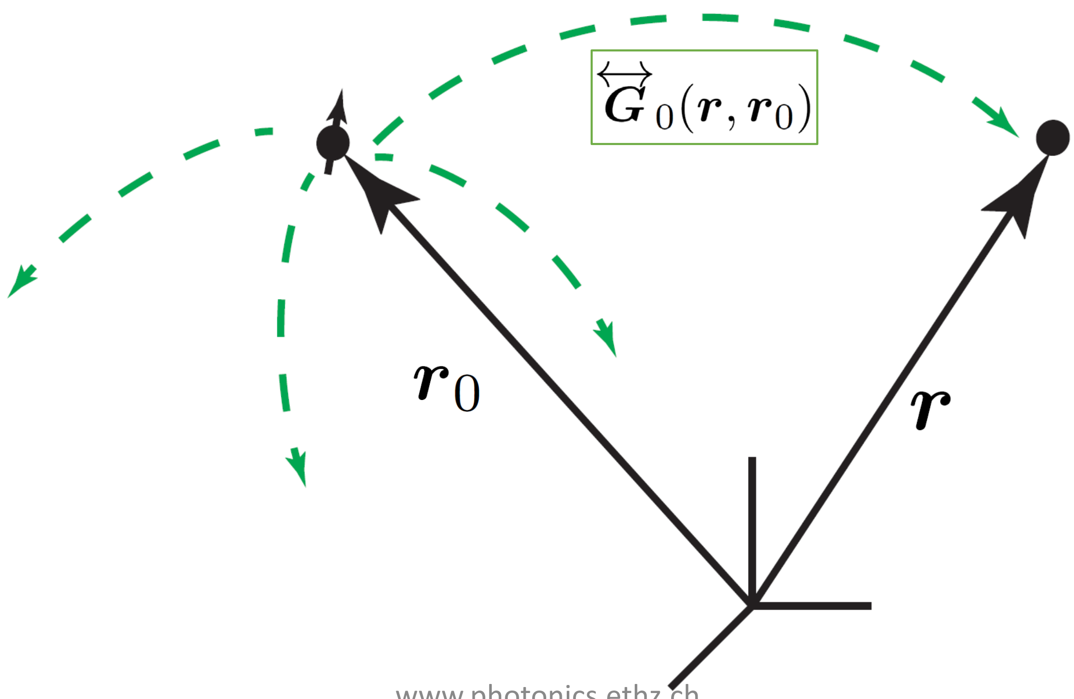
Radiated power depends on environment via refractive index!

The Green function

$$\mathbf{E}(\mathbf{r}) = \omega^2 \mu_0 \mu \overleftrightarrow{\mathbf{G}}(\mathbf{r}, \mathbf{r}_0)\, \mathbf{p}$$

Field at **r** generated by dipole at $\mathbf{r}_0$

In a homogeneous medium: $\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}, \boldsymbol{r}_0) = \overleftrightarrow{\boldsymbol{G}}_0(\boldsymbol{r}, \boldsymbol{r}_0)$

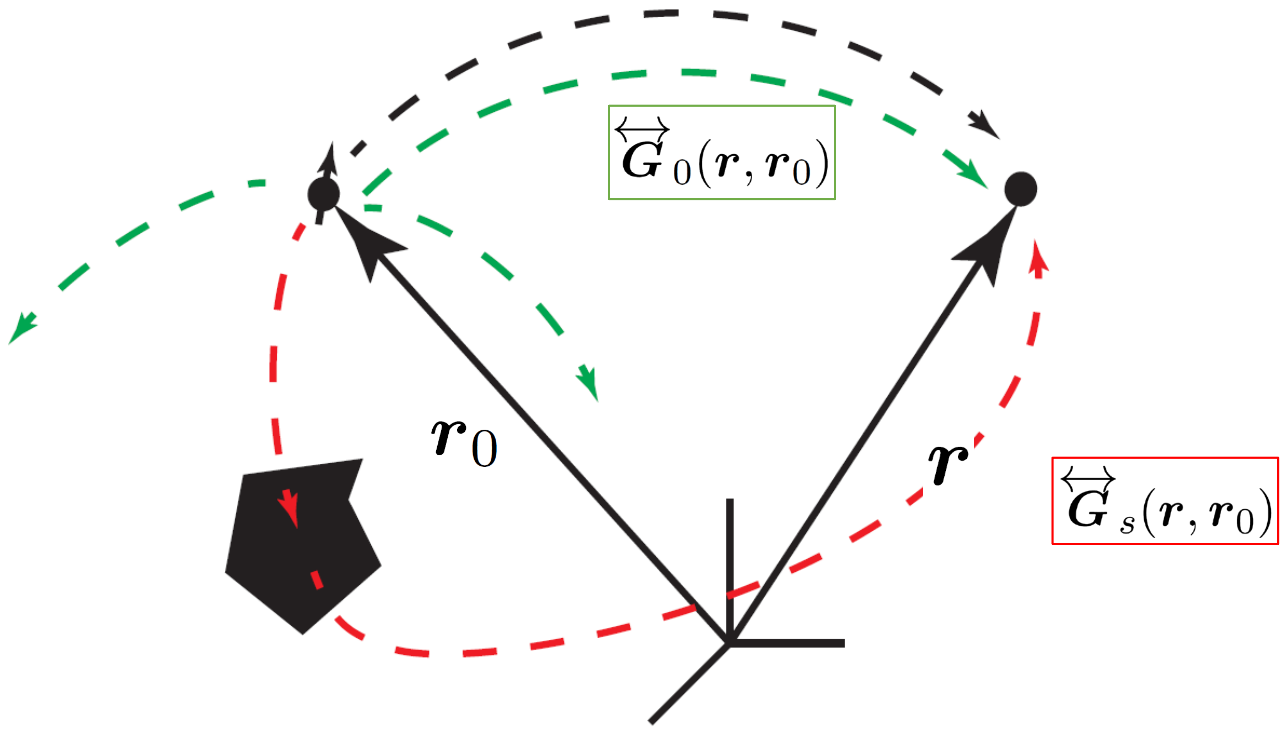


Fields in inhomogeneous environment

$$\mathbf{E}(\mathbf{r}) = \omega^2 \mu_0 \mu \overleftrightarrow{\mathbf{G}}(\mathbf{r}, \mathbf{r}_0)\, \mathbf{p}$$

Field at **r** generated by dipole at $\mathbf{r}_0$

In an inhomogeneous environment: $\overleftrightarrow{G}(\boldsymbol{r}, \boldsymbol{r}_0) = \overleftrightarrow{G}_0(\boldsymbol{r}, \boldsymbol{r}_0) + \overleftrightarrow{G}_s(\boldsymbol{r}, \boldsymbol{r}_0)$



Power radiated in inhomogeneous environment

$$\mathbf{E}(\mathbf{r}) = \omega^2 \mu_0 \mu \, \overleftrightarrow{\mathbf{G}}(\mathbf{r}, \mathbf{r}_0) \, \mathbf{p}$$

Field at **r** generated by dipole at **r**$_0$

In an inhomogeneous environment: $\overleftrightarrow{G}(\boldsymbol{r}, \boldsymbol{r}_0) = \overleftrightarrow{G}_0(\boldsymbol{r}, \boldsymbol{r}_0) + \overleftrightarrow{G}_s(\boldsymbol{r}, \boldsymbol{r}_0)$

Calculate Poynting vector flux through enclosing surface

Computationally costly!