Random rank-1 lattices and high-dimensional sparse FFT

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Abstract

Sampling schemes based on so-called rank-1 lattices allow for powerful methods for numerical integration and reconstruction of high-dimensional periodic functions. In order to efficiently reconstruct such functions, they should fulfill certain properties like special smoothness characteristics or (approximate) sparsity in frequency domain. Then, they can be well approximated by multivariate trigonometric polynomials consisting only of a relatively small number of oscillations with certain frequencies and corresponding Fourier coefficients. The hard task is determining the (often) unknown frequencies in a fast way using as few measurements of the function under consideration as possible. In one spatial dimension, several methods called sparse fast Fourier transform (sparse FFT) are available for solving this task. However, the high-dimensional case is much more challenging, especially if the number of possible frequencies, i.e. the cardinality of the search space, is very huge.

In this talk, new results on high-dimensional sparse FFT are presented, which use a recently developed sampling scheme called "random rank-1 lattices". This sampling method distinctly improves the required number of samples compared to the approach in [1]. The idea behind "random rank-1 lattices" is explained and theoretical results are presented.

Two approaches for the reconstruction of high-dimensional periodic functions based on "random rank-1 lattices" are discussed. The first approach utilizes a direct search for the frequencies belonging to the approximately largest Fourier coefficients. The number of samples only depends logarithmically on the number of possible frequencies and the arithmetic complexity only linearly. The second approach provides additional improvements for the case where the number of possible frequencies is extremely huge. Then, we propose to use a search method based on dimension-incremental projections from [1] using "random rank-1 lattices", which distinctly reduces the arithmetic complexity. The talk is concluded by presenting impressive numerical results for up to 30 spatial dimensions.

References

[1] L. Kämmerer, D. Potts and T. Volkmer: *High-dimensional sparse FFT based on sampling along multiple rank-1 lattices*, ArXiv e-prints, 2017; arXiv:1711.05152 [math.NA].