# THE ARITHMETIC OF CONNECTIONS

# MONTE VERITÀ, JULY 15-19, 2019

## Minicourses

JEAN-BENOÎT BOST – *Transcendence techniques and modules with integrable connections over number fields*

These lectures will be devoted to some classical results of transcendence theory and to their applications to the monodromy of modules with integrable connections over algebraic varieties defined over number fields. We will notably discuss the theorems of Schneider–Lang and of the Chudnovsky's and their proofs from a geometric perspective.

NICHOLAS KATZ – *Connections and monodromy: the finite field version*

We will pass from connections over $\mathbb{C}$ to local systems over $\mathbb{C}$ to local systems over finite fields, and talk about their monodromy groups.

## Talks

YVES ANDRÉ – *Parallel transport, transcendence, and the category of "bivector spaces"*

Pairs of vector spaces $V, V'$ over fields $k, k'$ together with an isomorphism $\iota$ after extension of scalars to some common bigger field form an interesting tannakian category. It was introduced fifteen years ago in the speaker's book on motives, in the context of Grothendieck's period conjecture. Another interesting context is that of (arithmetic) connections, where $V, V'$ are solution spaces at two $k-, k'$-rational points, and $\iota$ is parallel transport along a path. We shall discuss this category, and outline these two contexts and their intersection, with a view toward transcendental number theory.

YOHAN BRUNEBARBE – *o-minimal geometry and algebraicity of period maps*

In this talk I will introduce o-minimal geometry and illustrate its relevance to proving algebraicity of certain analytically defined objects. As an application, I will explain that the period maps associated to variations of pure Hodge structures are algebraic in corestriction to their image, as conjectured by Griffiths. This is joint work with Benjamin Bakker and Jacob Tsimerman.

HÉLÈNE ESNAULT – *$\ell$-arithmetic subloci of the moduli space of local systems*

We explain in rank one the motivicity of those (joint with Moritz Kerz) and speculate (with Michael Groechenig and Moritz Kerz) about the higher rank case.

MIRCEA MUSTAŢĂ – *Hodge filtration, minimal exponent, and local vanishing*

I will discuss a circle of ideas relating the Saito's minimal exponent of a singularity, the Hodge filtration on the localization along a regular function, the V-filtration of Malgrange and Kashiwara, and local vanishing for differential forms with log poles (all terms will be explained in the talk). This is based on joint work with Mihnea Popa.

FERNANDO RODRÍGUEZ VILLEGAS – *Mixed Hodge numbers of hypergeometric motives*

I will discuss in down to earth terms how the combinatorics of polytopes enters into describing the geometry of certain one parameter families of varieties. These varieties are associated to data that defines a classical hypergeometric series. But the relation goes deeper: the number of points of a given fiber over a finite field can be computed in terms of a finite field analogue of the hypergeometric series itself.

CLAUDE SABBAH – *Hodge structures and rigid local systems*

Rigid irreducible local systems on the punctured Riemann sphere are known to underlie variations of complex Hodge structures if their monodromies satisfy some unitarity property. The first part of the talk will focus on the methods of computation of the corresponding Hodge numbers, that can be considered as hidden invariants. In the second part, the analogous theory with irregular singularities will be developed and some explicit computations will be explained for the confluent hypergeometric differential equations.

MASHA VLASENKO – *Dwork crystals*

In his work on zeta functions for families of algebraic varieties Bernard Dwork discovered a number of remarkable p-adic congruences. In this lecture I present our joint work with Frits Beukers which explains the underlying mechanism of such congruences using only elementary definitions and arguments. We construct an explicit Cartier operator on geometric (Gauss–Manin) connections and show its applications in arithmetic.