

# **60 Years of SAT Solving**

## ***– Applications to Automotive Configuration –***

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# **Variant-rich automotive configuration**

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- Variant-rich „individualized mass production“
  - Configure 1 out of  $\sim 10^{30}$  cars, observing Boolean constraints
- 2 Levels: Product description (PD) + parts list (BoM)
  - **High Level PD:** Which configurations can be built?
    - Boolean constraints system,  $\sim 500\text{KB}$ , 1000s constraints,  $\sim 10^{30}$  solutions
    - Daimler: PÜ; VW: MBT; BMW: VRM; GM: VDS; Renault, Peugeot, ...
  - **Low Level BoM:** Which parts go into each configuration?
    - List of all (10,000s) parts for each model line (e.g. C Class, A4, Golf)
    - Boolean conditions (*if cond then part*) select the parts for each order
- **Mechanical Theorem Proving → Formal Verification**
  - verify properties, detect defects, answer queries, optimize
  - with respect to the full theoretical variance ( $\sim 10^{30}$  orders)



# Mercedes High-Level Product Description

Example: E-Class

- approx. 1.500 codes (options, countries, ..)
- approx. 3.000 rules in product overview (PÜ / PD)
- rule-based BOM with approx. 35,000 parts.
- B(code): condition for presence of code in order
- Z(code): condition for automatic addition of code



$$B(P09) = 297+540+543;$$

$$B(610) = (512 / 527 / 528)+608+ -978;$$

$$Z(P09) = (M271 + -M013 / M272) + 830 / \\ Z04+ -(M273 + Z27)$$

$$Z(682) = 623 / 830 / 513L$$

$$B(450) = L+965+ 670+837+ -P34+ \\ ((M271/M651)+(953/955)+(100A/200A)+(334/335)+(301/336/337) / \\ / M642+(2XXL/557L/571L)+(953/955)+(100A/200A)+(334/335)+(301/337));$$



# *Low-Level Product Description (BOM)*

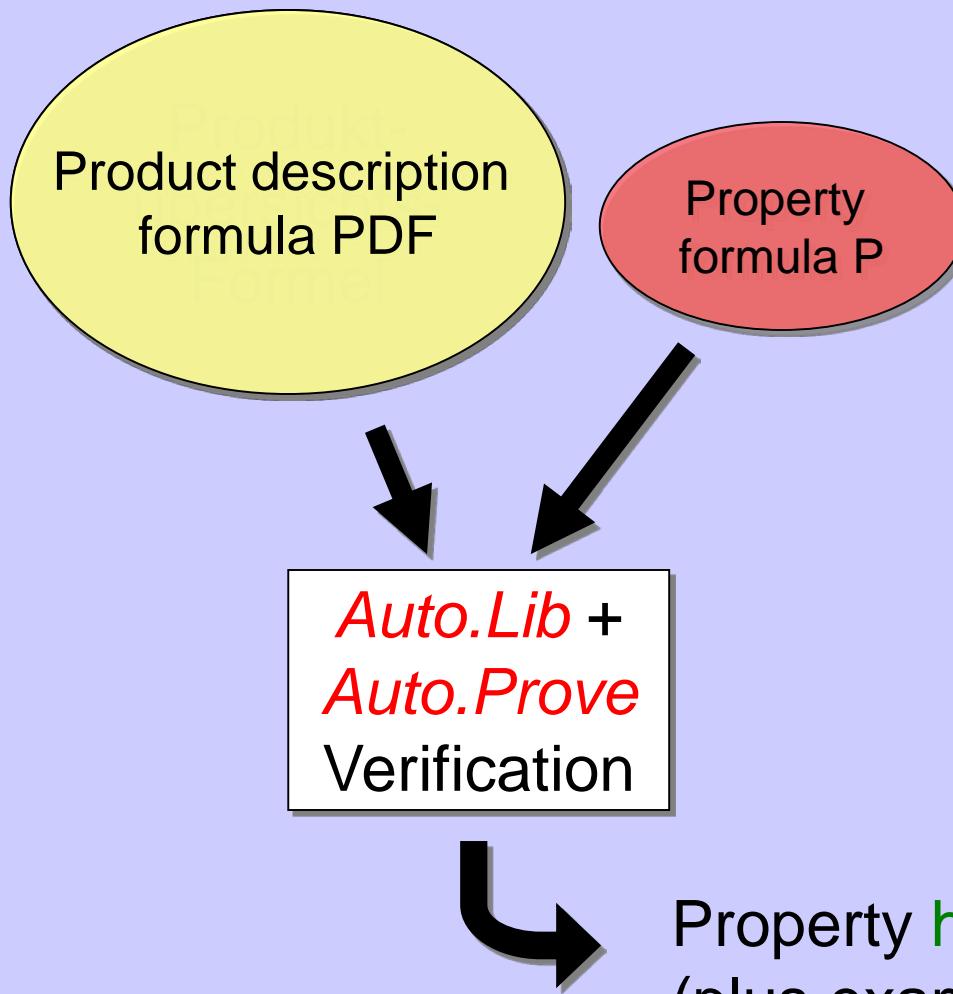
## ➤ BOM – Bill-of-Materials

- List of Materials (parts, software, colors) for entire car model line
- Grouped by Functionality (e.g. steering wheels, headlights,...)
- Within group: List of alternative parts (materials)
- List of Tupels: <Material, Boolean Selection Condition>

