Computing statistical solutions of hyperbolic conservation laws

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Abstract: An open question in the field of hyperbolic conservation laws is the question of well-posedness. Recent theoretical and numerical evidence have indicated that multidimensional systems of hyperbolic conservation laws exhibit random behavior, even with deterministic initial data. We use the framework of statistical solutions to model this inherit randomness. We review the theory of statistical solutions for conservation laws.

Afterwards, we introduce a convergent numerical method for computing the statistical solution of conservation laws, and prove that it converges in the Wasserstein distance through narrow convergence for the case of scalar conservation laws. For the scalar case, we validate our theory by computing the structure functions of the Burgers equation with random initial data. We especially focus on Brownian initial data, and the measurement of the scalings of the structure functions. The results agree well with the theory, and we get the expected convergence rate. We furthermore show that we can get faster computations using Multilevel Monte-Carlo for computing the statistical solutions of scalar conservation laws.

In the case of systems of equations, we test our theory against the compressible Euler equations in two space dimensions. We check our numerical algorithm against two ill-behaved initial data, the Kelvin-Helmholtz instability and the Richtmeyer-Meshkov instability. We observe Wasserstein convergence for the two point correlation marginals.