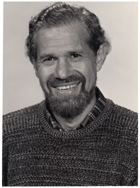
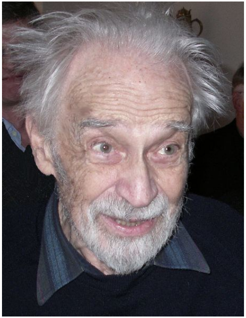
Dedicated to the memory of *Ernst Specker*

by a former student and lifelong admirer

This talk is on the





Kochen – *Specker*

Theorem

Engeler-Specker Symposium, ETH Zurich, February 22, 2020

JF

Summary

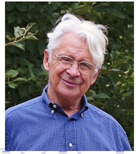
I will start my lecture by telling you a story about *Ernst Specker* that I learned from *Raoul Bott*. I will then tell you a few things I learned as an undergraduate student in Specker's course on linear algebra and by listening to him at a political gathering in down-town Zurich.

My main task is, however, to attempt to explain to you the *Kochen-Specker Theorem* concerning the **non-existence of hidden variables** in Quantum Mechanics (QM). Some mathematical problems that arose from this theorem, as well as some recent ideas of how to complete the structure of QM and unravel its message will be sketched.

I will end by telling you some anecdotes and recalling some of **Specker's non-scientific concerns**.

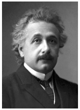
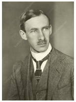
—

And, of course, I would like to express my heartfelt best wishes to *Erwin Engeler* for a happy continuation of his journey!



ETH in 1965 = "hub of the world" – *at least for me!*

Forefathers:

…       …

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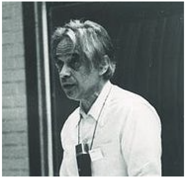
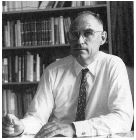
Mathematicians:

*  *     …

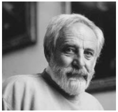
Theoretical Physicists:

    …

A story about Specker's stay at the Institute for Advanced Study in Princeton

In the first half of the 20^{th} Century, thanks to *Heinz Hopf*, ETH Zurich became a world centre of a rather new field in mathematics that *Henri Poincaré* had named **"Analysis situs"**, nowadays called *"algebraic topology"*. – After the completion of his PhD under the supervison of Hopf, Specker spent more than a year at the IAS in Princeton. He knew everything of relevance in algebraic topology. There he met the young Raoul Bott, an electrical engineer turned into a mathematician with an untamed curiosity in algebraic topology who would come up with a new conjecture almost every day and would then try it out on Specker. After he had succeeded in disproving several of Bott's conjectures, Specker proposed a *bet* to him: He would disprove *everyone* of Bott's conjectures within five minutes. For a while, Specker won the bet; but, after some time, the situation changed, and Bott became a famous topologist.

Here is another story I learned from Bott: *Specker and the fire flies ...*



Things I learned in Specker's lectures on linear algebra

Linear equations:

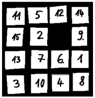
$$\begin{pmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix},$$

or

$$A\,\mathbf{x} = \mathbf{b}\,. \tag{1}$$

When does this equation always have a solution?

What is "$\det(A)$"? What does it have to do with Eq. (1)? What if the homogeneous equation has non-zero solutions? *kernel*, *co-kernel*, *...*

What is the group of permutations of n elements? What is a property common to all permutations of $\{1, \ldots, 15\}$ appearing in 

? ...

Why does a mirror image of your face exchange *Left* with *Right*, but not $\Uparrow$ with $\Downarrow$?

What is the shortest walk through a revolving door of a supermarket?

Etc., etc.

Things I learned from Ernst in the “Weisser Wind”

Nach den Globus-Kravallen: *”Zürcher Manifest”* - erschienen am 4.7.1968

Ein Aufruf zur Besinnung

Zürcher Manifest

Wir stellen fest:
In Zürich ist es zwischen Jugendlichen und der Polizei zu Kämpfen gekommen. Damit brachen auch in unserer Stadt Konflikte auf, wie sie sich gegenwärtig in Ost und West zeigen.

