Convolution quadrature for wave equations with transmission boundary condition

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Abstract: This talk presents the application of the *generalised Convolution Quadrature* (gCQ) [1, 2] to a 3D time-space geophysical problem that arises from the problem of evaluating the ice volume of glaciers in the Swiss Alps.

The geophysical problem is described mathematically by 3D wave equations in two disjoint bounded domains of different media, namely the ice domain and the air domain, which are related to each other via the transmission boundary condition. Let Ω_{ice} denote the domain of the ice with the boundary $\partial\Omega_{ice} = \Gamma_0 \cup \Gamma_{01}$, and let Ω_{air} denote the domain of the air with the boundary $\partial\Omega_{air} = \Gamma_{01} \cup \Gamma_1$. The boundary Γ_{01} denotes the interface between the ice and the air, while the boundary Γ_0 denotes the interface between the ice and the rock underneath. The goal is to solve the following 3D time-space problem by using retarded potential ansatzs [3]

$$\begin{aligned}
u_{tt}^{ice} - a_{ice}^2 \Delta u^{ice} &= 0 \quad \text{in } \Omega_{ice} \times [0,T], \\
u_{tt}^{air} - a_{air}^2 \Delta u^{air} &= 0 \quad \text{in } \Omega_{air} \times [0,T], \\
u &= g \quad \text{on } \Gamma_0 \quad \times [0,T], \\
[u]_{\Gamma_{01}} = \left[a^2 \frac{\partial u}{\partial n} \right]_{\Gamma_{01}} &= 0 \quad \text{on } \Gamma_{01} \quad \times [0,T], \\
\frac{\partial u}{\partial n} + \frac{1}{a_{air}} u_t &= 0 \quad \text{on } \Gamma_1 \quad \times [0,T], \\
u(0,x) = u_t(0,x) &= 0 \quad \text{in } \Omega,
\end{aligned}$$

where $\Omega := \Omega_{ice} \cup \Omega_{air}$.

With the single layer retarded potentials $\phi^{ice} : \Gamma_{ice} \times [0,T] \to C$ and $\phi^{air} : \Gamma_{air} \times [0,T] \to C$, the solutions u^{ice} and u^{air} can be expressed respectively by

$$\begin{aligned}
u^{ice}(x,t) &:= \int_{\Gamma_{ice}} \frac{\phi^{ice}\left(y, t - \frac{\|x-y\|}{a_{ice}}\right)}{4\pi \|x-y\|} ds_y \quad \forall (x,t) \in \Omega_{ice} \times [0,T], \\
u^{air}(x,t) &:= \int_{\Gamma_{air}} \frac{\phi^{air}\left(y, t - \frac{\|x-y\|}{a_{air}}\right)}{4\pi \|x-y\|} ds_y \quad \forall (x,t) \in \Omega_{air} \times [0,T].
\end{aligned}$$

The substitution of the solution ansatzs into the boundary conditions results in a 4×4 system of time-domain boundary integral equations. The unknowns ϕ^{ice} and ϕ^{air} in the system are then solved by using the gCQ based on the implicit

Euler method for temporal discretisation and the Galerkin boundary element method (BEM) of constant basis function for spatial discretisation.

The numerical simulation produces stable results, and shows the feasibility of solving real-world complex problems of such a type by using gCQ with BEM.

References

[1] María López Fernández and Stefan Sauter. *Generalized Convolution Quadrature with Variable Time Stepping, Part I.* IMA Journal of Numerical Analysis, 2013.

[2] María López Fernández and Stefan Sauter. *Generalized Convolution Quadrature with Variable Time Stepping, Part II.* Applied Numerical Mathematics, 2015.

[3] Stefan Sauter and Martin Schanz. *Convolution quadrature for the wave equation with impedance boundary condition.* Journal of Computational Physics, 2017.