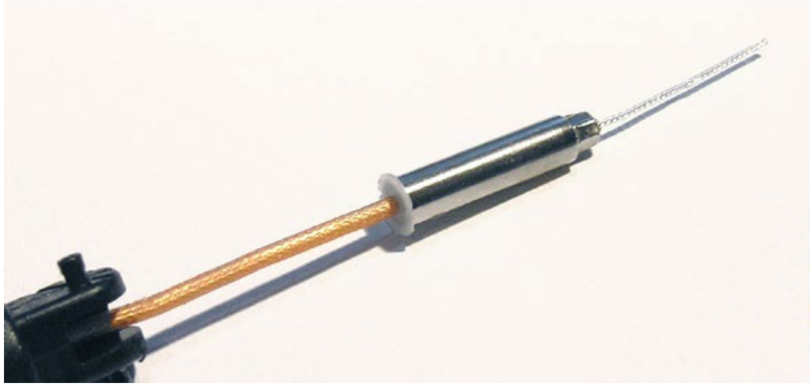
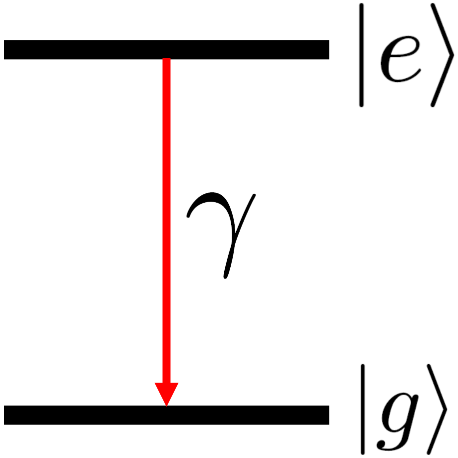
On the menu today

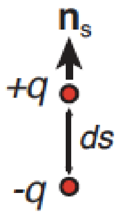
Radiation sources

- The electric dipole
- Green function
- Fields of electric dipole
- Power dissipated by an oscillating dipole
- The local density of optical states (LDOS)
- Decay rate of quantum emitters
- Decay rate engineering





A dipole – the elementary source of radiation



$$\mathbf{p}(t) = q(t)\, d\mathbf{s} \quad \longrightarrow \quad \boxed{\mathbf{j}(\mathbf{r},t) \;=\; \frac{\partial}{\partial t}\mathbf{p}(t)\,\delta(\mathbf{r}-\mathbf{r}_0)}$$

Harmonic time dependence:

$$\mathbf{p}(t) \;=\; \mathrm{Re}\{\mathbf{p}\,\exp[-i\omega t]\} \quad \longrightarrow \quad \mathbf{j}(\mathbf{r}) \;=\; -i\omega\,\mathbf{p}\,\delta(\mathbf{r}-\mathbf{r}_0)$$

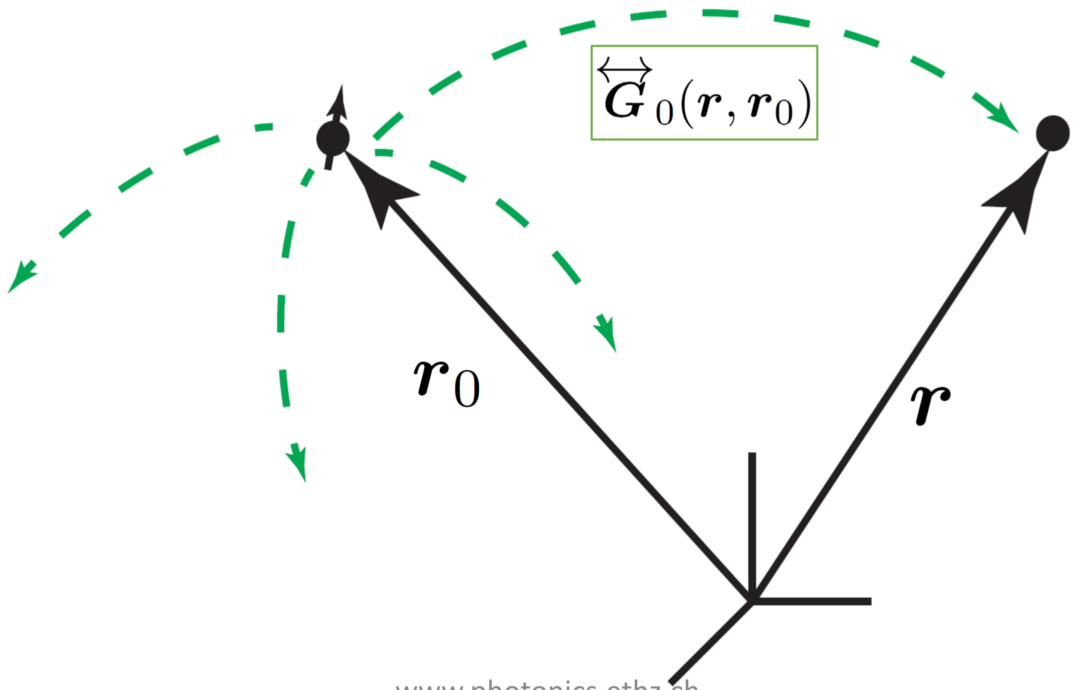
An oscillating dipole is a point-like time-harmonic current density.

The Green function

$$\mathbf{E}(\mathbf{r}) \;=\; \omega^2 \mu_0 \mu \, \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}_0)\,\mathbf{p}$$

Field at **r** generated by dipole at $\mathbf{r}_0$

In a homogeneous medium: $\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}_0) = \overleftrightarrow{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}_0)$

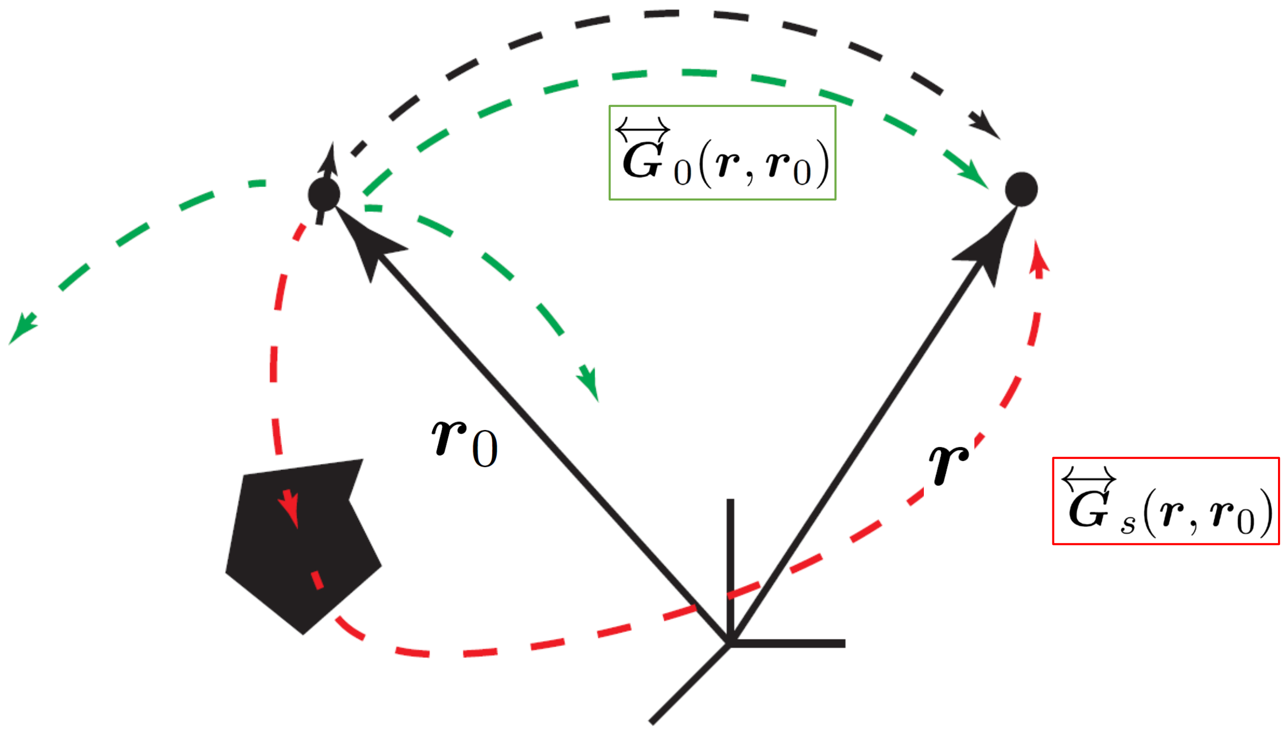


Fields in inhomogeneous environment

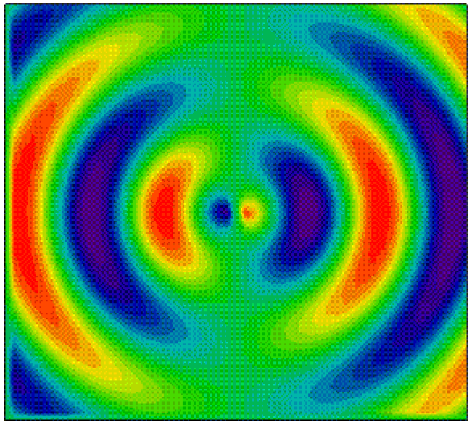
$$\mathbf{E}(\mathbf{r}) = \omega^2 \mu_0 \mu \, \overleftrightarrow{\mathbf{G}}(\mathbf{r}, \mathbf{r}_0)\, \mathbf{p}$$

