The domain decomposition method of Bank and Jimack as an optimized Schwarz method

Author and Presenter:
Parisa Mamooler (University of Geneva, Switzerland)

Co-authors:
Martin J. Gander (University of Geneva, Switzerland)
Gabriele Ciaramella (University of Konstanz, Germany)

Abstract: In 2001, Bank and Jimack introduced a new domain decomposition algorithm for the adaptive finite element solution of elliptic partial differential equations. The novel feature of this algorithm is that the subdomain problems are defined over the entire domain, consisting of a fine grid in the area where the subdomain is responsible for an accurate solution, and a coarse grid elsewhere. A convergence analysis of this algorithm was given in 2008 by Bank and Vassilevski. We are interested here in understanding what the precise contribution of the outer coarse mesh is to the convergence behavior of the domain decomposition method proposed by Bank and Jimack. We show for a two subdomain decomposition that the outer coarse mesh can be interpreted as computing an approximation to the optimal transmission condition represented by the Dirichlet to Neumann map, and thus the method of Bank and Jimack can be viewed as an optimized Schwarz method, i.e. a Schwarz method that uses Robin or higher order transmission conditions instead of the classical Dirichlet ones. In particular, we show that when applied to the Laplace equation in one spatial dimension, the algorithm of Bank and Jimack computes an optimal Robin parameter for any choice of the outer coarse mesh, and the method thus converges in two iterations in this case. We then present more general situations, and we illustrate our results with numerical experiments.

References

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