Convergence order of Euler-type schemes for SDEs in mathematical finance

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Abstract

Stochastic differential equations (SDEs) are essential for many models mathematical finance. In many cases the coefficients of these SDEs lack regularity properties that are assumed in the classical literature on numerical methods for SDEs. For example when solving stochastic control problems in mathematical finance by simulation one has to take into account that the control might depend on the controlled process in an irregular (non-Lipschitz) manner. Motivated by this problem we study the strong convergence rate of the Euler-Maruyama scheme for scalar SDEs with additive noise and irregular drift. We provide a framework for the error analysis by reducing it to a weighted quadrature problem for irregular functions of Brownian motion. By analysing the quadrature problem we obtain for abitrarily small $\epsilon > 0$ a strong convergence order of $(1 + \kappa)/2 - \epsilon$ for a non-equidistant Euler-Maruyama scheme, if the drift has Sobolev-Slobodeckij-type regularity of order κ.

In the multi-dimensional setting we allow the drift coefficient to be non-Lipschitz on a set of positive reach. We prove strong convergence of an Euler-type scheme, which uses adaptive step-sizing for a better resolution close to the discontinuity. We obtain a numerical method which has – up to logarithmic terms – strong convergence order 1/2 with respect to the average computational cost, which is the best we can expect.