Sparse-grid polynomial interpolation approximation and integration for parametric and stochastic elliptic PDEs with lognormal inputs

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Abstract

By combining a certain approximation property in the spatial domain, and weighted ℓ_2-summability of the Hermite polynomial expansion coefficients in the parametric domain obtained in [1] and [2], we investigate linear non-adaptive methods of fully discrete polynomial interpolation approximation as well as fully discrete weighted quadrature methods of integration for parametric and stochastic elliptic PDEs with lognormal inputs. We explicitly construct such methods and prove corresponding convergence rates of the approximations by them. The linear non-adaptive methods of fully discrete polynomial interpolation approximation are sparse-grid collocation methods which are certain sums taken over finite nested Smolyak-type indices sets $G(\xi)$ parametrized by $\xi > 0$, of mixed tensor products of dyadic scale successive differences of spatial approximations of particular solvers, and of successive differences of their parametric Lagrange interpolating polynomials. The Smolyak sparse grids in the parametric domain are constructed from the roots of Hermite polynomials or their improved modifications.

Moreover, they generate fully discrete weighted quadrature formulas in a natural way for integration of the solution to parametric and stochastic elliptic PDEs and its linear functionals, and the error of the corresponding integration can be estimated via the error in the Bochner space $L_1(\mathbb{R}^\infty, V, \gamma)$ norm of the generating methods where γ is the Gaussian probability measure on $\mathbb{R}^\infty$ and V is the energy space.

Our analysis leads to auxiliary convergence rates in parameter ξ of these approximations when ξ going to ∞. For a given $n \in \mathbb{N}$, we choose ξ_n so that the cardinality of $G(\xi_n)$ which in some sense characterizes computation complexity, does not exceed n, and hence obtain the convergence rates in increasing n, of the fully discrete polynomial approximation and integration.

References

[1] M. Bachmayr, A. Cohen, R. DeVore and G. Migliorati: *Sparse polynomial approximation of parametric elliptic PDEs. Part II: lognormal coefficients*, ESAIM Math. Model. Numer. Anal., 51:341-363, 2017.

[2] M. Bachmayr, A. Cohen, D. Dũng and C. Schwab: *Fully discrete approximation of parametric and stochastic elliptic PDEs*, SIAM J. Numer. Anal., 55:2151-2186, 2017.