Field at **r** generated by dipole at $\mathbf{r}_0$

In an inhomogeneous environment: $\overleftrightarrow{G}(\boldsymbol{r}, \boldsymbol{r}_0) = \overleftrightarrow{G}_0(\boldsymbol{r}, \boldsymbol{r}_0) + \overleftrightarrow{G}_s(\boldsymbol{r}, \boldsymbol{r}_0)$



Dipole fields in free space for z-oriented dipole



NF IF

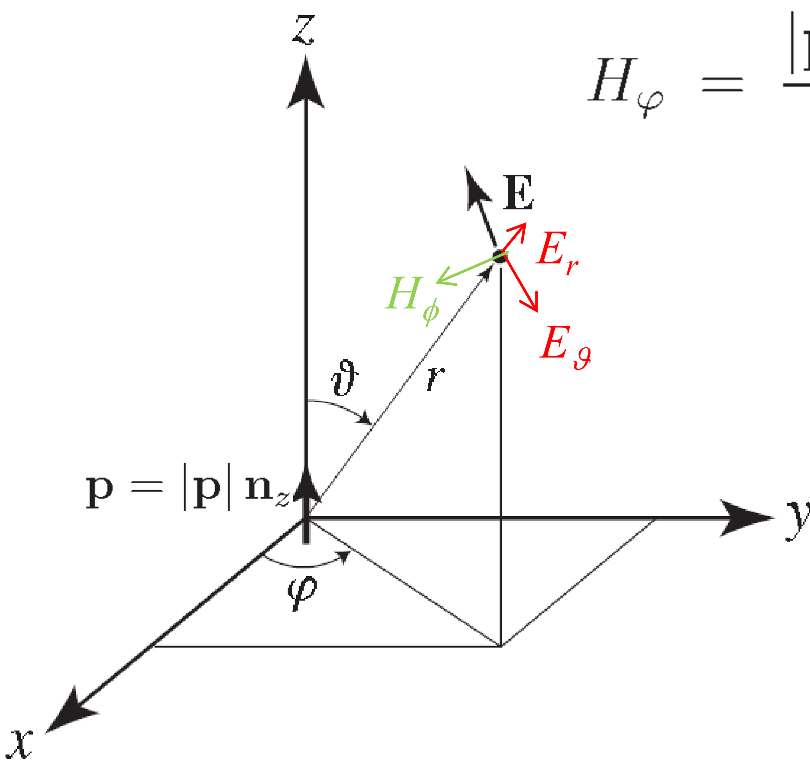
$$E_r = \frac{|\mathbf{p}|\cos\vartheta}{4\pi\varepsilon_0\varepsilon}\frac{\exp(\mathrm{i}kr)}{r}k^2\left[\frac{2}{k^2r^2} - \frac{2\mathrm{i}}{kr}\right],$$

NF IF FF

$$E_\vartheta = \frac{|\mathbf{p}|\sin\vartheta}{4\pi\varepsilon_0\varepsilon}\frac{\exp(\mathrm{i}kr)}{r}k^2\left[\frac{1}{k^2r^2} - \frac{\mathrm{i}}{kr} - 1\right],$$

IF FF

$$H_\varphi = \frac{|\mathbf{p}|\sin\vartheta}{4\pi\varepsilon_0\varepsilon}\frac{\exp(\mathrm{i}kr)}{r}k^2\left[-\frac{\mathrm{i}}{kr} - 1\right]\sqrt{\frac{\varepsilon_0\varepsilon}{\mu_0\mu}}$$



NB:

- There is no magnetic near-field
- Far-fields are transverse
- Intermediate field is 90° out of phase with near- and far-field

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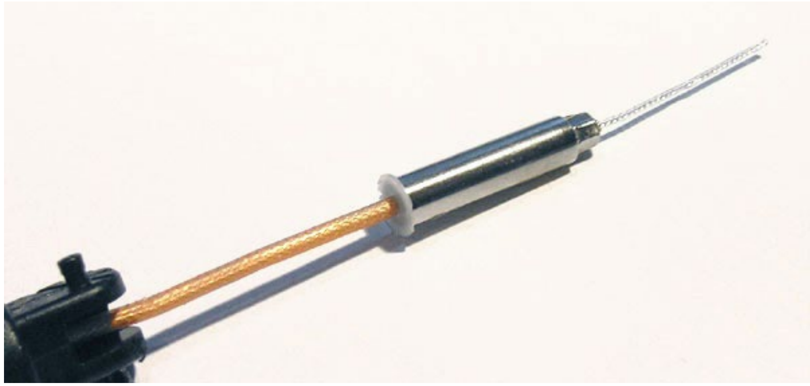
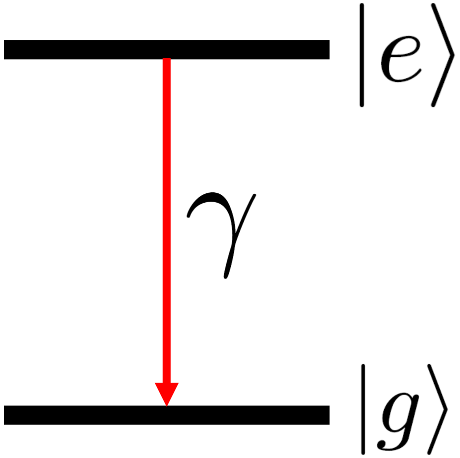




Power radiated in inhomogeneous environment

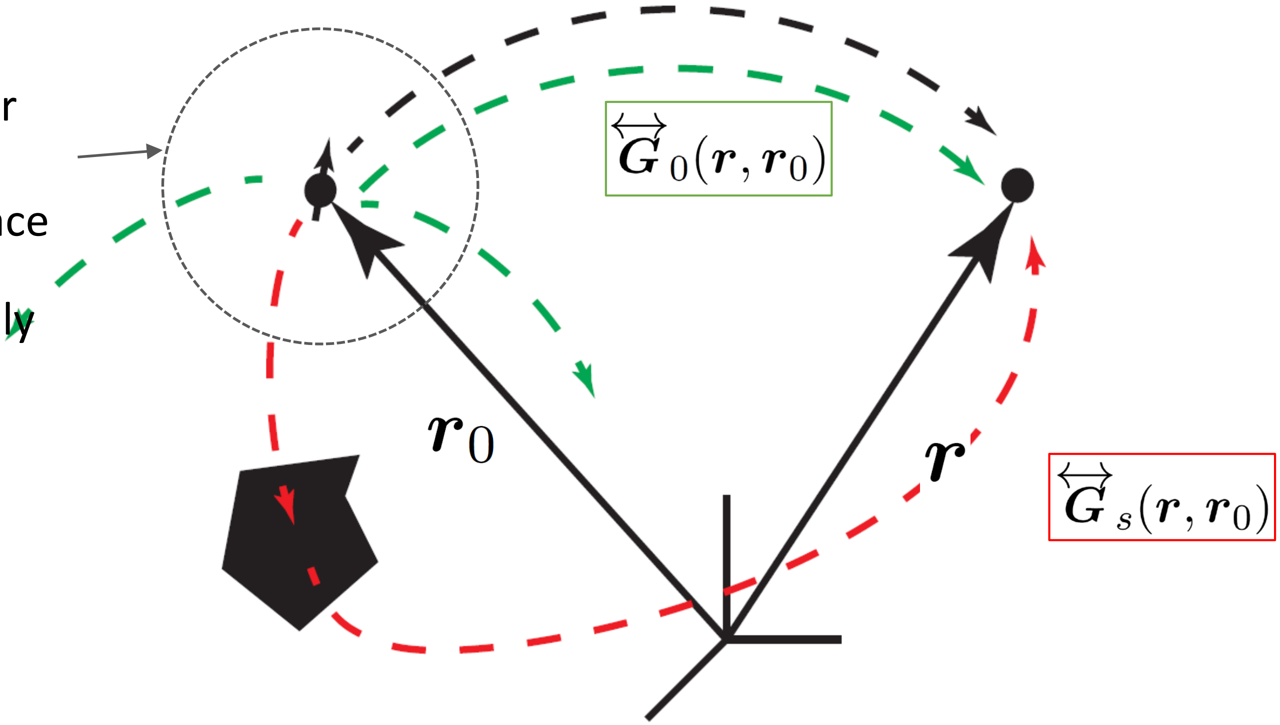
$$\mathbf{E}(\mathbf{r}) \;=\; \omega^2 \mu_0 \mu \, \overleftrightarrow{\mathbf{G}}\,(\mathbf{r}, \mathbf{r}_0)\; \mathbf{p}$$

Field at **r** generated by dipole at $\mathbf{r}_0$

In an inhomogeneous environment: $\overleftrightarrow{G}(\boldsymbol{r}, \boldsymbol{r}_0) = \overleftrightarrow{G}_0(\boldsymbol{r}, \boldsymbol{r}_0) + \overleftrightarrow{G}_s(\boldsymbol{r}, \boldsymbol{r}_0)$

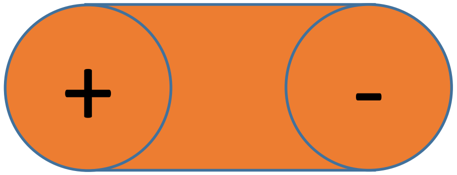
Calculate
Poynting vector
flux through
enclosing surface

Computationally
costly!



