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Theory of Induction Machines

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Fundamentals

Induction machines (also known as asynchronous machines) are by far the most common type of machines used for various applications around the globe. Induction machines consume approximately one third of the energy used in industrialized countries. They are used widely in industrial, commercial or residential settings. The main reason behind this is that induction machines are cheap and robust. The induction machine is one of the older electric machines with its invention being attributed to Tesla, then working for Westinghouse, in 1888.

Figure 1 shows the simplified structure of a three phase, two pole (one pole pair) induction machine. Although induction machines with different number of phases can also be built, three phase machines are used the most common ones and for that reason only the three phase machines will be analyzed here. The three phase winding placed in a number of slots in the stator. The rotor may also have three phase windings which are accessible from outside through slip rings and usually short circuited during operation of the machine. However, electrical power sources or additional impedances can be connected to the rotor winding using the slip rings, for different modes of operation such as start-up or braking. Another type of induction machine contains conductive bars in the rotor instead of windings. Those bars are connected to each other via short circuit rings as shown in Figure 2. This type of induction machine is called the cage rotor machine or squirrel cage machine, and it is more commonly used compared to the wound type of machine in motoring applications due to its robustness.

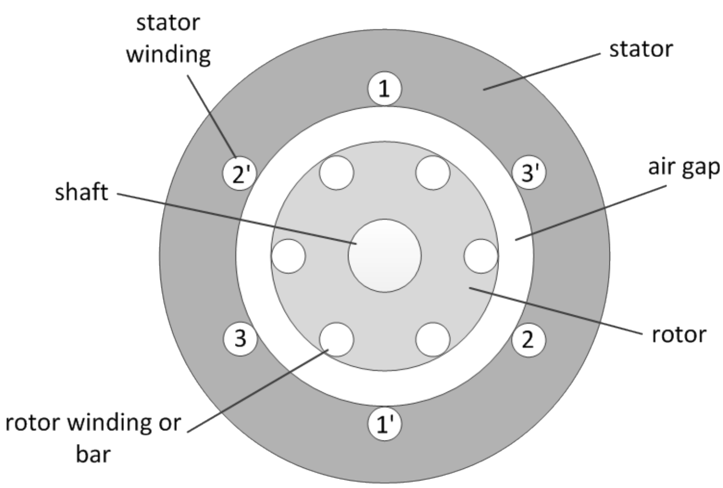

Figure 1. Simplified structure of the induction machine.

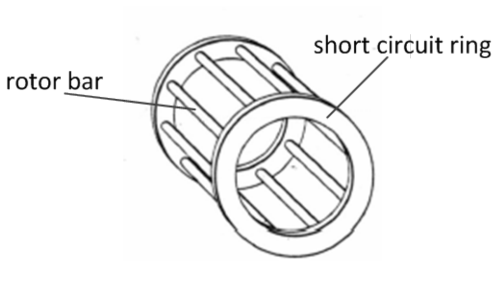

Figure 2. Cage rotor for induction machine.

Principle of Operation

Qualitative Description

Like all the electrical machines, induction machines can be used as motor or generator. However, this type of machine is predominantly used as a motor; therefore the motoring mode is explained first. The basic idea behind the operation of an induction machine is quite simple, and it can be explained as:

1. The three phase stator winding is connected to a three phase supply.
2. Three phase currents flow in the stator winding, producing a rotating magnetic field in stator. The speed of this rotating field is called the *synchronous frequency (n_s)*.

   $$n_S = \frac{60 \cdot fs}{p}$$

   where p is the pole pair number, f_s is the frequency of the stator voltage and n_s given in rpm.
3. The magnetic field passes the conductors (windings or bars) on the rotor and induces a voltage in those conductors.
4. Since the conductors are short circuited, current flows in the rotor conductors.
5. The rotor currents produce a second rotor magnetic field, which acts to oppose the stator magnetic field and also rotates at synchronous speed.
6. With two magnetic fields rotating at constant speed, a torque is induced.
7. The rotor flux density will lag the stator flux density (flux density lags current by 90° electrically), therefore the torque will be in the same direction as the rotation of the magnetic fields.
8. The torque accelerates the rotor until synchronous speed is reached, at which time there is no relative motion between the rotor conductors and the stator flux density. Since the relative velocity is zero, the induced voltage, rotor currents and flux density fall to zero and torque is also zero.