ZB	Benennung	Codebedingung
POS	ZB RAD 34R	
ZB	ZBU RAD MIT REIFEN 34R	
ZB	RAD / 7J X 16H2 + 205/55 R16 91V	34R+R01+V22;
ZB	RAD / 7J X 16H2 + 205/55 R16 91V	34R+R01+V22+(470/475);
ZB	RAD / 7J X 16H2 + 205/55 R16 91W	34R+R01+V50;
ZB	RAD / 7J X 16H2 + 205/55 R16 91W	34R+R01+V50+(470/475);
ZB	RAD / 7J X 16H2 + 205/55 R16 91H	34R+R01+V21;
ZB	RAD / 7J X 16H2 + 205/55 R16 91H	34R+R01+V21+(470/475);
ZB	RAD / 7J X 16H2 + 205/55 R16 91W	34R+R01+V50+M272+M35+M012;
ZB	RAD / 7J X 16H2 + 205/55 R16 91H AL...	34R+R02+V21;
ZB	RAD / 7J X 16H2 + 205/55 R16 91H M...	34R+645+V21;
ZB	RAD / 7J X 16H2 + 205/55 R16 91H M...	34R+645+V21+(470/475);
ZB	RAD / 7J X 16H2 + 205/55 R16 91H M...	34R+645+V21+M272+M35+M012;



# Verification by *Auto.Prove* Mechanical Prover



## Standard Checks on PD

- Codes which CANNOT be selected
- Codes which MUST be selected
- Per Model line, model, country ...

## Standard Checks on BOM

- No-Hits (part is missing for some car)
- Double-Hits (some car gets 2 parts)
- Orphaned parts (no car gets the part)

## Individual (free) Queries

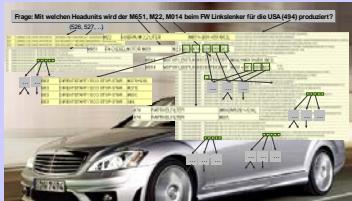
- Codes optional in US, ....
- Parts deliverable to Japan, ...
- → Configurator



# Query Example: Explaining PD Inconsistency

Ex: Country code 817L „Taiwan“ impossible for station wagon, model type Cxxx

Analysis of product description which yields contradiction and pinpoints the problem:



Auto.Prove

Code rule for 817L (Taiwan): L+M272+M30  
Code rule for M30 (displacement): M642 / R  
Code rule for M642 (engine): -M272

## Conclusion:

Taiwan (817L) requires L+M272+M30. Since engine displacement M30 with motor M272 requires right-hand steering R, no orders for Taiwan (817L) can be manufactured.  
→ The product overview is defective / incomplete and the wagon is impossible for Taiwan!

Defects in the product overview can potentially cause significant damage (Mio Euros):

- Delivery of non certified Wheel/Tyre combinations to Asia
- Delivery of non certified aggregate combinations to China
- Delivery of headunits to Asia which can only display Latin characters



# *Our SAT based configurator framework **Auto.Config***

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## ➤ Car configuration task

- User enters some option combination A+B+...
- User selects any option family F in any order (e.g. motors, seats, wheels ..)
- *Auto.Config* computes paths to success (SAT Solving the config rules)
  - which options in F can still be selected (which are implied / which are impossible)
  - and which BoM parts are already implied / which are impossible / still possible
- User selects an option O in F and repeats the process with A+B+O

## ➤ Re-configuration task

- User selects an impossible option O
- *Auto.Config* computes (using a MaxSAT algorithm)
  - minimal change to order (undo previous selections) so O can be selected

## ➤ Weak learning AI?

- we program only the *Config* framework, then load (learn?) PD and BoM **rules**
- If *Config* is wrong, **error can be in programming, or in the rules**



# Example: Configuration with Auto.Config

- Start with a model
- *Auto.Config* computes arbitrary PD-valid order
- Select any option family
- **Bold:** *Auto.Config* computes options available for this model
- Grey: *Auto.Config* computes options not available for this model