Power radiated by a dipole

What is the resistance we have to work against to keep the dipole current oscillating?



Power dissipated in volume V (c.f. Poynting's theorem):

$$P = \frac{\mathrm{d}W}{\mathrm{d}t} = \int \mathrm{d}V\, \boldsymbol{j}(\boldsymbol{r},t)\cdot \boldsymbol{E}(\boldsymbol{r},t)$$

Cycle averaged (monochromatic case):

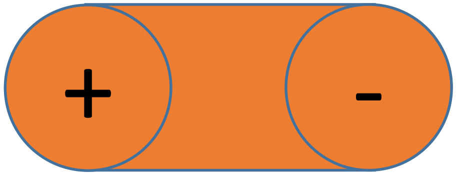
$$\langle P \rangle = \frac{\omega}{2} \mathrm{Im}\left[\boldsymbol{p}^* \cdot \boldsymbol{E}(\boldsymbol{r}_0)\right]$$

$$\langle P \rangle = \frac{\omega}{2} |\boldsymbol{p}|^2 \left\{\boldsymbol{n}_p^{\intercal}\, \mathrm{Im}\left[\omega^2 \mu_0 \mu \overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_0, \boldsymbol{r}_0; \omega)\right] \boldsymbol{n}_p\right\}$$

The energy radiated by a dipole equals the work done by the dipole's own field on the dipole itself!

Power radiated by a dipole

What is the resistance we have to work against to keep the dipole current oscillating?



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Cycle averaged (monochromatic case):

$$\langle P\rangle = \frac{\omega}{2}\mathrm{Im}\left[\boldsymbol{p}^{*}\cdot\boldsymbol{E}(\boldsymbol{r}_0)\right]$$

$$\langle P\rangle = \frac{\omega}{2}\left|\boldsymbol{p}\right|^{2}\left\{\boldsymbol{n}_p^{\intercal}\,\mathrm{Im}\left[\omega^{2}\mu_0\mu\,\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_0,\boldsymbol{r}_0;\omega)\right]\boldsymbol{n}_p\right\}$$

The environment determines the radiated power via the imaginary part of the GF at the origin.

Power radiated in inhomogeneous environment

$$\mathbf{E}(\mathbf{r}) = \omega^2 \mu_0 \mu \overleftrightarrow{\mathbf{G}}(\mathbf{r}, \mathbf{r}_0)\, \mathbf{p}$$

$$\boldsymbol{E}(\boldsymbol{r}_0) = \omega^2 \mu_0 \mu \overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_0, \boldsymbol{r}_0) \boldsymbol{p}$$

$$\overleftrightarrow{\boldsymbol{G}} = \overleftrightarrow{\boldsymbol{G}}_0 + \overleftrightarrow{\boldsymbol{G}}_s$$

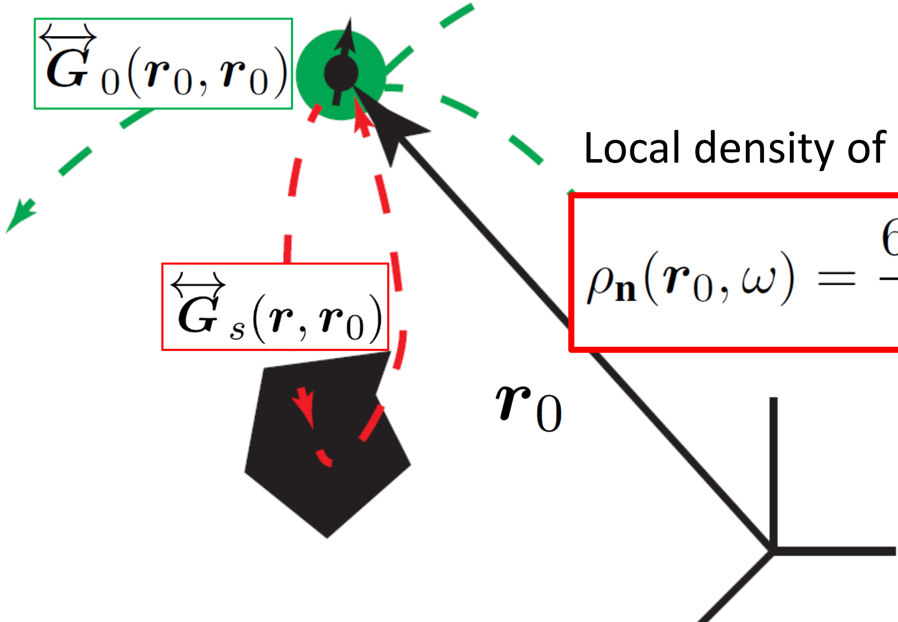
In an inhomogeneous environment:

$$\langle P \rangle = \frac{\omega}{2} \mathrm{Im}\left[\boldsymbol{p}^* \cdot \boldsymbol{E}(\boldsymbol{r}_0)\right]$$

Dipole does work against its own field!

$$\langle P \rangle = \frac{\omega}{2} |\boldsymbol{p}|^2 \left\{ \boldsymbol{n}_p^\intercal \, \mathrm{Im}\left[\omega^2 \mu_0 \mu \overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_0, \boldsymbol{r}_0)\right] \boldsymbol{n}_p \right\}$$

$$\langle P \rangle = \frac{\pi \omega^2}{12 \epsilon \epsilon_0} |\boldsymbol{p}|^2 \; \rho_{\mathbf{n}}(\boldsymbol{r}_0, \omega)$$



Local density of optical states

$$\rho_{\mathbf{n}}(\boldsymbol{r}_0, \omega) = \frac{6 \omega n^2}{\pi c^2} \left\{ \boldsymbol{n}_p^\intercal \, \mathrm{Im}\left[\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_0, \boldsymbol{r}_0; \omega)\right] \boldsymbol{n}_p \right\}$$

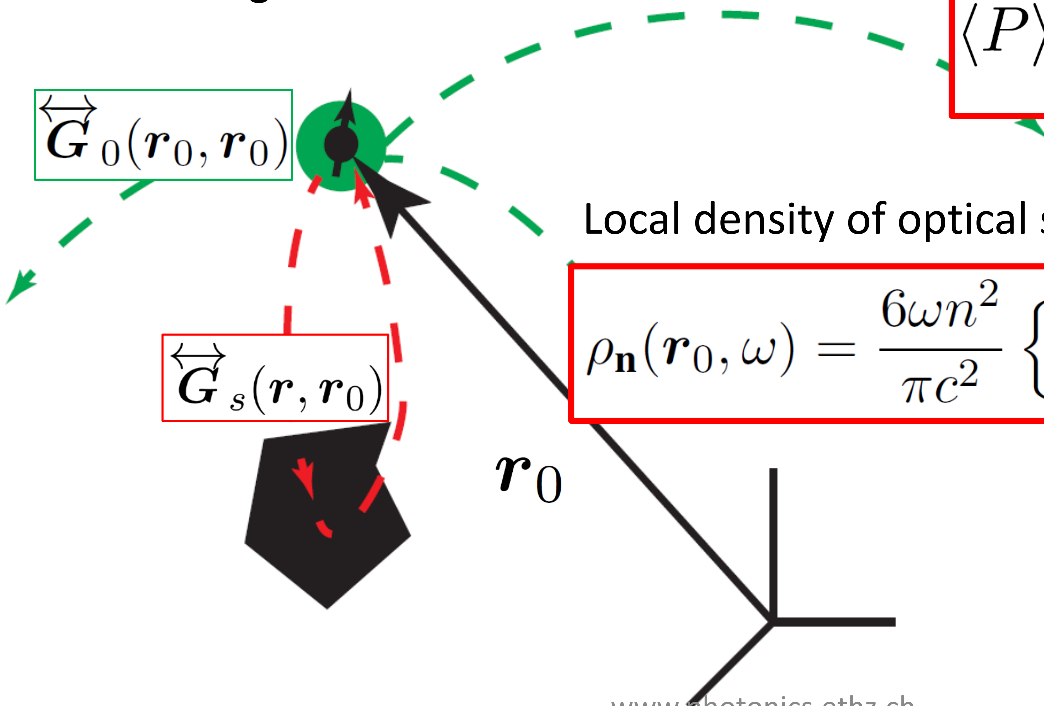
Prefactor differs from author to author

Power radiated in inhomogeneous environment

- The power dissipated by a dipole depends on it's environment and is proportional to the local density of optical states (LDOS).
- The LDOS is (besides prefactors) the imaginary part of the Green's function evaluated at the origin (i.e., the position of the dipole).
- Controlling the boundary conditions (and thereby the LDOS) allows us to control the power radiated by a dipole!