Wir folgern:
Die Zürcher Ereignisse dürfen nicht isoliert beurteilt werden. Sie sind eine Folge unzulänglicher Gesellschaftsstrukturen. Sie als Krawalle abzutun und die Beteiligten nur als randalierende Taugenichtse und Gaffer hinzustellen, ist oberflächlich.

Wir sind überzeugt:
Eine Ursache der Krise ist die Unbeweglichkeit unserer Institutionen. Diese Unbeweglichkeit wendet sich gegen den Menschen. Sie verhindert die Anpassung an die sich wandelnden Bedürfnisse der Menschen und die Entfaltung schöpferischer Minderheiten.

Wir erinnern:
Wesentliche Umwälzungen sind immer von Minderheiten ausgegangen. So fand 1848 der Liberalismus gerade in der Jugend leidenschaftliche Anhänger. Diese Minderheit – damals Revoluzzer genannt – bewahrte die Unabhängigkeit der Schweiz und schuf unseren Bundesstaat.

Wir warnen:
Einen kulturellen Konflikt lösen weder Prügel und Verbote noch Besänftigung durch gönnerhafte Angebote. «Wohltätigkeit ist das Ersaufen des Rechts im Mistloch der Gnade» (Pestalozzi). Unterdrückung der Konflikte treibt die Jugend auf die Barrikaden.

Wir fordern:
1. Bereitstellung eines zentral gelegenen, autonom verwalteten Diskussionsforums für Jung und Alt.
2. Verzicht auf Sanktionen wie Relegation von Studenten und Schülern, Entzug von Stipendien, Ausweisung von Ausländern, Entlassungen, sofern nicht schwerwiegende Delikte vorliegen.
3. Wiederherstellung des verfassungsgemässen Demonstrationsrechts.
4. Fortsetzung der Gespräche mit allen Minderheiten.
5. Einladung zur Meinungsäusserung aller Konfliktparteien durch Presse, Radio und Fernsehen.
6. Unverzügliche Bildung einer wissenschaftlichen Arbeitsgruppe mit dem Auftrag, die tieferen Ursachen des Konflikts zu erforschen und praktische Vorschläge auszuarbeiten.

Forderung 3 inzwischen erfüllt

Ernst Specker →

Ausschreitungen in Zürich

Zeugenaufruf

Wir möchten eine umfassende Dokumentation aufbauen, die den Anwälten von Geschädigten oder Angeschuldigten zur Verfügung steht und die wissenschaftlich ausgewertet werden soll.

Wir bitten alle Personen, insbesondere auch Ärzte, Journalisten und Anwälte, uns Beobachtungen über Vorgänge vom 29./30. Juni 1968 in Zürich und deren Auswirkungen mitzuteilen ohne Rücksichtnahme darauf, welche Konfliktspartei allenfalls belastet wird.

Ihre Informationen, insbesondere Namen und Personenbeschreibungen, werden nur mit Ihrer ausdrücklichen Genehmigung an Anwälte weitergegeben oder für Publikationen verwendet.

Arbeitsgruppe Zürcher Manifest
Dokumentationsstelle

8022 Zürich Postfach 190
Tel. 051 45 02 63 von 17—21 Uhr

Unter den vielen Unterzeichnern des Zürcher Manifestes befinden sich:

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Prof. Dr. Hans Biäsch
Prof. Marcel Beck
Prof. Dr. Hans Burla
Prof. Dr. Kurt von Fischer
Prof. Dr. med. Etienne Grandjean
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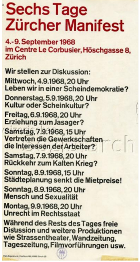


Sechs Tage Zürcher Manifest

4.-9. September 1968
im Centre Le Corbusier, Höschgasse 8, Zürich

Wir stellen zur Diskussion:

Mittwoch, 4.9.1968, 20 Uhr
Leben wir in einer Scheindemokratie?

Donnerstag, 5.9.1968, 20 Uhr
Kultur oder Scheinkultur?

