Deep neural network approximation of high-dimensional functions inspired by quasi-optimal polynomial methods

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Abstract

We show the existence of a class of deep neural networks with ReLU activation functions (DNNR) which approximates tensor products of polynomials. By considering existing analysis of quasi-optimal polynomial approximations, our proposed DNNR is shown to approximate a large class of high-dimensional functions. When an estimate of the bounds of the polynomial coefficients is known, this network achieves a rate of approximation comparable to that of quasi-optimal methods which are sub-exponential in the number of polynomials, M. Furthermore, the complexity of the network which achieves this sub-exponential rate is shown to be algebraic in M. Our proof is constructive and species a set of parameters so that the DNNR is a piecewise linear approximation of a polynomial. Numerically, these parameters can be used to set the initial state of a DNNR which can then be trained to approximate a high-dimensional function. We consider a numerical experiment which compares global polynomial sparse grid approximations of some non-smooth high-dimensional functions to our improved DNNR approximations by using the samples of the target function at a set of sparse grid points as training data. After training the error of each approximation is empirically estimated. Our results indicate that neural networks trained in this manner produce better approximations than certain sparse grid approximations.

References

[1] J. Daws Jr. and C. Webster: *A Polynomial-Based Approach for Architectural Design and Learning with Deep Neural Networks*, arXiv:1905.10457, 2019.

[2] H. Tran, C. Webster and G. Zhang: *Analysis of quasi-optimal polynomial approximations for parameterized PDEs with deterministic and stochastic coefficients*, Numer. Math., 137:451-493, 2017.