Exponential ReLU DNN expression of holomorphic maps in high dimension

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Abstract

We consider the approximation of analytic maps $u : [-1,1]^d \to \mathbb{R}$ by deep ReLU neural networks, for a possibly large input dimension $d \in \mathbb{N}$. We assume quantitative control of the domain of holomorphy, i.e. u admits a holomorphic extension to a Bernstein polyellipse $\mathcal{E}_{\rho_1} \times \ldots \times \mathcal{E}_{\rho_d} \subset \mathbb{C}^d$ with semiaxis sums $\rho_i > 1$, containing $[-1,1]^d$. We establish exponential convergence in terms of the neural network size N, as the error in Lipschitz norm $W^{1,\infty}([-1,1]^d)$ is shown to decay as $O(\exp(-bN^{1/(d+1)}))$ for a constant $b > 0$ determined by $\{\rho_j\}_{j=1}^d$. In addition, we discuss deep neural networks with a so-called "rectified power unit" (RePU) activation function, which approximate u with convergence of the order $O(\exp(-cN^{1/d}))$ for $c > 0$ depending on $\{\rho_j\}_{j=1}^d$.

References

[1] J.A.A. Opschoor, Ch. Schwab and J. Zech: *Exponential ReLU DNN expression of holomorphic maps in high dimension.* Technical Report 2019-35, Seminar for Applied Mathematics, ETH Zürich, Switzerland, 2019.