Freitag, 6.9.1968, 20 Uhr
Erziehung zum Jasager?

Samstag, 7.9.1968, 15 Uhr
Vertreten die Gewerkschaften die Interessen der Arbeiter?

Samstag, 7.9.1968, 20 Uhr
Rückkehr zum Kalten Krieg?

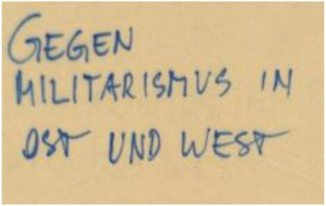
Sonntag, 8.9.1968, 15 Uhr
Städteplanung senkt die Mietpreise!

Sonntag, 8.9.1968, 20 Uhr
Mensch und Sexualität

Montag, 9.9.1968, 20 Uhr
Unrecht im Rechtsstaat

Während des Rests des Tages freie Disussion und weitere Produktionen wie Strassentheater, Wandzeitung, Tageszeitung, Filmvorführungen usw.

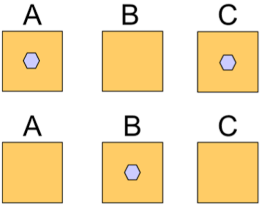
Wandzeitung



!

King Asarhaddon's wise man from Ninive who taught at the school for prophets, named Arba'ilu

Ernst Specker's parable of the suitors of the wise man's daughter:
2 out of 3 boxes, A, B, C, are *either* empty *or* contain, each, a gem.



A	B	C
1	1	0
1	0	1
0	1	1
1	0	0
0	1	0
0	0	1

"Classical-like" prediction
("properties" exist in space-time
before mesurement):
$x \in \{1,0\}$, $y \in \{1,0\}$
$P(x, y \mid x \neq y) = 2/3$
$P(x, y \mid x = y) = 1/3$
whatever pair of boxes is opened

"Quantum-like" prediction
$P(x, y \mid x \neq y) = 1$
$P(x, y \mid x = y) = 0$
whatever pair of boxes is opened.

↑ *Contextuality*
for a "Bell pair"

The Assyrian prophet's contest

Illustration by *A. Suarez*

$$A_f \Rightarrow B_e \Rightarrow C_f\,, \quad \text{but: } A_f \Rightarrow C_e\,, \text{ etc.}$$

Yet, **not all** of A, B, C can be verified simultaneously! $\longrightarrow \cdots$

“Die Logik nicht gleichzeitig entscheidbarer Aussagen”

Ernst Specker, 1960

La logique est d’abord une science naturelle. - F. Gonseth

“Kann die Beschreibung eines quantenmechanischen Systems durch Einführung von zusätzlichen – fiktiven – Aussagen so erweitert werden, dass im erweiterten Bereich die klassische Aussagenlogik gilt ... ? [meaning that all statements/results of experiments on the system could be embedded in a Boolean lattice.]

Die Antwort auf diese Frage ist **negativ**, *ausser im Fall von Hilbertschen Räumen der Dimension 1 und 2. ... Ein elementargeometrisches Argument zeigt, dass eine solche Zuordnung* (such an embedding) **unmöglich** *ist, und dass daher über ein quanten-mechanisches System (von Ausnahmefällen abgesehen) keine konsistenten Prophezeiungen möglich sind.”*

In his paper, Specker does not present any details concerning the “elementargeometrische Argument”. They were provided in the famous paper by Kochen and Specker, seven years later, which I paraphrase next.

"The Problem of Hidden Variables in Quantum Mechanics"

Simon Kochen and Ernst Specker, 1967

Question: $\exists$ a hidden-variables theory recovering the predictions of quantum mechanics; or, in other words, can the predictions of quantum mechanics be embedded in a Boolean lattice?

