Finding of maximum volume square and rectangular submatrices in block matrix case

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Abstract

This work is devoted to finding submatrices with certain extreme property, so-called D-optimality criterion, which is extremely useful in a variety of tasks, such as recommender systems [5] or wireless communication [4]. We expand the criterion to the block matrix case (when one physical entity corresponds to several rows, which are connected together, and this entity cannot be chosen partially: it is either fully selected, or not selected at all). The proposed approach allows finding square and rectangular block D-optimal submatrices of a matrix by greedy updates and extensions of the initial square submatrix. Algorithm for selecting the last one is also proposed and based on PLUQ decomposition of a matrix. This work originates from the results of [1] and [3], which are concerned with finding D-optimal submatrices in a standard case, and appears to be their natural expansion to the block case.

As a practical application, we consider selection nodes for multivariate function approximation with use of its derivatives. One way to approximate a function is to construct a polynomial expansion. Coefficients of the former can be found with the Least Squares Method (LSM). Values of derivatives give additional equations by extending the matrix and right-hand side of LSM, which is the case of utilizing block matrices. Thus, we reduce the number of points where to evaluate the function. Based on the notion that using D-optimal submatrices in the LSM has a positive effect on the approximation quality (see [2] in a square case, minimizing variance in overdetermined case) we use proposed algorithm to select the proper nodes. Accuracy of approximation is compared with well-known samplings such as Latin Hypercubes Sampling (LHS), Sobol and random.

References

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