In an inhomogeneous environment:

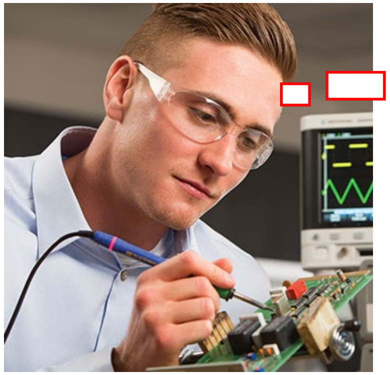
$$\langle P \rangle = \frac{\pi \omega^2}{12 \epsilon \epsilon_0} |\boldsymbol{p}|^2 \; \rho_{\mathbf{n}}(\boldsymbol{r}_0, \omega)$$



Local density of optical states

$$\rho_{\mathbf{n}}(\boldsymbol{r}_0, \omega) = \frac{6 \omega n^2}{\pi c^2} \left\{ \boldsymbol{n}_p^{\intercal} \, \mathrm{Im} \left[\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_0, \boldsymbol{r}_0; \omega) \right] \boldsymbol{n}_p \right\}$$

Power radiated in inhomogeneous environment

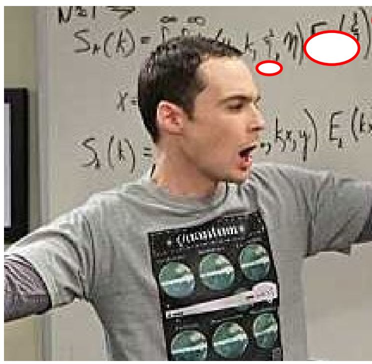


Radiation resistance!

Power dissipated in an electrical circuit:

$$P \propto I^2 \cdot R$$

current resistance

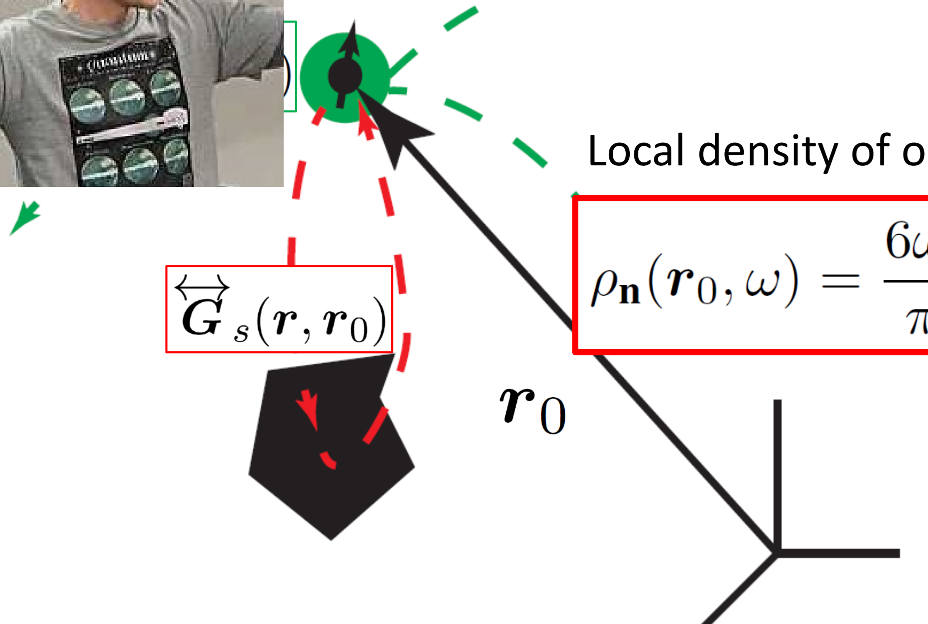


LDOS!

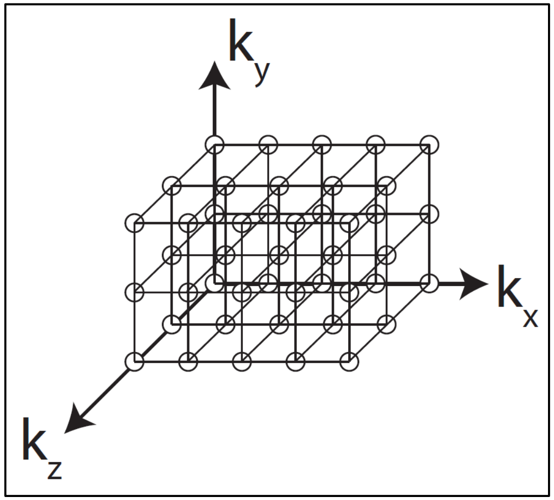
$$\langle P \rangle = \frac{\pi \omega^2}{12 \epsilon \epsilon_0} |\boldsymbol{p}|^2 \; \rho_{\mathbf{n}}(\boldsymbol{r}_0, \omega)$$

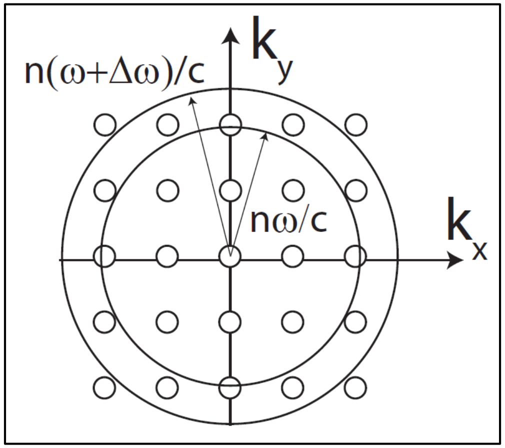
Local density of optical states

$$\rho_{\mathbf{n}}(\boldsymbol{r}_0, \omega) = \frac{6 \omega n^2}{\pi c^2} \left\{ \boldsymbol{n}_p^{\intercal} \operatorname{Im}\left[\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_0, \boldsymbol{r}_0; \omega) \right] \boldsymbol{n}_p \right\}$$



Why is it called density of states?



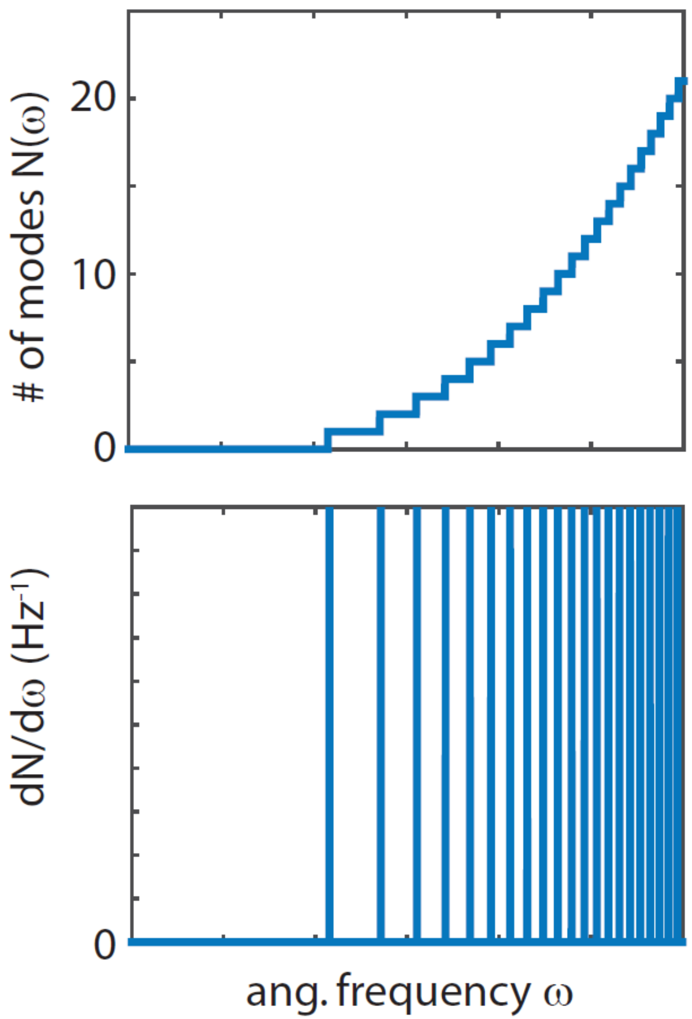


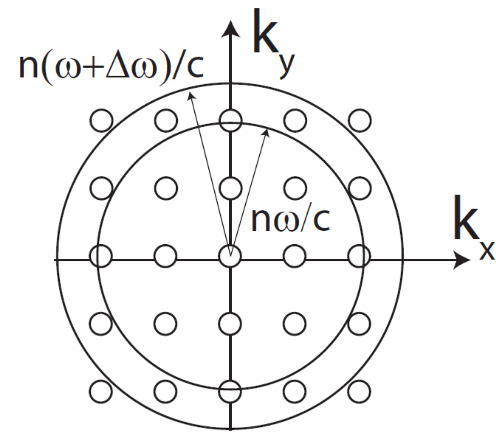
Expression “density of states” becomes clear when considering modes of resonator.

Determine DOS for free space by counting states in cubic resonator and letting resonator size become much larger than the wavelength.