Let S be a physical system to be described quantum-mechanically. Its Hilbert space of pure state vectors is denoted by $\mathfrak{H}$; ...
If the answer to the above question were *"yes"* this would imply that $\exists$ a measure space $(\Omega, \mathfrak{F})$ and maps f and ρ,

$$f : A = A^* \in B(\mathfrak{H}) \mapsto f_A : \Omega \to \mathbb{R}, \tag{1}$$
$$\rho : \Psi \in \mathfrak{H} \mapsto \rho_{[\Psi]} = \text{probability measure on } (\Omega, \mathfrak{F}),$$

with the following properties.

(P1) Preservation of expectation values: For every $A = A^* \in B(\mathfrak{H})$,

$$\|\Psi\|^{-2} \langle \Psi, A\,\Psi \rangle = \int_\Omega f_A(\omega)\, d\rho_{[\Psi]}(\omega)$$

Properties of a putative embedding in a Boolean lattice

(P2) If $u : \mathbb{R} \to \mathbb{R}$ is an arbitrary bounded measureable function then

$$f_{u(A)} = u \circ f_A.$$

<u>Note</u>: (P1) and (P2) are compatible with each other (check!); and (P1) and (P2) imply the following fact:

(P3) Given any abelian algebra $\mathfrak{M}$ of commuting self-adjoint operators acting on $\mathfrak{H}$, then

$$f : A \in \mathfrak{M} \mapsto f_A \in L^\infty(\Omega)$$

is an algebra homomorphism; i.e.,

$$f_{A_1 \cdot A_2} = f_{A_1} \cdot f_{A_2}, \quad \forall A_1, A_2 \text{ in } \mathfrak{M}.$$

(Easy to prove if $\dim(\mathfrak{H}) < \infty$!)

The Kochen-Specker Theorem

As already noticed by Specker in 1960, a hidden-variables theory satisfying (P1) - (P3) exists if $\dim(\mathfrak{H}) = 1$ or 2, (QM of a spin-$\frac{1}{2}$ object – nowadays called "Qbit", which sounds more interesting).

Theorem. (Kochen & Specker, 1967)
If $\dim(\mathfrak{H}) \geq 3$ a hidden-variables theory satisfying (P1)-(P3) does **not** exist.

Proof. We consider a particle, whose spin degree of freedom is described by a vector operator, $\vec{S}$, acting on the Hilbert space $\mathfrak{H} = \mathbb{C}^3 \simeq \mathbb{R}^3 \otimes \mathbb{C}$, (i.e., the particle has spin 1). Let $(\vec{n}_1, \vec{n}_2, \vec{n}_3)$ be the standard ortho-normal basis in $\mathbb{R}^3$ and set $S_j := \vec{S} \cdot \vec{n}_j$, $j = 1, 2, 3$. Then

$$S_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \; S_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \; S_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

One thus observes that the operators $P_j := 1 - S_j^2$, $j = 1, 2, 3$, are three mutually commuting orthogonal projections of rank 1, with $\sum_{j=1}^{3} P_j = 1$.

Arbitrary orthonormal bases in $\mathbb{R}^3$

More generally, for an arbitrary vector $\vec{e}$ in S^2, $P(\vec{e}) := 1 - (\vec{S} \cdot \vec{e})^2$ is an orthogonal projection projecting onto the one-dimensional subspace of $\mathfrak{H}$ spanned by $\vec{e}$. $\big[$Thus, the matrix elements of $P(\vec{e})$ in the basis $(\vec{n}_1, \vec{n}_2, \vec{n}_3)$ are given by $P(\vec{e})_{ij} = e_i e_j\,,\ \forall i, j\,.\big]$

For an *arbitrary* orthonormal basis, $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$, one then finds that

$$\sum_{j=1}^{3} P(\vec{e}_j) = 1\,, \quad P(\vec{e}_i) \cdot P(\vec{e}_j) = \delta_{ij} P(\vec{e}_i)\,. \tag{2}$$

The projections $\{P(\vec{e}_j)\}_{j=1}^{3}$ are functions of a single self-adjoint operator

$$A := \sum_{j=1}^{3} \alpha_j P(\vec{e}_j)\,, \quad \alpha_1 < \alpha_2 < \alpha_3\,. \tag{3}$$

generating a maximally abelian subalgebra of $B(\mathfrak{H}) = \mathbb{M}_3(\mathbb{C})$.

