Constructing least-squares multivariate polynomial approximation

Author and Presenter: Tao **Zhou** (Chinese Academy of Sciences, China)

Co-authors: John **Jakeman** (Sandia Lab, USA)
Akil **Narayan** (University of Utah, USA)

Abstract

Polynomial approximations constructed using a least squares approach is a ubiquitous technique in numerical computations. One of the simplest ways to generate data for the least squares problems is with random sampling of a function. We discuss theory and algorithms for stability of the least-squares problem using random samples. The main lesson from our discussion is that the intuitively straightforward ("standard") density for sampling frequently yields suboptimal approximations, whereas sampling from a non-standard density either by the so-called induced distribution or the asymptotic equilibrium measure, yields near-optimal approximations. We present recent theory that demonstrates why sampling from such measures is optimal, and provide several computational experiments that support the theory. New applications of the equilibrium measure sampling will also be discussed.

References

[1] J. Jakeman, A. Narayan and T. Zhou: *A Christoffel function weighted least squares algorithm for collocation approximations*, Mathematics of Computation, 306(86):1913-1947, 2017.

[2] L. Guo, Y. Liu and T. Zhou: *Data-driven polynomial chaos expansions: A weighted least-square approximation*, Journal of Computational Physics, 381:129-145, 2019.

[3] L. Guo, A. Narayan and T. Zhou: *Constructing least-squares polynomial approximations*, submitted to Sirew, 2019.