Density of states in the lossless resonator

How many modes in frequency band [ω, ω+Δω] and resonator volume V?





$$\rho(\omega) = \frac{1}{V}\frac{dN(\omega)}{d\omega}$$

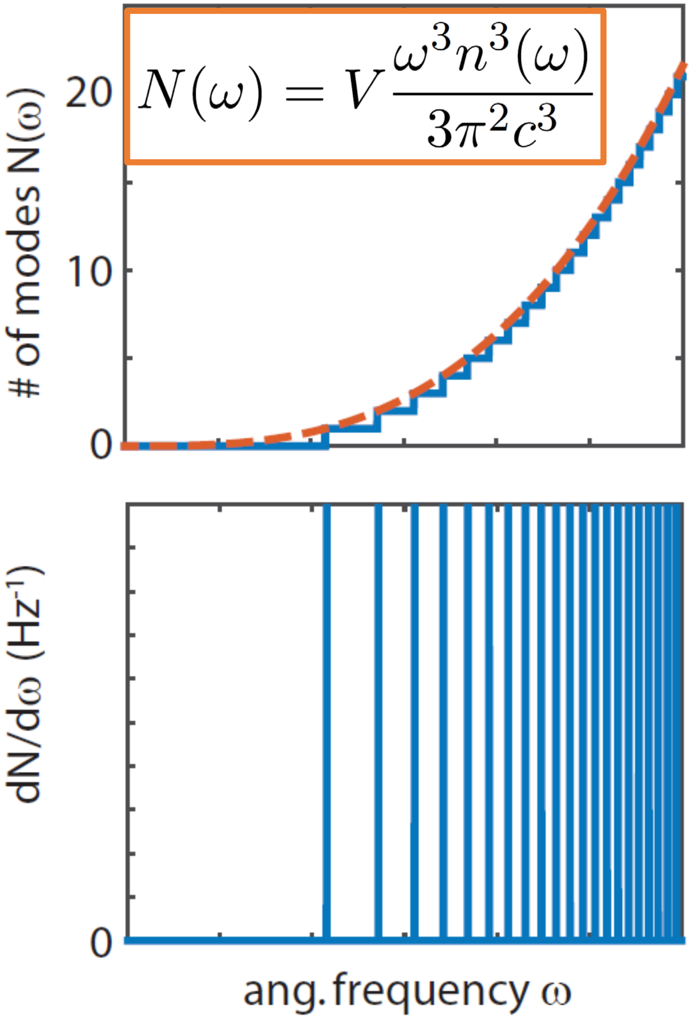
V: resonator volume

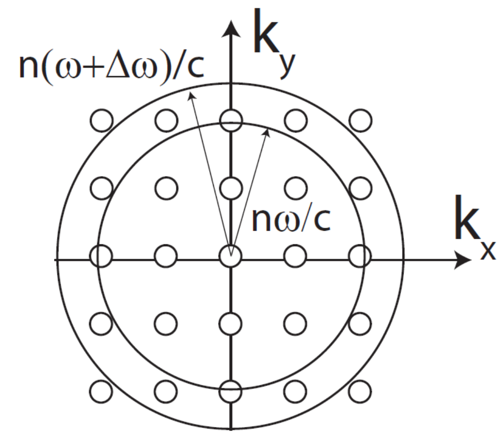
N: number of modes

Density of states in the lossless resonator

How many modes in frequency band [ω, ω+Δω] and resonator volume V?

$$N(\omega) = V\frac{\omega^3 n^3(\omega)}{3\pi^2 c^3}$$





$$\rho(\omega) = \frac{1}{V}\frac{dN(\omega)}{d\omega}$$

V: resonator volume

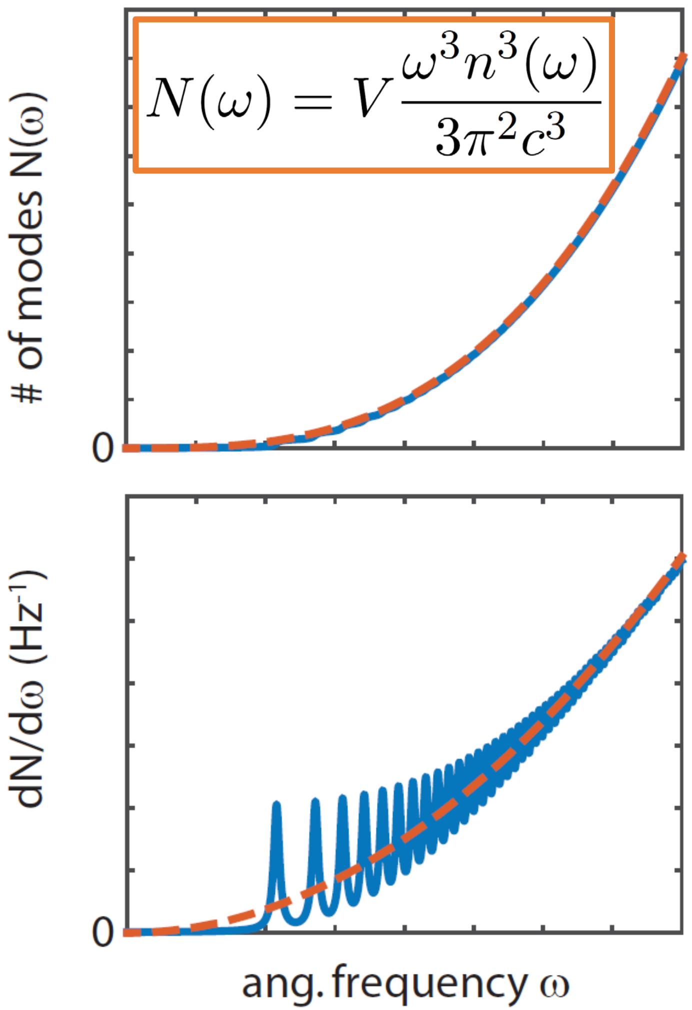
N: number of modes

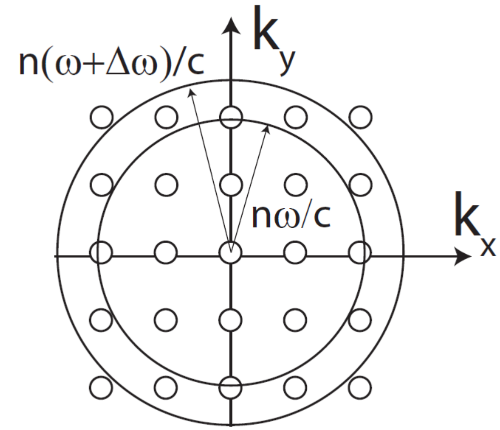
In free space (large resonator):

$$\rho(\omega) = \frac{\omega^2 n^3(\omega)}{\pi^2 c^3}$$

Density of states in a realistic resonator

How many modes in frequency band [ω, ω+Δω] and resonator volume V?





$$\rho(\omega) = \frac{1}{V}\frac{dN(\omega)}{d\omega}$$

V: resonator volume

N: number of modes

In free space (large resonator):

$$\rho(\omega) = \frac{\omega^2 n^3(\omega)}{\pi^2 c^3}$$

Power radiated in inhomogeneous environment

Via the local density of states (LDOS)

- Radiated power depends on location of source within its environment
- Radiated power depends on frequency of source
- Radiated power depends on orientation of source

The LDOS can be interpreted as a radiation resistance

$$\langle P \rangle = \frac{\pi \omega^2}{12 \epsilon \epsilon_0} |\boldsymbol{p}|^2 \ \rho_{\mathbf{n}}(\boldsymbol{r}_0, \omega)$$

In analogy with $P = I^2 \cdot R$

Power enhancement by photonic system

Normalize emitted power to power emitted in free space:

$$\frac{P}{P_0} = \frac{\boldsymbol{n}_p^\intercal \,\mathrm{Im}\left[\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_0, \boldsymbol{r}_0; \omega)\right] \boldsymbol{n}_p}{\boldsymbol{n}_p^\intercal \,\mathrm{Im}\left[\overleftrightarrow{\boldsymbol{G}}_0(\boldsymbol{r}_0, \boldsymbol{r}_0; \omega)\right] \boldsymbol{n}_p} = 1 + \frac{\boldsymbol{n}_p^\intercal \,\mathrm{Im}\, \overleftrightarrow{\boldsymbol{G}}_s \boldsymbol{n}_p}{\mathrm{Im}\, G_0}$$

Depending on the sign (phase) of the scattered field returning to the dipole, it enhances or suppresses power dissipation.

Warning: The term LDOS (enhancement) is used sloppily to refer to $\frac{P}{P_0}$, $\boldsymbol{n}_p^\intercal \,\mathrm{Im}\left[\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_0, \boldsymbol{r}_0; \omega)\right] \boldsymbol{n}_p$, $\frac{\boldsymbol{n}_p^\intercal \,\mathrm{Im}\, \overleftrightarrow{\boldsymbol{G}}_s \boldsymbol{n}_p}{\mathrm{Im}\, G_0}$, $\mathrm{Im}\left[\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_0, \boldsymbol{r}_0; \omega)\right]$, $\boldsymbol{n}_p^\intercal \,\mathrm{Im}\, \overleftrightarrow{\boldsymbol{G}}_s \boldsymbol{n}_p$

