Risk average optimal control problem for elliptic PDEs with uncertain coefficients

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Abstract: We consider a risk averse optimal control problem for an elliptic PDE with uncertain coefficients. The control is a deterministic distributed forcing term and is determined by minimizing the expected L^2-distance between the state (solution of the PDE) and a target deterministic function. An L^2-regularization term is added to the cost functional (see e.g. [1]).

We consider a finite element discretization of the underlying PDE and derive an error estimate on the optimal control.

Concerning the approximation of the expectation in the cost functional and the practical computation of the optimal control, we analyze and compare two strategies.

In the first one, the expectation is approximated by either a Monte Carlo estimator, and a steepest descent algorithm is used to find the discrete optimal control.

The second strategy, named Stochastic Gradient (see e.g. [2, 3]) is again based on a steepest-descent type algorithm. However the expectation in the computation of the steepest descent is approximated with independent Monte Carlo estimators at each iteration using possibly a very small sample size. The sample size and possibly the mesh size in the finite element approximation could vary during the iterations. We present error estimates and complexity analysis for both strategies and compare them on few numerical test cases.

References

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