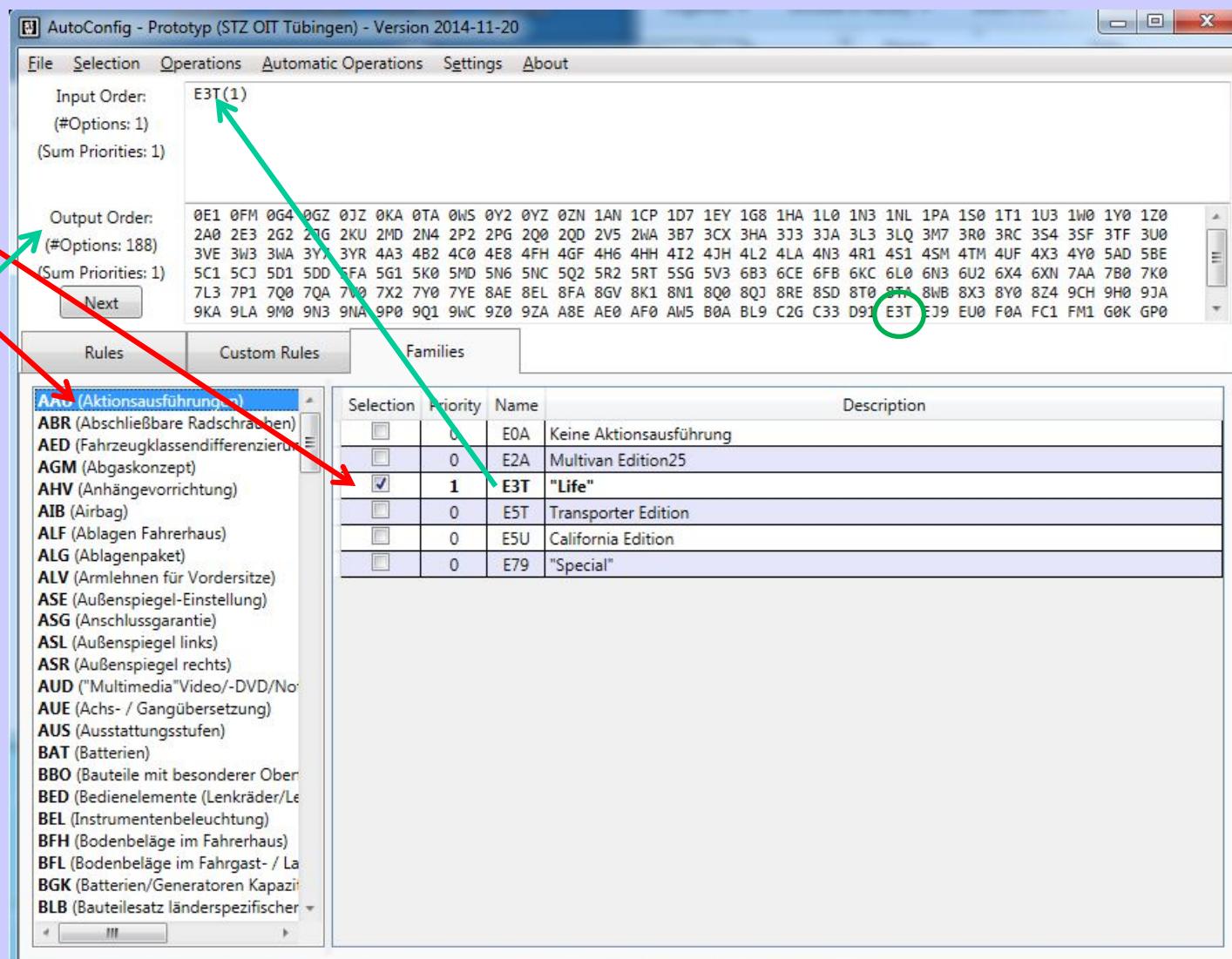
The screenshot shows the AutoConfig software interface. At the top, there's a menu bar with File, Selection, Operations, Automatic Operations, Settings, and About. Below the menu is a section for Input Order, Output Order, and a large list of options. The Output Order section shows 188 options with sum priorities of 0. A green arrow points from the text "computes arbitrary PD-valid order" to the Output Order list. Below this is a table titled "Families" with columns for Selection, Priority, Name, and Description. The table lists several car models (E0A, E2A, E3T, E5T, E5U, E79) with priority 0. A red arrow points from the text "Select any option family" to the "Families" table. Another green arrow points from the text "Grey: Auto.Config computes options not available for this model" to the "Families" table, where the first row (E0A) is highlighted in grey.

Selection	Priority	Name	Description
<input type="checkbox"/>	0	E0A	Keine Aktionsausführung
<input type="checkbox"/>	0	E2A	Multivan Edition25
<input type="checkbox"/>	0	E3T	"Life"
<input type="checkbox"/>	0	E5T	Transporter Edition
<input type="checkbox"/>	0	E5U	California Edition
<input type="checkbox"/>	0	E79	"Special"



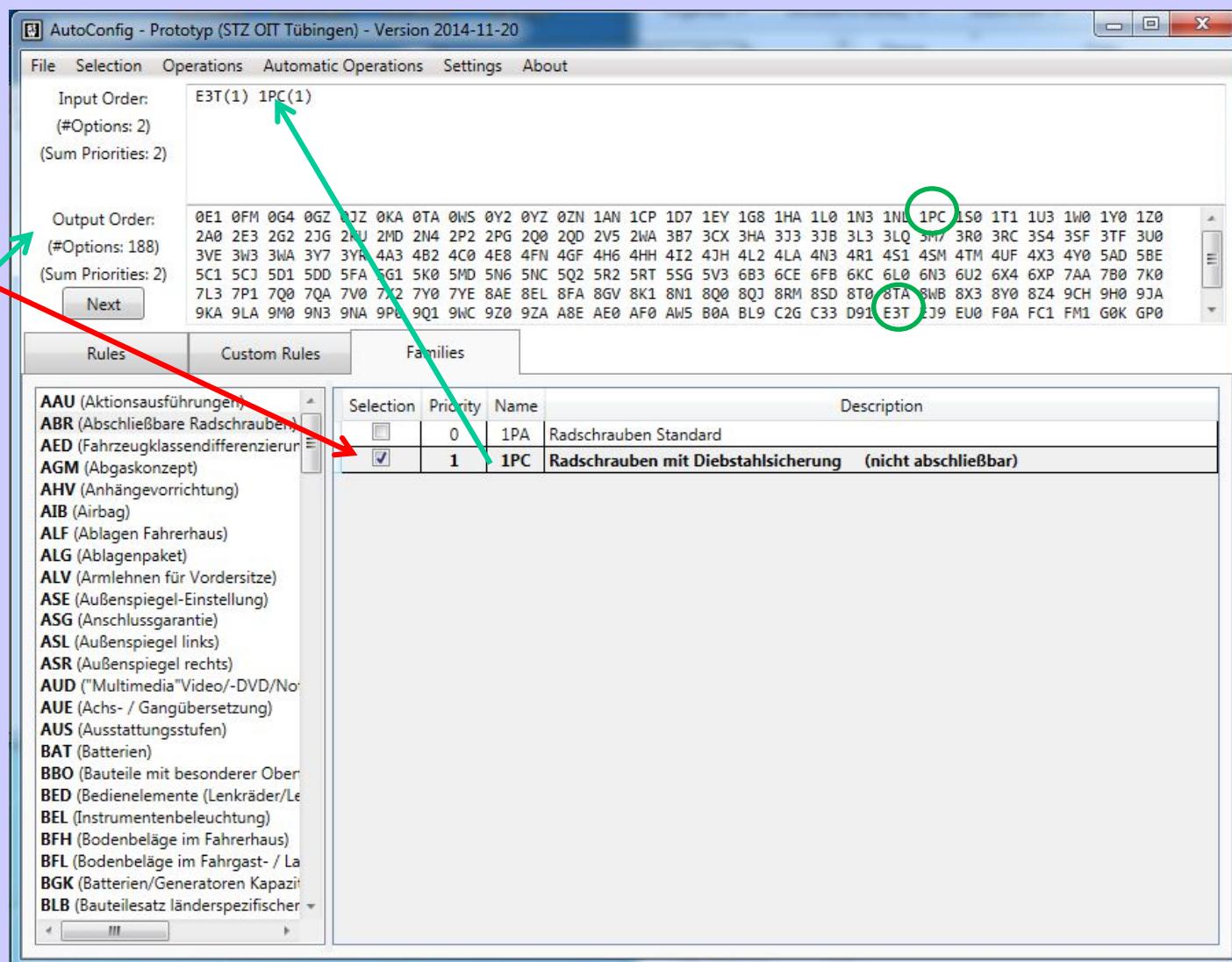
# Example: Configuration with Auto.Config