and more

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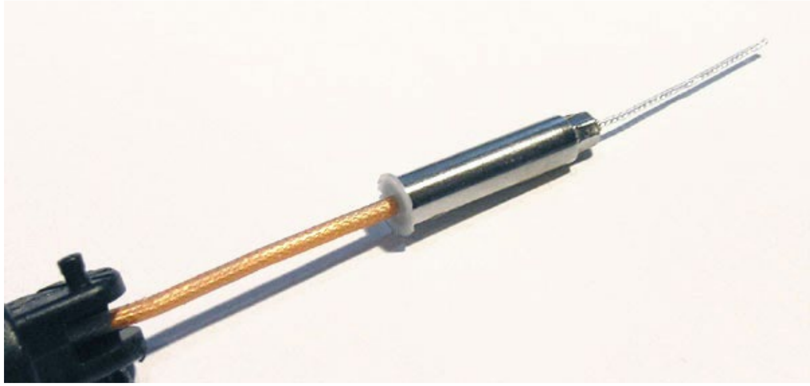
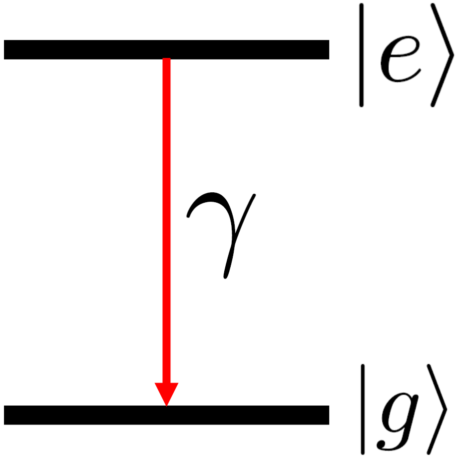
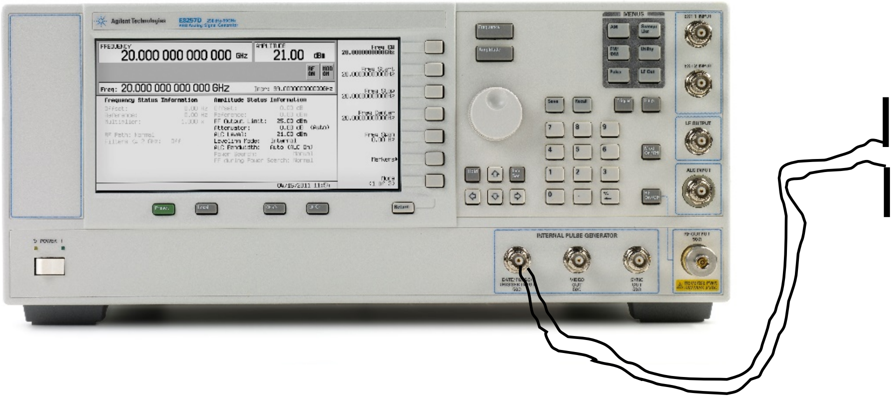




Radiation sources

Radiating source up to GHz:



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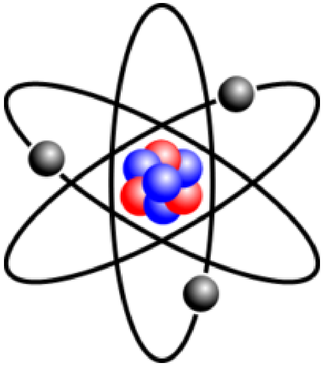
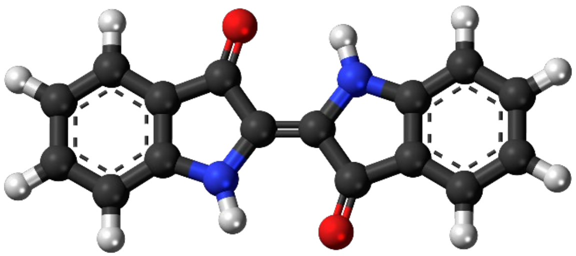
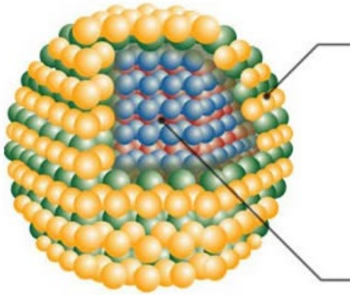
Light sources

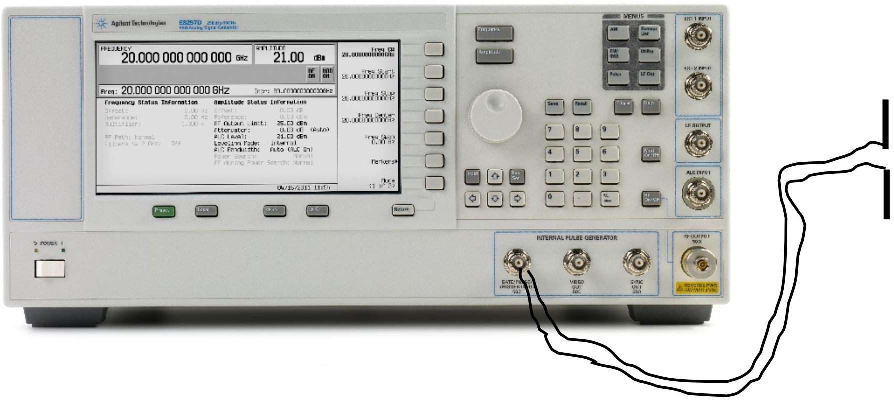
Radiating sources at 1000 THz (visible):


Atoms


Dye molecules


Quantum dots

Radiating source up to GHz:



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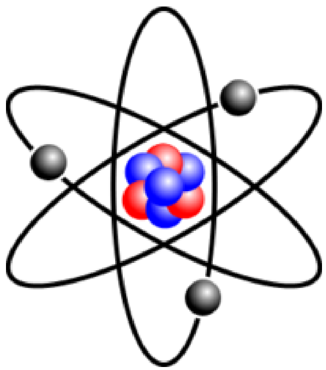
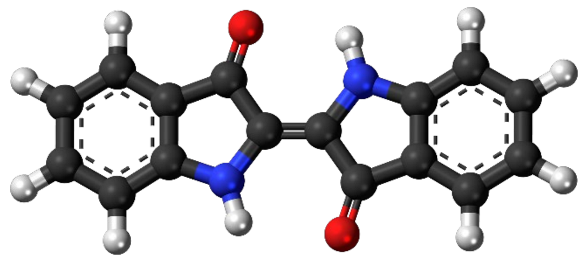
Quantum emitters

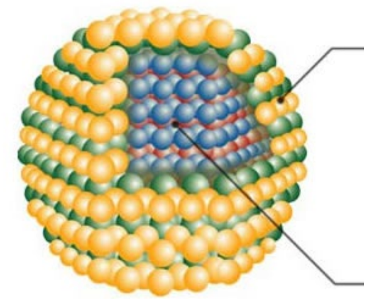
Radiating sources at 1000 THz :



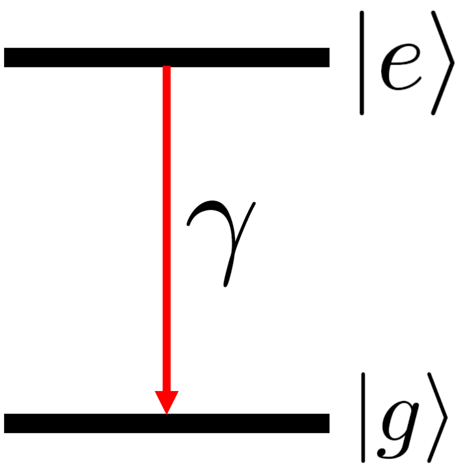
Atoms



Dye molecules



Quantum dots

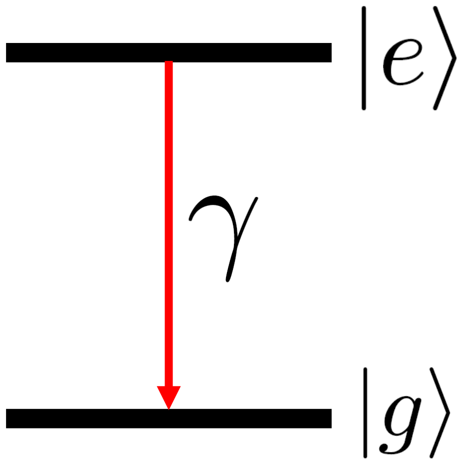


Optical emitters have discrete level scheme (in the visible)

Let’s focus on the two lowest levels

How long will the system remain in its excited state?

Decay rate of a quantum emitter

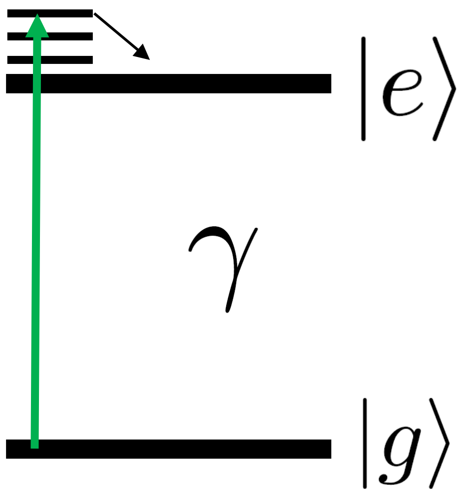


$$p_e(t) = p(0)\ \exp\left[-\gamma t\right]$$

Probability to find the system in the excited state decays exponentially with rate γ.

How can we measure the population of the excited state?

