Construction of rank-1 lattice rules by a smoothness-independent component-wise digit-by-digit construction

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Abstract

In this talk, we introduce a component-wise digit-by-digit method (CBC-DBD) for the construction of rank-1 lattice rules in spaces of periodic functions with prescribed decay for the Fourier coefficients and associated smoothness $\alpha > 1$. This method is an extension of the construction algorithm established by Korobov in [1] to the weighted function space setting. We show that the introduced CBC-DBD algorithm constructs lattice rules with $N = 2^n$ points which achieve the almost optimal worst-case error convergence rates in the studied function spaces. Due to the usage of a quality function which is independent of the smoothness α, the algorithm constructs lattice rules which achieve these convergence rates uniformly for all smoothness parameters $\alpha > 1$. The involved proof technique is essentially different from common proofs for component-wise construction methods. Furthermore, we derive conditions on the weights under which the mentioned error bounds are independent of the dimension. The CBC-DBD algorithm can be implemented in a fast manner such that the construction only requires $\mathcal{O}(sN \ln N)$ operations, where $N = 2^n$ is the number of lattice points and s denotes the dimension, making it competitive with the common fast CBC algorithm, see, e.g., [2]. This fast construction is easy to implement and does not rely on the use of fast Fourier transformations (FFTs). Numerical results confirm our theoretical findings.

References

[1] N.M. Korobov: *On the computation of optimal coefficients*, Dokl. Akad. Nauk SSSR, 26:590-593, 1982.

[2] D. Nuyens and R. Cools: *Fast component-by-component construction of rank-1 lattice rules with a non-prime number of points*, J. Complexity, 22(1):4-28, 2006.