- Select any option family
- Select any available option
- *Auto.Config* re-computes PD-valid order with selected option
- **Bold:** *Auto.Config* computes options available for this input order
- Grey: *Auto.Config* computes options not available for this input order



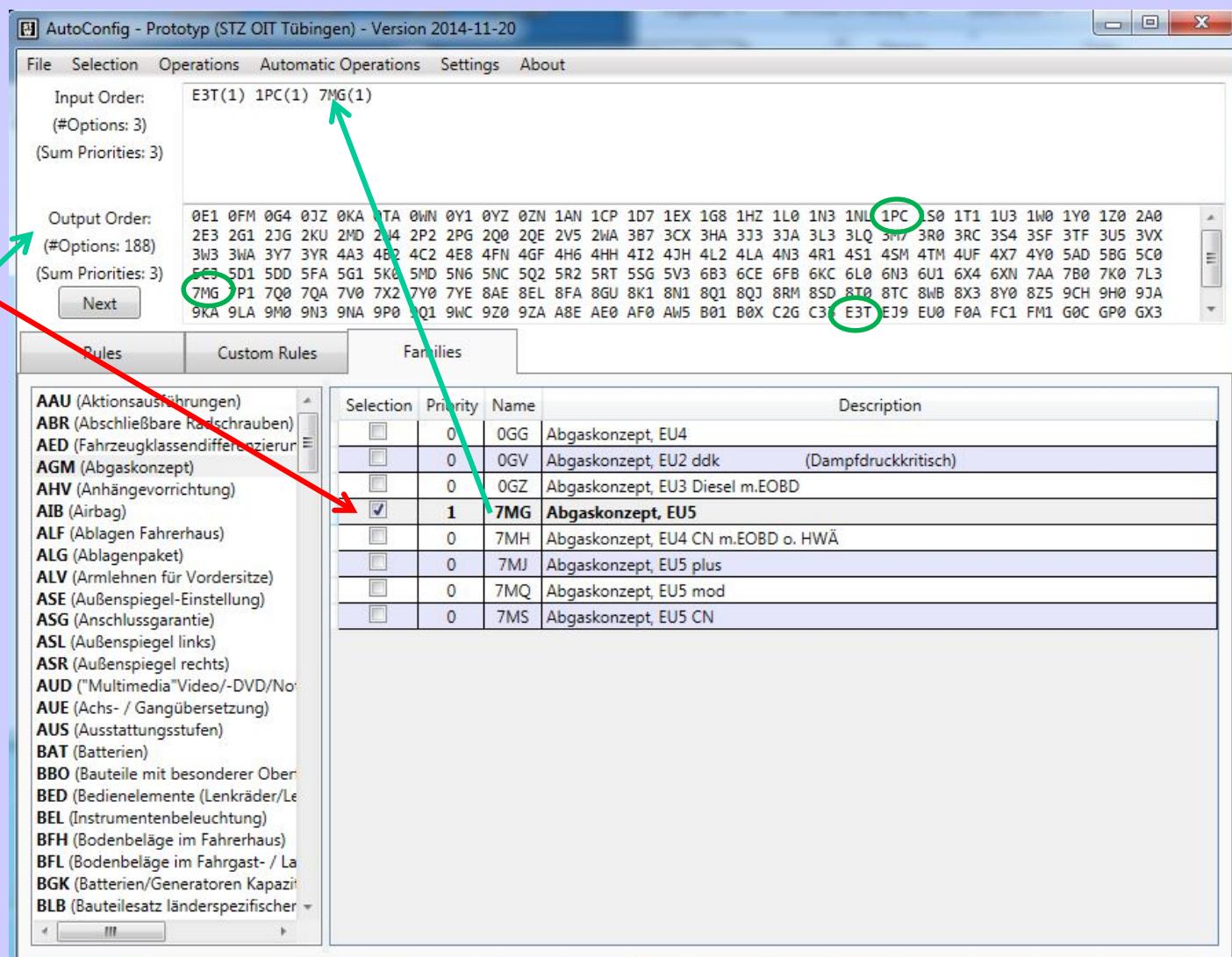
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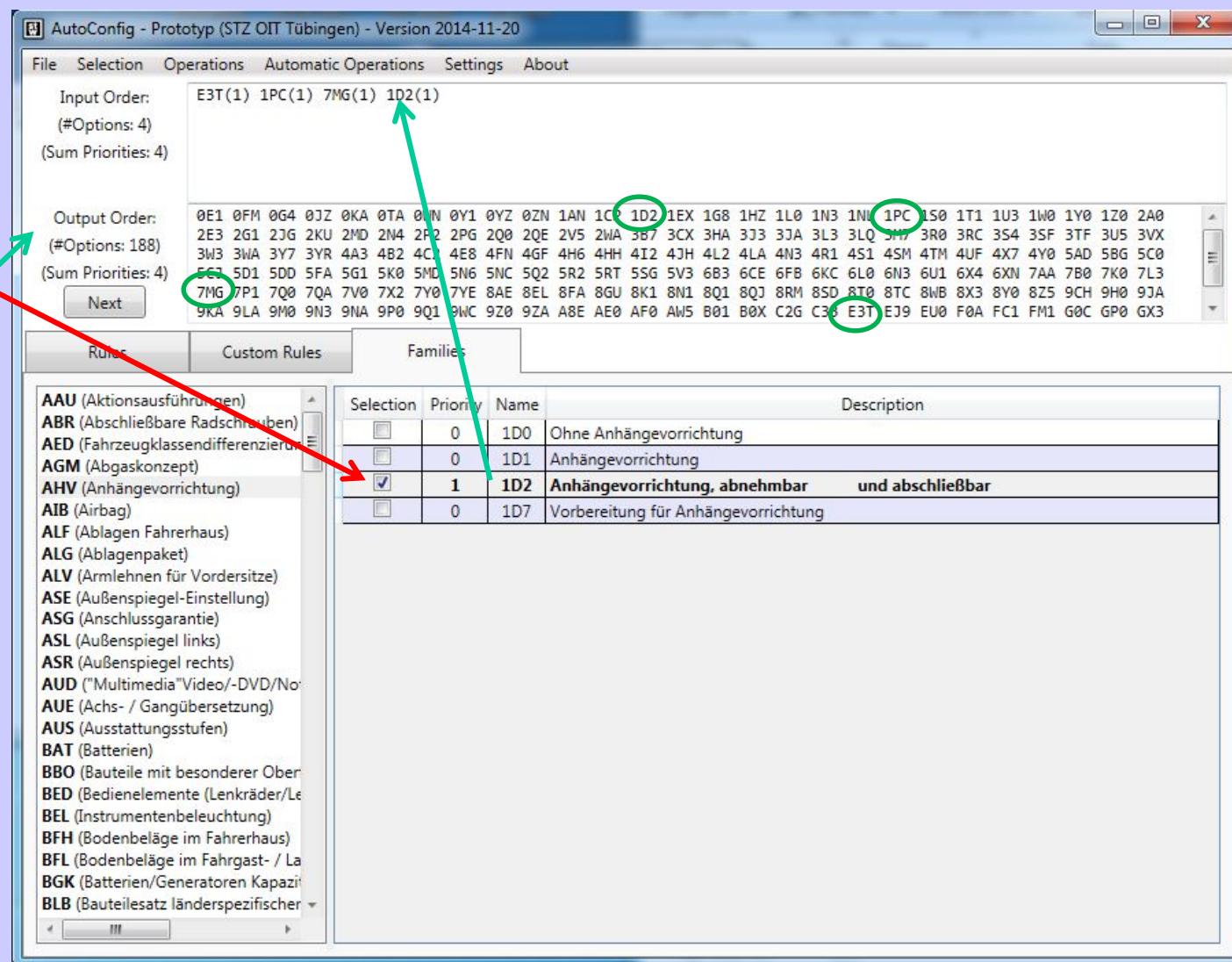
# Example: Configuration with Auto.Config

