**Multiple lattice rules for multivariate $L_\infty$ approximation in the worst-case setting**

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**Abstract**

We consider multivariate periodic functions that belong to reproducing kernel Hilbert spaces. A recent work [1] of Cobos, Kühn, and Sickel proved that the worst case approximation error measured in the sup-norm, where one is allowed to use at most $n$ linear information of the function in order to compute the approximation, can be realized by the exact Fourier partial sum that involves the potentially $n$ most important trigonometric monomials. Note that the afore-mentioned importance only depends on the reproducing kernel Hilbert space, since we consider a worst case approximation error.

We present an approach for approximating this Fourier partial sum by means of a specific sampling strategy that is based on rank-1 lattice sampling and investigate the corresponding worst case sampling error, i.e., the approximation error with respect to the required number of sampling values. It turns out that our sampling strategy requires a number of approximately $9n\ln n$ sampling values in order to approximate the Fourier partial sum such that the worst case sampling error is almost as good as the worst case approximation error of the exact Fourier partial sum - up to some multiplicative factor $3\ln n + 1$. In particular, this relation of the approximation errors to the corresponding sampling errors does not depend on the dimension and holds even in pre-asymptotic settings. Further applications of the general theoretical result improve known estimates of the worst case sampling error for rank-1 lattices substantially including those that incorporates tractability considerations.

Besides the extremely convincing approximation properties, a further great advantage of the suggested sampling method is that the computation of the approximants can be realized using a highly efficient FFT approach. Thus, we present an excellently suitable sampling strategy even for reasonable possibly pre-asymptotic settings.

## References

[1] F. Cobos, T. Kühn and W. Sickel: *Optimal approximation of multivariate periodic Sobolev functions in the sup-norm*, J. Funct. Anal., 270:4196-4212, 2016.