Multilevel quasi-Monte Carlo methods for a random elliptic eigenvalue problem

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Abstract

Motivated by uncertainty quantification for the neutron diffusion criticallity problem, we will study an elliptic eigenvalue problem with coefficients that depend on infinitely many stochastic parameters. The stochasticity in the coefficients causes the eigenvalues and eigenfunctions to also be stochastic, and so our goal is to compute the expectation of the minimal eigenvalue. In practice, to approximate this expectation, one must:

1) truncate the stochastic dimension;

2) discretise the eigenvalue problem in space (e.g., by finite elements); and

3) apply a quadrature rule to estimate the expected value.

In this talk, we will present a multilevel (quasi-)Monte Carlo method for approximating the expectation of the minimal eigenvalue, which is based on a hierarchy of finite element meshes and truncation dimensions. To improve the sampling efficiency over Monte Carlo we will use a randomly shifted lattice rule to generate the sampling points, for which the sampling error can converge at a rate of $1/N$ as compared with the Monte Carlo rate of $1/\sqrt{N}$. Also, to make each eigenproblem solve on a given level more efficient, we utilize the two-grid finite element method for eigenproblems to obtain the eigenvalue on the fine mesh from the coarse eigenvalue (and eigenfunction) with a single linear solve.