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# Example: Configuration with Auto.Config

- Select any option family
- Select any available option
- *Auto.Config* re-computes PD-valid order with selected option
- **Bold:** *Auto.Config* computes options available for this input order
- **Grey:** *Auto.Config* computes options not available for this input order



# Example: Configuration with Auto.Config

- Select any option family
- Grey: options not available for this input order
- **Bold:** available options in family for this input order

The screenshot shows the AutoConfig software interface. At the top, it displays the input order: E3T(1) 1PC(1) 7MG(1) 1D2(1). Below this, the output order is listed with various option codes. A red line from the first bullet point points to the input order. A green line from the second bullet point points to the output order. A red arrow from the third bullet point points to the list of families on the left.

**Input Order:**  
E3T(1) 1PC(1) 7MG(1) 1D2(1)  
(#Options: 4)  
(Sum Priorities: 4)

**Output Order:**  
OE1 0FM 0G4 0JZ 0KA 0TA 0WN 0Y1 0YZ 0ZN 1AN 1CP 1D2 EX 1G8 1HZ 1L0 1N3 1N\_ 1PC S0 1T1 1U3 1W0 1Y0 1Z0 2A0  
2E3 2G1 2JG 2KU 2MD 2N4 2P2 2PG 2Q0 2QE 2V5 2WA 3B7 3CX 3HA 3J3 3JA 3L3 3LQ 3M7 3R0 3RC 3S4 3SF 3TF 3U5 3VX  
3W3 3WA 3Y7 4A3 4B2 4C2 4E8 4FN 4GF 4H6 4I2 4JH 4L2 4LA 4N3 4R1 4S1 4M 4T 4UF X7 4Y0 5AD 5BG 5C0  
5C1 5D1 5DD 5FA 5G1 5K0 5MD 5N6 5NC 5Q2 5R2 5RT 5SG 5V3 6B3 6CE 6FB 6KC 6L0 6N3 6U1 6V1 6XN 7AA 7B0 7K0 7L3  
**7MG** P1 7Q0 7QA 7V0 7X2 7Y0 7YE 8AE 8EL 8FA 8GU 8K1 8N1 8Q1 8QJ 8RM 8SD 8T0 8TC 8WB 8X3 8Y0 8Z5 9CH 9H0 9J0  
9KA 9LA 9M0 9N3 9NA 9P0 9Q1 9NC 9Z0 9ZA A8E AE0 AF0 AW5 B01 B0X C2G C33 E3T E39 EU0 F0A FC1 FM1 G0C GP0 GX3

