Sampling posterior distributions governed by partial differential equations using surrogate models

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Abstract: Solutions of inverse problems arising in various engineering fields usually involve intensive computations, including repetitive numerical solutions of partial differential equations. Unknown parameters represent e.g. boundary conditions, source or material in a domain of interest.

An observation that corresponds to an unknown combination of input parameters values is given and the aim is to determine or to describe mathematically the unknown parameters. In the case of the Bayesian approach, the vector of input parameters is treated as a random vector and the aim is to describe its joint probability distribution. The Bayesian approach expects that the observed data are corrupted by noise with a given probability distribution. Furthermore, information about the unknown parameters available from experiences is incorporated in the form of a prior distribution.

The resulting posterior probability density function of the vector of unknown parameters depends on the solution of the complex mathematical model that describes the related forward problem. Therefore, it cannot be expressed analytically or sampled directly. Standard Markov chain Monte Carlo methods are computationally demanding because they require a high number of evaluations of the forward problem. To reduce the number of evaluations, we focus on sampling methods based on the delayed acceptance Metropolis-Hastings algorithm, see [1]. Proposed procedures work with surrogate models that are iteratively refined during the sampling process. For the construction of surrogate models, we use two different approaches - the stochastic collocation method and radial basis functions.

The use of proposed procedures is presented on a model problem. The vector of observations is simulated and corrupted by additive noise, instead of real measurements.

References

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