Coupling Finite Elements and auxiliary sources

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Abstract: We propose to solve boundary value problems stemming from Maxwell's equations by using two numerical techniques in different domains: the Finite Element Method (FEM) and the Multiple Multipole Program (MMP). MMP is a Trefftz method for computational electromagnetics, successfully employed in the code OpenMaXwell for many years.

FEM and MMP enjoy complementary capabilities. FEM requires a mesh of the computational domain of interest. This is expensive, but can treat nonlinear materials, nonsmooth shapes, or complicated geometries. At the same time, FEM allows a purely local construction of the discrete system of equations.

On the other hand, MMP is a boundary method using global basis functions that solve exactly the PDE: only integrals on a hypersurface have to be computed, and the obtained linear combination is valid in the whole domain where the PDE holds. MMP performs well where the electromagnetic field is smooth, i.e. in the free space far from physical sources and material interfaces.

Thus, a natural way to combine the strengths of these methods arises when one needs to simulate the electromagnetic field of components with nonlinear or nonsmooth properties surrounded by free space: use FEM on a mesh defined on the components and MMP in the unbounded domain outside. The boundary between the FEM and MMP domains can be nonphysical if one surrounds the components by a conforming mesh of an "air box" also modeled by FEM.

The interface conditions on the surface of the FEM domain are key to accurate coupled FEM–MMP solutions. Integrating by parts the variational form of the PDE solved by FEM, surface integrals appear, through which one can impose interface conditions by substituting the ansatz of MMP. However, one interface condition cannot be imposed in this way.

We have explored four ways to include the additional condition. The first approach relies on optimizing a functional for the additional condition, subject to a constraint expressed by the variational form of FEM. The second approach introduces a weak formulation of the additional condition, where MMP basis functions are chosen as test functions. The third and fourth approaches are based on the mortar element method and Discontinuous Galerkin, respectively.

Convergence tests have been performed for scalar and vector configurations of Maxwell's equations with exact solutions, obtaining the expected results. Moreover, non-trivial physical configurations have been simulated. An extension to transient Maxwell's equations will be considered in future research.

References

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