**Rules**   **Custom Rules**   **Families**

Selection	Priority	Name	Description
<input type="checkbox"/>	0	4UC	Airbag für Fahrer
<input type="checkbox"/>	0	4UE	Airbag für Fahrer und Beifahrer
<input type="checkbox"/>	5	<b>4UF</b>	Airbag für Fahrer und Beifahrer mit Beifahrer-Airbag-Deaktivierung

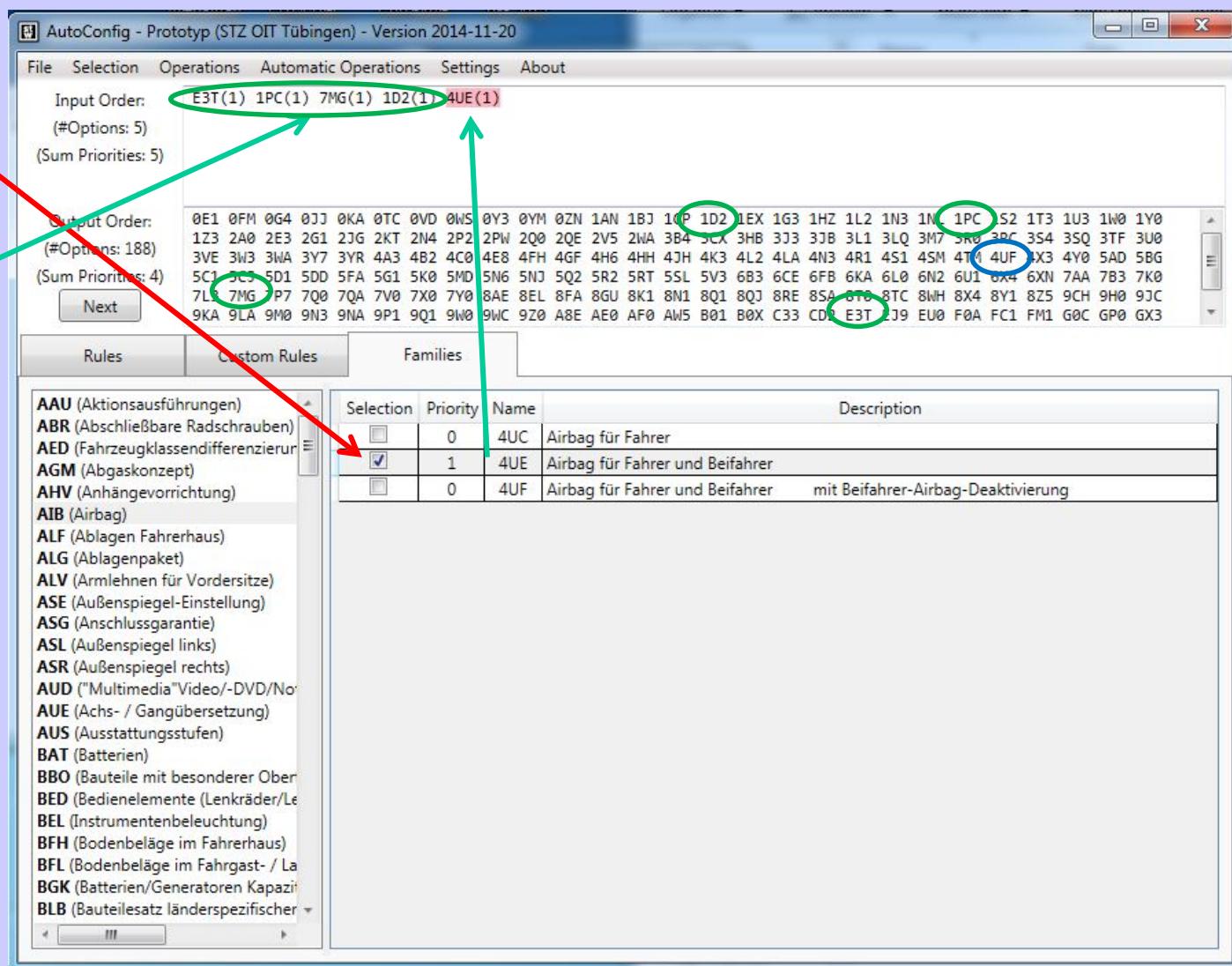
**Families:**

- AAU (Aktionsausführungen)
- ABR (Abschließbare Radschrauben)
- AED (Fahrzeugklassendifferenzierer)
- AGM (Abgaskonzept)
- AHV (Anhängevorrichtung)
- AIB (Airbag)**
- ALF (Ablagen Fahrerhaus)
- ALG (Ablagenpaket)
- ALV (Armlehnen für Vordersitze)
- ASE (Außenspiegel-Einstellung)
- ASG (Anschlussgarantie)
- ASL (Außenspiegel links)
- ASR (Außenspiegel rechts)
- AUD ("Multimedia"-Video-/DVD-/No-
- AUE (Achs- / Gangübersetzung)
- AUS (Ausstattungsstufen)
- BAT (Batterien)
- BBO (Bauteile mit besonderer Ober-
- BED (Bedienelemente (Lenkräder/Le-
- BEL (Instrumentenbeleuchtung)
- BFH (Bodenbeläge im Fahrerhaus)
- BFL (Bodenbeläge im Fahrgast- / La-
- BGK (Batterien/Generatoren Kapazit
- BLB (Bauteilesatz länderspezifischer