   The relationship of the rotor speed to the synchronous speed is generally expressed in a relative manner, which is called the slip.

   $$s = \frac{n_S - n_R}{n_S}$$

where s is the slip and n_R is the rotor speed.

Equivalent circuit

As described above, induction machines (as the name suggests) operate on the principle of induced currents. There are two magnetic fields, one from the rotor and one from the stator, but the rotor field is induced by the stator field. Therefore, we can think of the induction machine as a rotating transformer. The stator acts like the primary winding of a transformer creating the initial field, inducing voltages and currents in the rotor which acts like the secondary winding of a transformer. The main differences to a stationary transformer are that the secondary rotates and the amplitude and the frequency of the induced voltage on the secondary depend on the speed.

A wound rotor induction machine can actually be used as a variable frequency transformer, transferring power between two AC grids with different frequencies.

The per-phase equivalent circuit model for an induction machine in steady state operation supplied by a balances three-phase supply is based on the transformer model shown below.

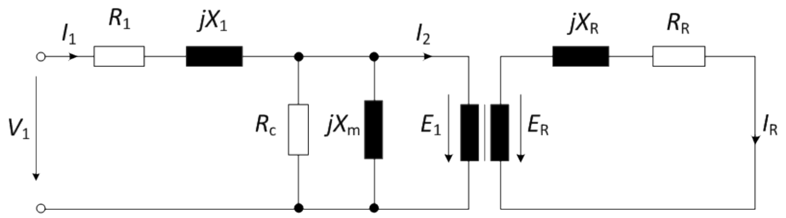

Figure 3. Transformer-like 1-phase equivalent circuit of the induction machine.

In the diagram above,

- V_1 = Phase RMS voltage
- I_1 = Stator Phase current
- R_1 = Stator winding resistance
- X_1 = Stator winding leakage reactance
- X_m = Magnetizing reactance
- R_c = Core loss resistance
- E_1 = Air gap voltage
- I_2 = Rotor current referred to stator
- E_R = Induced rotor voltage (Actual)
- I_R = Rotor induced current (Actual)
- X_R = Rotor leakage reactance (Actual)
- R_R = Rotor Resistance (Actual)

As mentioned above, induced rotor voltage E_R and rotor leakage reactance X_R both depend on slip. To simplify the model we can define them both in terms of their values when the speed is zero, slip s = 1.

$$E_R = sE_{R0}$$
$$X_R = sX_{R0}$$

where E_{R0} is the induced voltage at standstill, X_{R0} rotor leakage reactance at standstill. With this, the actual rotor current I_R can be written as:

$$I_R = \frac{E_R}{R_R + jX_R} = \frac{sE_{R0}}{R_R + jsX_{R0}} = \frac{E_{R0}}{\frac{R_R}{s} + jX_{R0}}$$

and the equivalent circuit can be re-drawn as shown below.

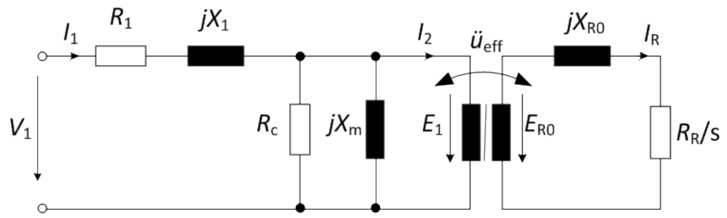

Figure 4. Equivalent circuit with modified rotor side.

In the above diagram, the effective turns ratio $ü_{eff}$ is constant and equal to the effective turns ratio at standstill. In a would rotor machine, $ü_{eff}$, X_{R0} and R_R can be measured. In a cage machine these parameters cannot be directly determined, as there is no method to directly measure voltages or currents on the rotor. To overcome this difficulty, the rotor (secondary) circuit can be referred to the stator (primary) side, as shown below.

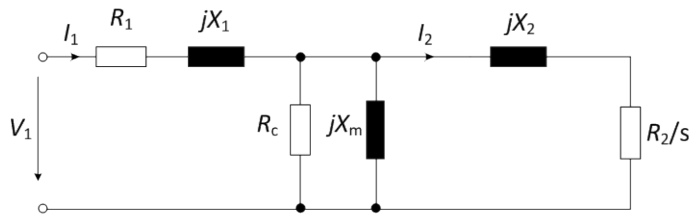

Figure 5. Full equivalent circuit of the induction machine.