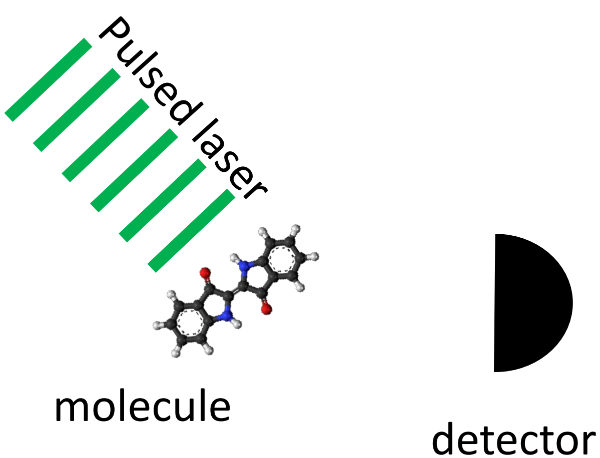
Fluorescence lifetime measurements



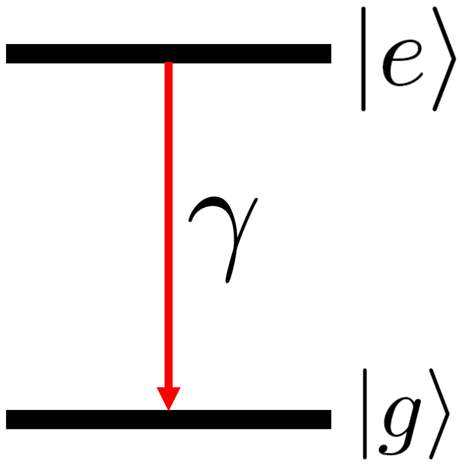
$$p_e(t) = p_e(0)\ \exp\left[-\gamma t\right]$$

The probability to detect a photon at time t is proportional to p(t)!

1. Prepare system in excited state with light pulse at t=0



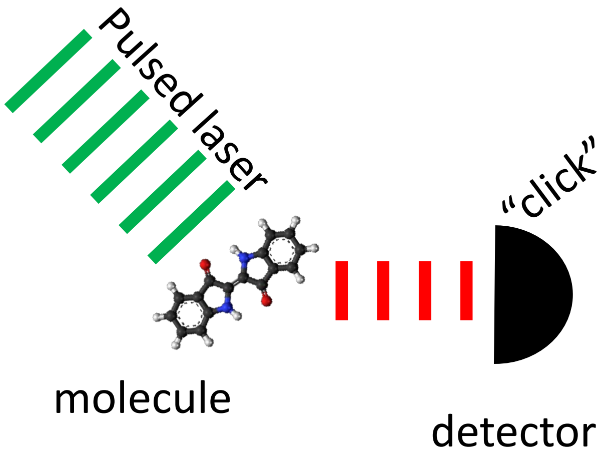
Fluorescence lifetime measurements

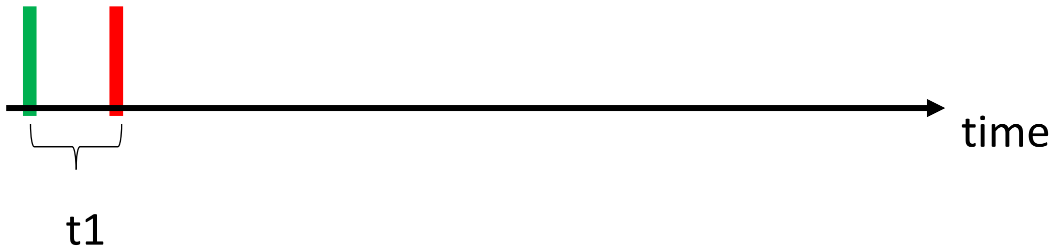


$$p_e(t) = p_e(0)\ \exp\left[-\gamma t\right]$$

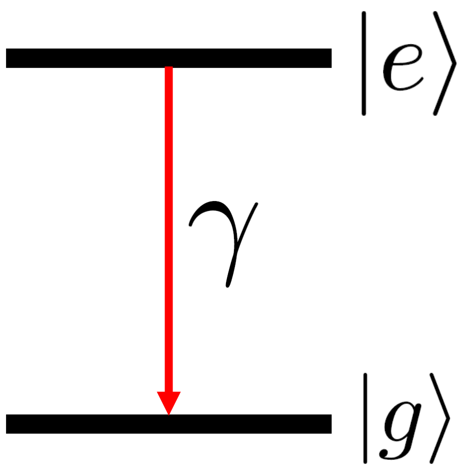
The probability to detect a photon at time t is proportional to p(t)!

1. Prepare system in excited state with light pulse at t=0
2. Record time delay t1





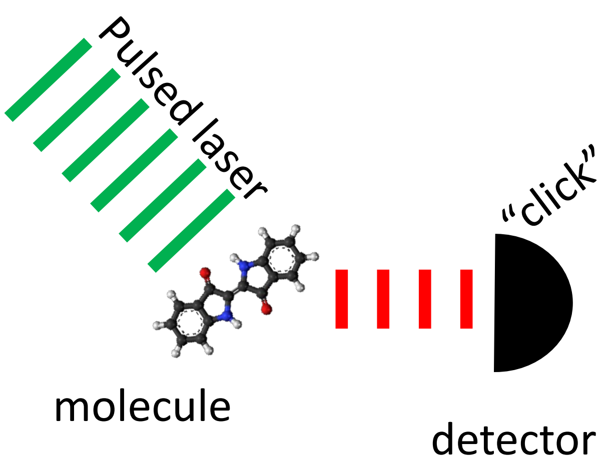
Fluorescence lifetime measurements

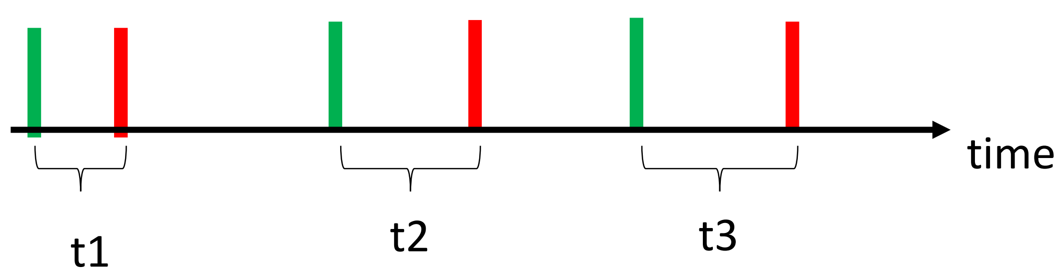


$$p_e(t) = p_e(0)\ \exp\left[-\gamma t\right]$$

The probability to detect a photon at time t is proportional to p(t)!

1. Prepare system in excited state with light pulse at t=0
2. Record time delay t1
3. Repeat experiment many times
4. Histogram arrival times





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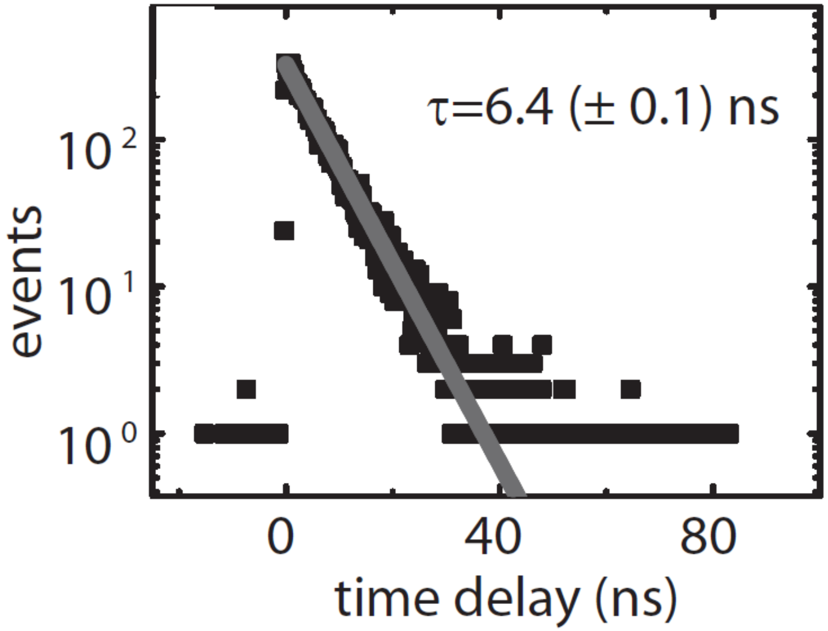