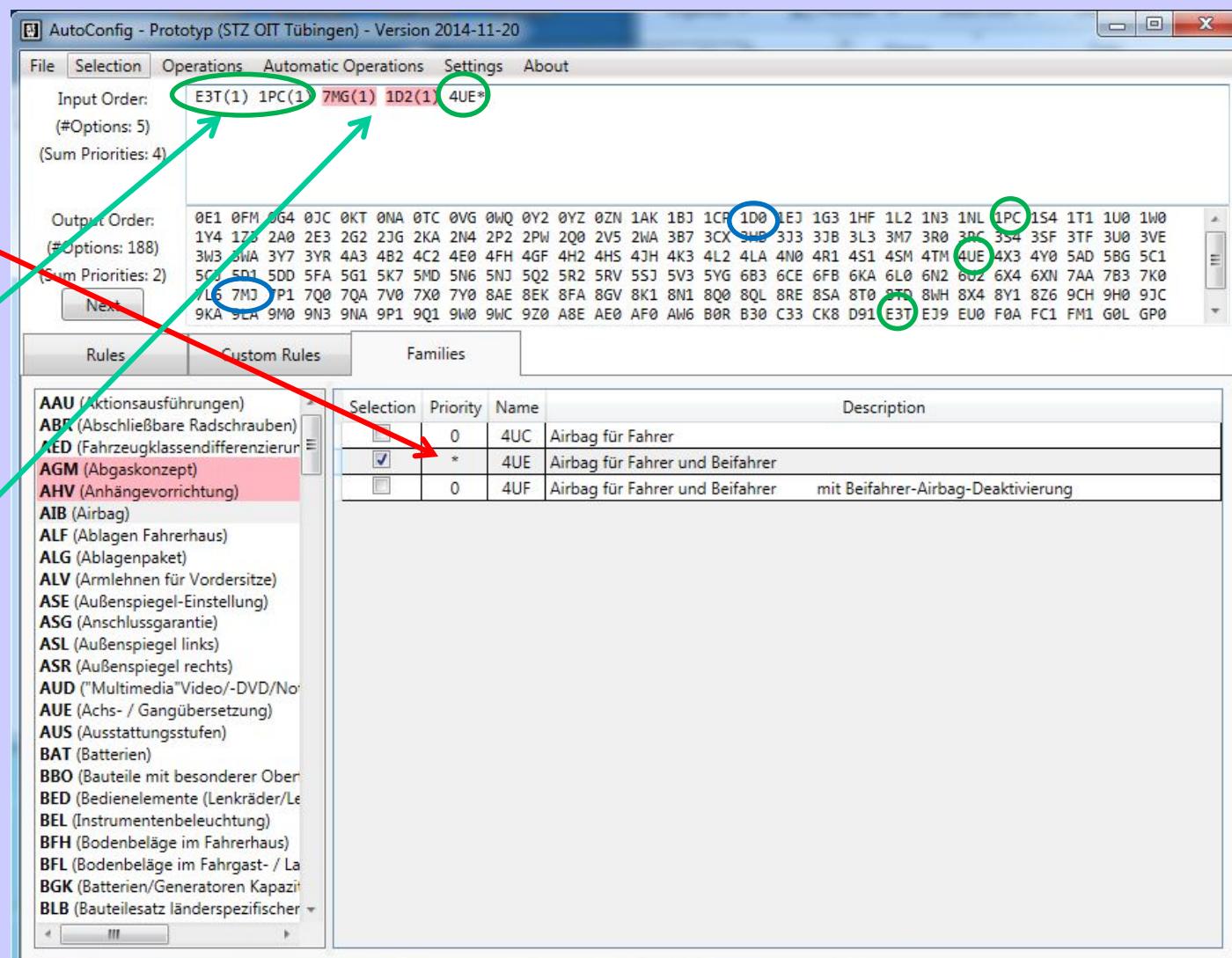
# Optimization Ex.: Re-Configuration with Auto.Config

- Select an option **not** available
- *Auto.Config* computes **maximum** subset of **PD-legal** option selections
- *Auto.Config* computes **minimum** subset of **impossible** option selections
- *Auto.Config* computes new order containing the PD-legal options with **maximum sum of input weights** (here always 1)



# Optimization Ex.: Re-Configuration with Auto.Config

- Re-Configuration
- Fix the unavailable option 4UE(\*)
- *Auto.Config* computes **maximum** subset of PD-legal option selections
- *Auto.Config* computes **minimum** subset of impossible option selections



# Optimization Ex.: Re-Configuration with Auto.Config

- Re-Configuration
- Fix the unavailable option 4UE(\*)
- *Auto.Config* computes **maximum** subset of PD-legal option selections
- *Auto.Config* computes **minimum** subset of impossible option selections
- *Auto.Config* computes new order **maximizing the weight** of the PD-legal input options

