Algorithms and complexity for stochastic integration in various function classes

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Abstract

Inspired by a recent paper of Eisenmann and Kruse [1] we study algorithms for and the complexity of stochastic integration with respect to the Wiener sheet measure $\int_{[0,1]^d} f(t)dW_t$ of stochastic functions f with the following types of regularity: f is assumed to belong to $L_u(\Omega, X)$, where $1 \le u < \infty$ and X is

- a Sobolev space $X = W_p^r([0,1]^d)(r \in \mathbb{N}, 1 \le p \le \infty)$, or
- a Besov space $X = B_{pp}^r([0,1]^d)(r \in \mathbb{R}; 0 < r < \infty, 1 \le p \le \infty)$ (also called Sobolev-Slobodeckij space), or
- a Bessel-potential space $X = H_p^r([0,1]^d)(r \in \mathbb{R}; 0 < r < \infty; 1 < p < \infty)$.

In all cases it is assumed that $r/d > 1/p - 1/2$. Information about f consists of function values while that about W_t may be function values or scalar products with polynomials of a given degree. Both deterministic and randomized algorithms are considered. We determine the order of the complexity, which includes finding and analyzing algorithms of optimal order and proving matching lower bounds. This extends results from [1], where upper bounds for the one-dimensional case $X = B_{pp}^r([0,1])$ with $0 < r < 2$, $2 \le p < \infty$ were established, and from [2], where only deterministic integrands were considered.

References

[1] M. Eisenmann and R. Kruse: *Two quadrature rules for stochastic Itô-integrals with fractional Sobolev regularity*, arXiv:1712.08152.

[2] S. Heinrich: *Complexity of stochastic integration in Sobolev classes*, J. Math. Anal. Appl., 476:177-195, 2019.