A fatal assumption

We now **assume** that $\exists$ a hidden-variables theory satisfying properties (P1), (P2) and (P3).

Since $P(\vec{e})^2 = P(\vec{e})$, it follows from (P2) that

$$P(\vec{e}) \mapsto f_{P(\vec{e})} =: \chi_{\vec{e}} \tag{4}$$

is a characteristic function on Ω. Eq. (2) implies that, for an arbitrary orthonormal basis $(\vec{e}_1, \vec{e}_2, \vec{e}_3)$,

$$\sum_{j=1}^{3} \chi_{\vec{e}_j} = 1, \qquad \text{a.e. on } \Omega\,. \tag{5}$$

For any point $\omega \in \Omega$,

$$\varphi_\omega(\vec{e}) := \chi_{\vec{e}}(\omega) \tag{6}$$

defines a function on S^2 with the following properties (which hold for a.e. $\omega \in \Omega$):

Strange functions on the unit sphere in $\mathbb{R}^3$

(i) It takes only the values 0 and 1, i.e.,

$$\varphi_\omega(\vec{e}) = 0 \ \text{ or } \ 1, \ \text{for any unit vector } \ \vec{e} \in S^2.$$

(ii) If $\vec{e}$ belongs to any orthonormal basis $\{\vec{e}_1 \equiv \vec{e}, \vec{e}_2, \vec{e}_3\}$ of $\mathbb{R}^3$ then the value, $\varphi_\omega(\vec{e})$, of φ_ω on $\vec{e}$ should be **independent** of the choice of $\vec{e}_2$ and $\vec{e}_3$, and

$$\sum_{j=1}^{3} \varphi_\omega(\vec{e}_j) = 1\,.$$

This follows from Eqs. (5) and (6).

(iii) Properties (i) and (ii) imply that the function φ_ω is an *additive measure on the lattice of orthogonal projections* acting on $\mathbb{C}^3 = \mathbb{R}^3 \otimes \mathbb{C}$, for almost all $\omega \in \Omega$.

Das "elementargeometrische Argument"

The evaluation of a function φ_ω with properties (i) - (iii) on finitely many unit vectors in $\mathbb{R}^3$, which give rise to finitely many orthonormal bases in $\mathbb{R}^3$, leads to the *contradiction* that, for some unit vectors $\vec{e}$, $\varphi_\omega(\vec{e}) = 0$ **and** $\varphi_\omega(\vec{e}) = 1$, depending on which completion of $\vec{e}$ to an orthonormal basis of $\mathbb{R}^3$ is considered – *"contextuality"*.

Kochen and Specker have found an explicit construction of finitely many unit vectors in S^2 leading to this contradiction. By now the best variant of their construction appears to require only 18 unit vectors.

There is an abstract proof of the claim that functions φ_ω on S^2 with properties (i) - (iii) do **not** exist, which is based on *Gleason's* theorem:[1]

Property (iii) says that the function φ_ω is an additive measure on the lattice of projections, for a.e. $\omega \in \Omega$. Gleason's theorem then says that

$\exists$ a *density matrix* $\Phi_\omega > 0$, with $\mathrm{tr}(\Phi_\omega) = 1$, such that

$$\varphi_\omega(\vec{e}) = \mathrm{tr}\big(\Phi_\omega\, P(\vec{e})\big) = \langle \vec{e}, \Phi_\omega\, \vec{e} \rangle \, .$$

This shows that $\exists$ a unit vector $\vec{e}$ such that $0 < \varphi_\omega(\vec{e}) < 1$. But this **contradicts** property (i) !

[1] I am grateful to *N. Straumann* for having explained this argument to me.

Connection to Kakutani's theorem[2]

We note that Gleason's theorem apparently implies that the functions $\varphi_\omega(\vec{e})$ are **continuous** in $\vec{e}$.