In the above circuit, $R_2=ü^2_{eff}\,R_R$ (the rotor resistance referred to the stator) and $X_2=ü^2_{eff}\,X_{R0}$ (the rotor leakage reactance referred to the stator).

The symbols used in induction machine models vary depending on the text and the context in which the circuit is being used. R_i, R_o, R_{fe}, R_m can all be found as references to the iron loss resistance. In some texts (especially from Europe), R_2, X_2 refer to actual rotor values with R'_2, X'_2 used for referred values. In drives texts, it is common to find R_s, R_r for stator resistance and rotor resistance referred to the stator.

Power and torque

The input power to a three-phase induction machine is given by:

$$P_{in} = 3 \cdot V_1 I_1 \cos\theta$$

where Θ is the phase angle between V_1 and I_1. The output power can be found by subtracting the losses from the input power. Important losses in an induction machine are

- stator and rotor Joule losses (also known as copper losses)
- Core (iron) losses,
- Friction and windage losses.

The power transferred to the rotor is called the "Air gap Power". In the equivalent circuit in Figure 5, if R_c is omitted (iron losses are neglected), air gap power can be written as:

$$P_{gap} = 3 \cdot \frac{I_2^2 R_2}{s}$$

To find the power converted to the mechanical system the rotor joule loss must be subtracted from the total rotor power:

$$P_{mech} = 3 \cdot \frac{I_2^2 R_2}{s} - 3 \cdot I_2^2 R_2 = 3 \cdot I_2^2 R_2 \frac{(1-s)}{s}$$

Torque, by definition is power divided by angular speed:

$$T = \frac{P_{mech}}{\omega_m}$$

Torque – speed curve and operating regions

If the equivalent circuit in Figure 5 is solved and the torque is calculated using the equation above for different slip values, the torque-slip curve shown in Figure 6 is obtained.

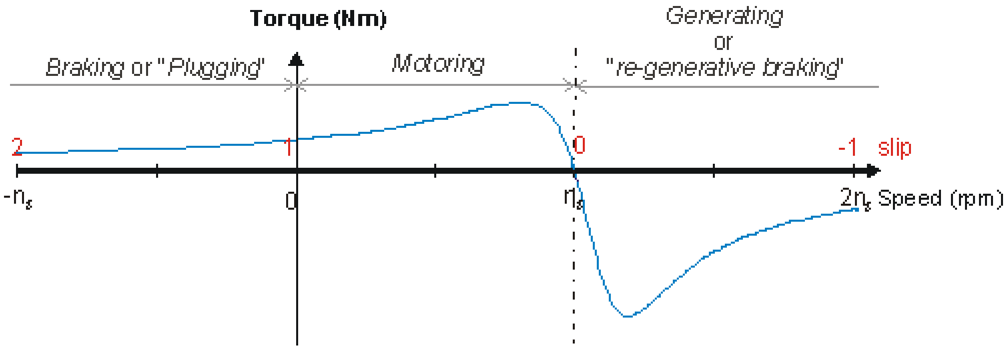

Figure 6. Torque – slip (speed) curve of an induction machine.

The three modes of operation are:

1. Braking, $n_r < 0$, $s > 1$

   Torque is positive while speed is negative. The air gap power is positive. i.e. the power is flowing from the stator to the rotor and also into the rotor from the mechanical system. This operation is also called plugging.

   This mode of operation can be used to quickly stop a machine. If a motor is travelling forwards it can be stopped by interchanging the connections to two of the three phases. Switching the two phases changes the direction of motion of the stator magnetic field, effectively putting the machine into braking mode in the opposite direction.

2. Motoring, $0 < n_r < n_s$, $1 > s > 0$

   Torque and speed are in the same direction. This is the most common mode of operation.

3. Generating, $n_r > n_s$, $s < 0$

In this mode, again torque is positive while speed is negative. However, unlike plugging, power flows from the mechanical system, to the rotor circuit, then across the air gap to the stator circuit and external electrical system.

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