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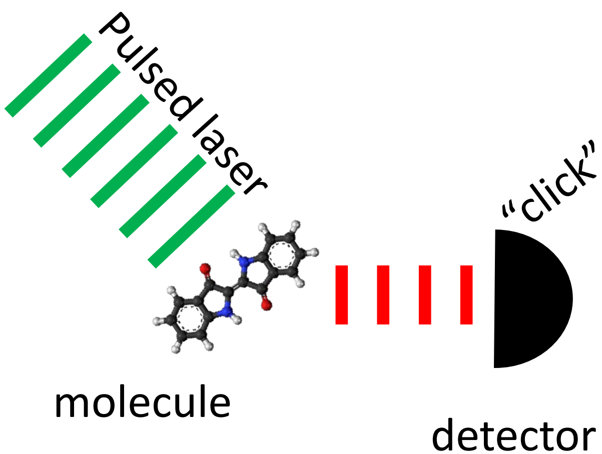
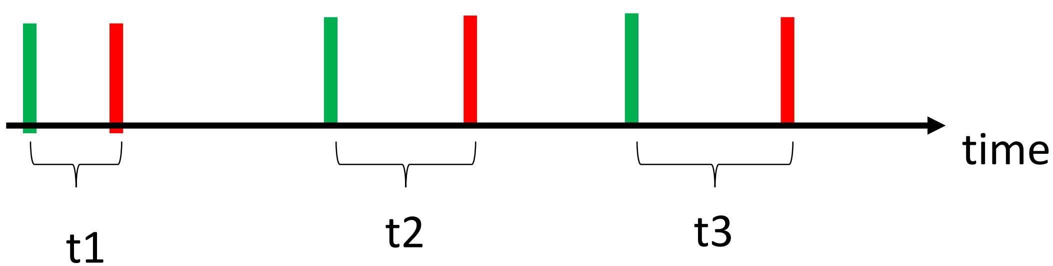
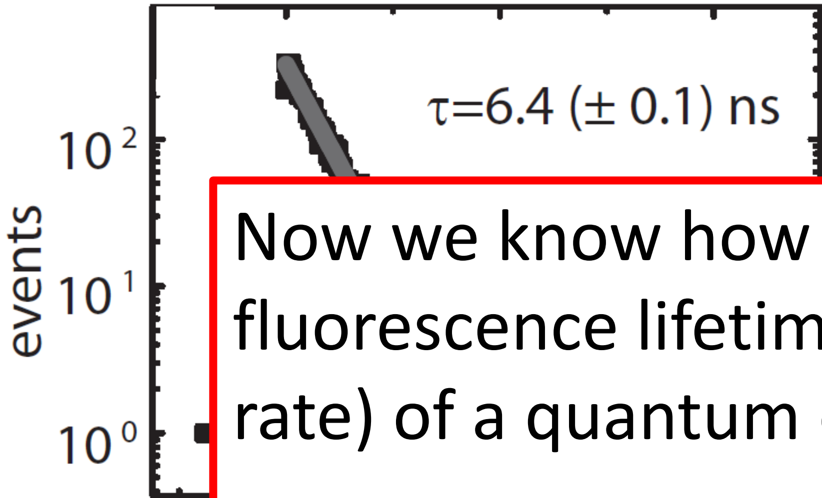




Fluorescence lifetime measurements



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Now we know how to measure the fluorescence lifetime (spontaneous emission rate) of a quantum emitter.

Can we also calculate it?

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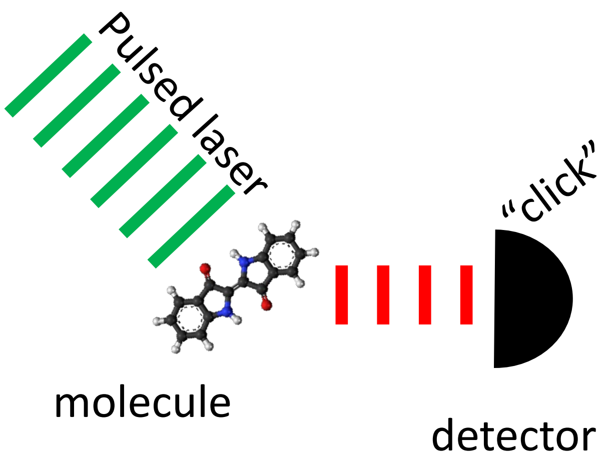
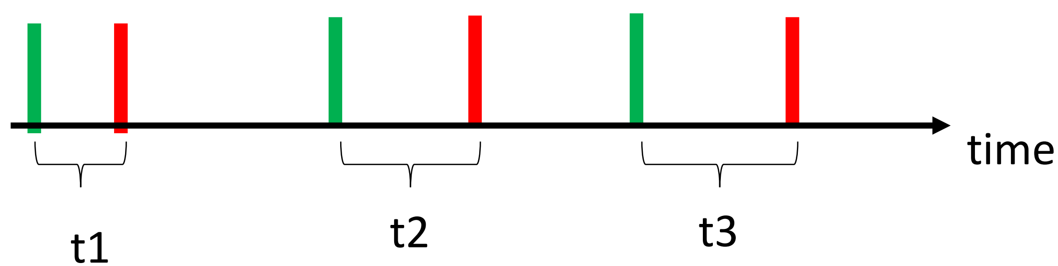
4. Histogram arrival time delays





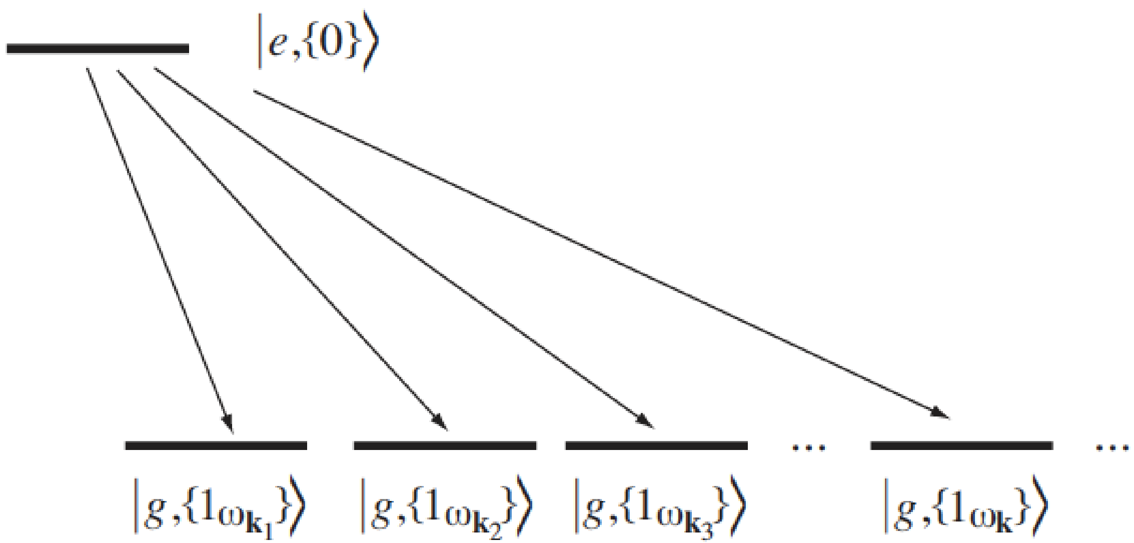
Calculation of decay rate γ

Fermi's Golden Rule:

$$\gamma = \sum_f \frac{2\pi}{\hbar} \left| \langle f | \hat{\mathscr{H}} | i \rangle \right|^2 \delta(E_i - E_f)$$

Initial state (excited atom, no photon): $|i\rangle = |e, 0\rangle$

Final state (de-excited atom, 1 photon in state k at frequency omega): $|f\rangle = |g, 1_{\omega_{\mathbf{k}}}\rangle$



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Interaction Hamiltonian: $\hat{\mathcal{H}} = -\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{E}}$

Sum over final states is sum over photon states (**k**) at transition frequency ω.

$$\gamma = \frac{\pi\omega}{3\hbar\epsilon_0} |\hat{\boldsymbol{p}}|^2 \rho_\mathbf{n}(\boldsymbol{r}_0, \omega)$$

Atomic part:
transition dipole moment (quantum)

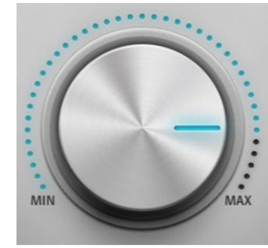
$$|\hat{\boldsymbol{p}}|^2 = |\langle g | \hat{\boldsymbol{p}} | e \rangle|^2$$

Field part:
Local density of states (classical)

$$\rho_\mathbf{n}(\boldsymbol{r}_0, \omega) = \frac{6\omega}{\pi c^2} \left\{ \boldsymbol{n}_p^\intercal \operatorname{Im}\left[\overleftrightarrow{\boldsymbol{G}}(\boldsymbol{r}_0, \boldsymbol{r}_0; \omega) \right] \boldsymbol{n}_p \right\}$$

Decay rate engineering

$$\gamma = \frac{\pi\omega}{3\hbar\epsilon_0} |\hat{\boldsymbol{p}}|^2 \rho_{\mathbf{n}}(\boldsymbol{r}_0, \omega)$$



Emitter

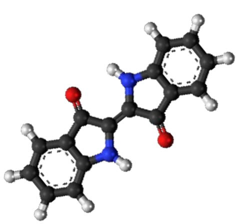
Transition dipole moment: Wave function engineering by synthesizing molecules, and quantum dots

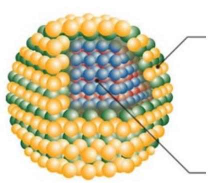
Environment

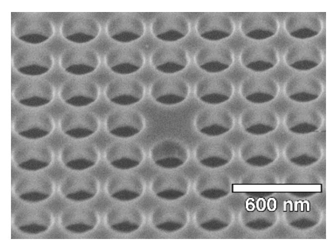
LDOS: Electromagnetic mode engineering by shaping boundary conditions for Maxwell's equations

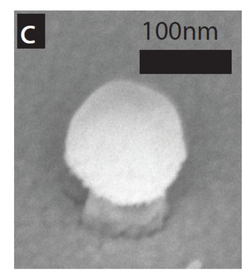
Chemistry, material science

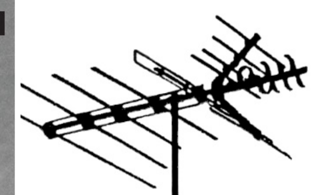
Physics, electrical engineering











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