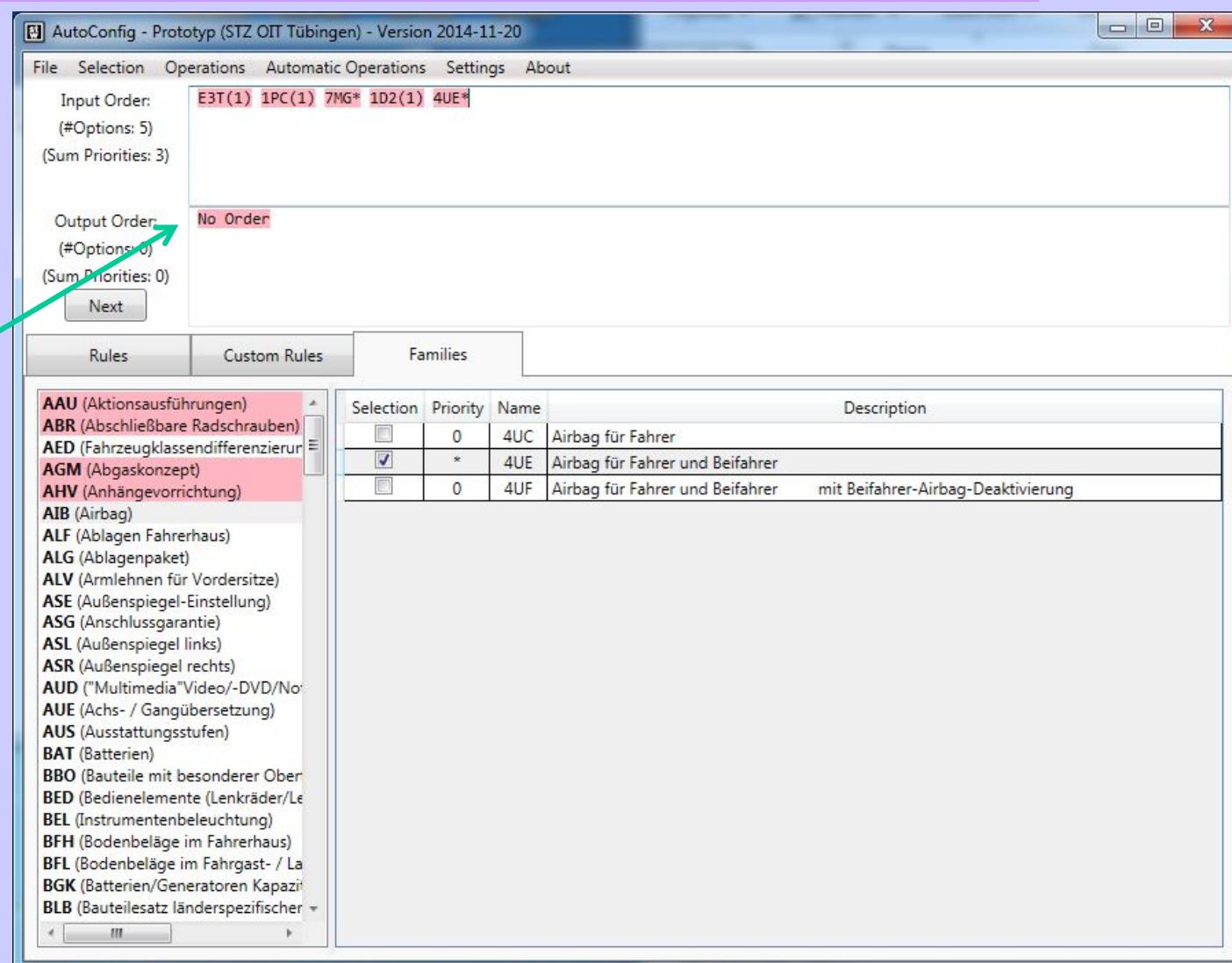
The screenshot shows the AutoConfig software interface. At the top, the menu bar includes File, Selection, Operations, Automatic Operations, Settings, and About. The 'Selection' tab is active. In the main area, there are two sections: 'Input Order:' and 'Output Order:'. The 'Input Order:' section shows a list of options: E3T(1) 1PC(1) 7MG(1) 1D2(1) 4UE\*. The 'Output Order:' section shows a much longer list of options, with several circled in green: 100, 1D0, EJ, 1G3, 1HF, 1L2, 1N3, 1NL, 1PC, 1S4, 1T1, 1U0, 1W0, 5C1, 5D1, 5DD, 5FA, 5G1, 5K7, 5MD, 5N6, 5N9, 5Q2, 5R2, 5RV, 5SJ, 5V3, 5YG, 6B3, 6CE, 6FB, 6KA, 6L0, 6N2, 6S2, 6X4, 6XN, 7AA, 7B3, 7K0, 7L1, 7M1, P1, 7Q0, 7Q4, 7V0, 7X0, 7Y0, 8AE, 8EK, 8FA, 8GV, 8K1, 8N1, 8Q0, 8QL, 8RE, 8SA, 8T0, 8T4, 8WH, 8X4, 8Y1, 8Z6, 9CH, 9H0, 9JC, 9KA, 9LA, 9M0, 9N3, 9NA, 9P1, 9Q1, 9W0, 9WC, 9Z0, A8E, AE0, AF0, AW6, B0R, B30, C33, CK8, D91, E3T, E39, EU0, F0A, FC1, FM1, G0L, GP0. Below these lists are buttons for 'Next' and 'Rules', 'Custom Rules', and 'Families'. On the left, a scrollable list of option families is shown, with 'AGM (Agaskonzept)' highlighted in pink. To the right, a table lists selected options with their priorities and descriptions:

Selection	Priority	Name	Description
<input type="checkbox"/>	0	4UC	Airbag für Fahrer
<input checked="" type="checkbox"/>	*	4UE	Airbag für Fahrer und Beifahrer
<input type="checkbox"/>	0	4UF	Airbag für Fahrer und Beifahrer mit Beifahrer-Airbag-Deaktivierung



# Optimization Ex.: Re-Configuration with Auto.Config

- Re-Configuration
- Fix (\*) the unavailable options 7MG, 4UE
- *Auto.Config* finds contradiction:  
no legal order with  
7MG + 4UE



# *Other Problems*

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- **Compute all combinations of a set of materials**
  - given the materials in a subset of the BoM (e.g. driver's seat, rear axle, ...)
  - From each BoM position pick exactly one material variant
  - Compute all consistent combinations which can occur in configurable vehicles
  - Example: All seats or all axles which can occur in a model line
- **Compute software upgrades**
  - given software variants and their upgrade dependencies
  - compute possible upgrades / compute all cars with possible upgrades
- **Compute homologation (certification) requirements**
  - given a sales program for a market
  - compute all homologation relevant materials for that market
- **Change the documentation method**
  - convert the formulas for a car model line from one documentation method to another, preserving the set of legal configurations.



# Literature

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