Old and new tensor formats for PDE driven applications in many dimensions

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Abstract

Rank-structured tensor approximation of functions and operators by using the traditional canonical (CP), Tucker and tensor train (TT) formats allows the numerical calculus with linear complexity scaling in dimension. Further data-compression to the logarithmic scale can be achieved by using the quantized-TT (QTT) approximation. The novel range-separated (RS) tensor format is capable for the efficient low-rank approximation in the numerical modeling of many-particle systems in $\mathbb{R}^d$. We discuss how the tensor numerical methods apply to the solution of complicated multidimensional problems in the PDE driven applications, and for the efficient representation and analysis of large multi-dimensional data. In particular, we describe

- The RS tensor decomposition of the Dirac delta and elliptic operator inverse with application to the Poisson-Boltzmann equation in bio-molecular modeling;
- An example on how the QTT and RS tensor formats apply in machine learning for modeling and analysis of scattered multi-dimensional data;
- How the tensor numerical methods apply in stochastic homogenization of the elliptic PDEs in random media, and in the control problems constrained by PDEs in $\mathbb{R}^d$.

The details can be found in the research monograph [1], and in the recent papers [2-5].

References

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