Thus, let us consider an arbitrary real-valued, **continuous** function, φ, on the n-dimensional sphere S^n in $\mathbb{R}^{n+1}$ centered at the origin $\mathcal{O}$. *Dyson's* variant of *Kakutani's* theorem says that $\exists\, n+1$ points, $x_1, x_2, \ldots, x_{n+1}$, on S^n such that the $n+1$ unit vectors $\left\{\vec{e}_j := \overline{\mathcal{O}\, x_j} \mid j = 1, 2, \ldots, n+1\right\}$ are mutually orthogonal, and

$$\varphi(\vec{e}_1) = \varphi(\vec{e}_2) = \cdots = \varphi(\vec{e}_{n+1}).$$

For $n = 2$, this contradicts properties (i) and (ii) of the functions φ_ω !

Remarks:

1. Ultimately, all the theorems asserting that there does not exist a hidden-variables theory reproducing the predictions of quantum mechanics exploit, in one way or another, the obvious fact that there does not exist a homomorphism from a non-commutative algebra *into* an *abelian* algebra. *Mermin's* version of Kochen-Specker makes this particularly clear.

[2] See F. J. Dyson, Ann Math. **54**, 534-536 (1951)

Open problems

2. *Gleason's* theorem can be generalized as follows: Additive measures on the lattice of orthogonal projections of a **general** von Neumann algebra are given by *normal states* on the von Neumann algebra[3].

3. It would be interesting to extend *Huaxin Lin's* theorem on almost commuting self-adjoint operators to the setting of pairs, (ω, X), of a state, ω, on a von Neumann algebra $\mathfrak{M}$ and an operator $X \in \mathfrak{M}$ with the property that $ad_X(\omega)$ has a tiny norm.

4. *Bell's inequalities*: Consider correlations between outcomes of some family of *commuting* measurements on two "distant" systems. It turns out that if *quantum-mechanical* correlations are shrunk by a constant ${K_G}^{-1} < 1$ the resulting values lie inside the range of corresponding *classical* correlations; (Tsirelson's theorem). Here, $K_G \approx 1.782$ is the *Grothendieck constant* appearing in the theory of tensor products. – However, as Bell's inequality shows, the range of quantum-mechanical correlations is *strictly larger* than the range of corresponding classical correlations; ($\nearrow$ blackboard!)

[3] see, e.g., *L. J. Bunce* & *J. D. Maitland Wright*, BAMS, **26**, 288-293 (1992)

The four pillars of full-fledged Quantum Mechanics

So far, we have considered a torso of QM, because we have not studied the roles played by *"time"*, *"events"*, and *"evolution"* in QM, yet.

Quantum Theory of *isolated physical systems* rests on 4 pillars:

(i) *"Potential events possibly happening at time t or later"* are represented by families of disjoint orthogonal projections in a v.N. algebra $\mathcal{E}_{\geq t}$. The *descending filtration* of algebras $\left\{\mathcal{E}_{\geq t}\right\}_{t\in\mathbb{R}}$ satistfies the *"Principle of Diminishing Potentialities"*: $\mathcal{E}_{\geq t'} \subsetneq \mathcal{E}_{\geq t}, \ \forall t' > t$.

(ii) *"States at time t"* are additive measures on the lattice of projections in $\mathcal{E}_{\geq t} \equiv$ normal states on $\mathcal{E}_{\geq t}$, $t \in \mathbb{R}$; ($\nearrow$ 2.)

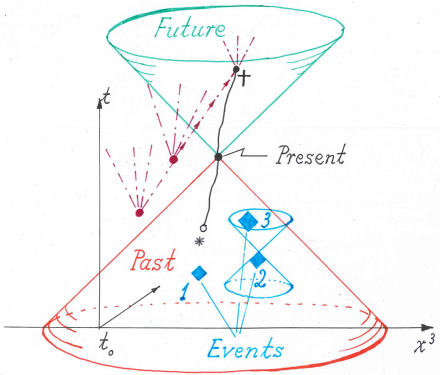
(iii) *Heisenberg-picture "time evolution"* (of operators by conjugation with a unitary propagator) acts on $\left\{\mathcal{E}_{\geq t}\right\}_{t\in\mathbb{R}}$ by shift of t; and

(iv) Definition of *"(actual) events"* ($\nearrow$ 3.) ... $\rightarrow$ understanding the *stochastic time evolution* of states[4], which branch when an "event" happens, the branching probabilities being given by *Born's* rule.

