A Quasi-Monte Carlo Method for PDE-constrained optimization under uncertainty

Author and Presenter: Philipp **Guth** (University of Mannheim, Germany)

Co-authors: Vesa **Kaarnioja** (UNSW Sydney, Australia)
Frances **Kuo** (UNSW Sydney, Australia)
Claudia **Schillings** (University of Mannheim, Germany)
Ian **Sloan** (UNSW Sydney, Australia)

Abstract

In this work we apply a quasi-Monte Carlo (QMC) method to an optimal control problem constrained by an elliptic partial differential equation (PDE) equipped with an uncertain diffusion coefficient. In particular the optimization problem is to minimize the expected value of a tracking type cost functional with an additional penalty on the control. The uncertain coefficient in the PDE is parametrized by a countably infinite number of terms via a Karhunen-Loève expansion (KLE) and the expected value is considered as an infinite-dimensional integral in the corresponding parameter space.

We discretize the optimization problem by truncating the KLE after s terms, approximating the expected value by an n-point QMC rule in s dimensions and approximating the solution of the PDE using finite elements (FE). It is shown that the discretization error of the solution to the optimization problem is bounded by the discretization error of the adjoint state. For the convergence analysis the latter is decomposed into truncation error, QMC error and FE error, which are then analysed separately. Numerical experiments confirming our theoretical convergence results will be presented.