This view of QM (in particular (i) and (iv)) is recent and remains to be fully appreciated by the community at large. – It shows that ...

[4] i.e., "keine konsistenten Prophezeiungen möglich ..."

… Quantum Mechanics is intrinsically irreversible!



Past = History of Events (Facts) / Future = Ensemble of Potentialities

Quantum Mechanics implements this fundamental dichotomy.

—

QM remains a great source of interesting math problems!

Some personal reminscences

Ernst Specker was born on 11/02/1920 and died on 10/12/2011. Notice that he was born on a day, the 11^{th}, that is a *prime number*. (Incidentally, the year of his death, 2011, is prime, too.)

I was born on 04.07.1946; i.e., I was born on a day that is definitely **not** a *prime number*.

Back in 1976, I applied for a job in the math department of ETH. On that occasion, Ernst asked me on what day I was born. When I disclosed my birth date to him he said: *"I am afraid you will never become a good mathematician"*. Why? *"Because ..."* – Luckily I didn't get the job.[5]

In any event, some years later, in 1982, I became a colleague of Ernst's, albeit in the physics department of ETH. We slowly developed amicable social contacts. And I also became aware of his interests in questions of religious faith and of his sermons in the *"Predigerkirche"*. – Later, he gave me the booklet of his sermons as a present. I particularly like *"Eine äthiopische Apostelgeschichte"*, which tells the story of *Apezemak*.

[5] Much later I heard that, after my talk, Ernst had expressed a surprisingly positive opinion about me, but felt the job was not suitable for me. – I think he was right!

Specker's interests in questions of faith

His interests in questions transcending rationality and logics are already apparent in his 1960 paper on quantum logics:

"Die Schwierigkeiten, die durch Aussagen entstehen, welche nicht zusammen entscheidbar sind, treten besonders deutlich hervor bei Aussagen über ein quantenmechanisches System. ... In einem gewissen Sinne gehören aber auch die scholastischen Spekulationen über die "Infuturabilien" hierher, das heisst die Frage, ob sich die göttliche Allwissenheit auch auf Ereignisse erstrecke, die eingetreten wären, falls etwas geschehen wäre, was nicht geschehen ist." –

In: "Die Logik nicht gleichzeitig entscheidbarer Aussagen"

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At the end of a lecture for a general public by the theologian *Pierre Bühler* Ernst asked the speaker: *"Glauben Kinder an den Osterhasen?"*

Bühler was perplexed. – I suppose that Ernst wanted to indicate that **faith** (Glauben) is not about *"truth"* and *"existence"*, and that these categories have no place in debates of religious revelations.

"immer besser"

Evidently it is **not** factual that there is an easter-bunny hiding eggs in the garden. Yet, the *"idea of the easter bunny"* bringing eggs, symbols of life and fertility, to the children has meaning and impact and **"exists"** as an element in *Plato's* **"Realm of Ideas"** ("World of Universals").

Roughly fifteen years ago, my wife and I ran into Ernst somewhere in this wonderful building. Not knowing what to talk about, my wife asked: *"How are you doing, Ernst?"* Whereupon he answered:

"Immer besser!"

And then he explained to us why he gave this answer ...; which converted a light conversation into one where a piece of worldly wisdom was communicated by someone who had reached the state of serenity. –

I imagine that the conditions of his "being", freed from the dungeon of "time" and "causality", would never stop improving were there "time" in the "World of Universals", where he resides. Though Ernst has reached *"the state of the flame when it is extinct"* (*Buddha*), "eggs" in the form of his thought-provoking ideas keep being (re-)discovered!

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I thank you for your attention!