Ernst Specker and the Fundamental Theorem of Quantum Mechanics

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Alles Weltvertrauen fängt an mit den Namen, zu denen sich Geschichten erzählen lassen (Hans Blumenberg).

"Grazie, lieber Giovanni Sommaruga! Herr Professor Engeler, meine Damen und Herren. Ich möchte damit beginnen, zu danken. Ich danke Ihnen, Herr Engeler und natürlich auch Ernst Specker, der heute nicht mehr unter uns sein kann, dafür wie sie unser denken inspiriert haben, auf eine sehr herausfordernde Art und Weise. Ich danke den Organisatoren dieses faszinierenden Anlasses. In diesen Dank möchte ich auch die Organisatoren des entsprechenden Anlasses vor zehn Jahren mit einbeziehen, namentlich *Janos Makovsky*. Jener Anlass war in der Tat für mein Leben ein Markstein. Ich wusste damals, dass *Jürg Fröhlich* im Publikum sein würde; dies war aufregend aber auch im wahrsten Sinn bedrohlich.

Ich hatte also Angst, das Resultat war dann ganz unerwartet, dass Jürg mehr darüber hören wollte. Bei unserem ersten Gespräch war ich es, der seine deterministiche Sicht nicht teilen konnte, und ich glaube, dass es heute genau umgekehrt ist: so rotieren wohl zwei Planeten umeinander, die nicht ineinander stürzen und trotzdem zusammen bleiben: sie tauschen immer mal wieder die Plätze.

Ich erinnere mich auch, dass an jenem Workshop vor zehn Jahren zum ersten Mal *Ämin* aufgetaucht ist, gleichsam aus dem nichts, und permanent unterschätzt, auch von mir. Er hat voriges Jahr den Preis gewonnen für die beste Informatikdiss in D, AUT, CH, mit einer naturphilosophichen Arbeit über "Kausalität" verletzende Schlaufen.

Arne Hansen macht den zweiten Teil unseres gemeinsamen Vortrages. Er ist in den letzten Jahren vielen Denkfäden gefolgt; seine grossartige Diss *gestaltet*, ja *schafft* im besten Sinne die Vergangenheit, in dem sie die

Wurzeln in sie verfolgt des Baums der "Erkenntnis," *hier* im wahrsten — also gemäss *Nietzsche* leider verlogensten — Sinns; wobei Arne gegen diese Verlogenheit angeht, als wären es Windmühlen: er hat damit in meinen Augen grossen Erfolg).

When I held the talk ten years ago, I reported from my research at the time, on Bell correlations in quantum mechanics, and how those could be used for cryptography: The non-classicality of quantum behaviour *à la Bell* means that classical information is insufficient for explaining the link, so the information did not exist before, so it must be fresh upon measurement, so an outside adversary cannot know it: This is the reasoning by *Barrett, Hardy, Kent*, that continues to be beautiful; it looks shakier now in my eyes.

The feedback to my talk was most generous, even for a *Saturday*: Klaus Hepp was smiling, Gian-Michele Graf as I now know found it probably *post-modern*, Jürg was silent and after wanted to hear more, and Ernst Specker himself came up to me and said: "Sie haben Recht, das ist das ist dasjenige meiner Resultate, welches mich am längsten überleben wird." (Obviously, I had not said that at all, who would I be?)

My talk had been turning around Speckers text from 1960 in German, "Über die Logik nicht gleichzeitig entscheidbarer Aussagen." What I got from that text is the described above: "Consistent predictions about the measurement outcomes (classical information) of a quantum mechanical system are impossible." In a sense, a set of predictions does not have to be falsified by an experiment, but suffers *intrinsic* contradictions. It is self-contractictory, not logically, rather *geometrically* in some specific sense (called "contextual").

Connected to that was what I did *not* get in Specker's text: The story of the overprotective seer and his daughter whose suitors are to identify two boxes among three, where each one contains a diamond or not, with the same content. Trivially, (at least) one such pair must exist according to *Dirichlets Schubfachprinzip.* They always fail, until the smart daugther really likes on of the guys and interferes favorably. I have to admit that I even thought: "Such a behavior is even impossible quantum-mechanically — Specker hasn't got it." Shame on me! But not what I thought: Specker's point *was*, that it is impossible quantum-mechanically, as I plan to argue for today, setting a certain contrast to how Jürg interpreted the story in Munich, and just before this talk.

Warum also hatte ich nicht verstanden, was Specker sagen wollte? Weil es nicht war, was ich erwartete, dass er sagen würde: So scheitert Kommunikation oft. Ich hatte erwartet, dass die Geschichte seine Aussage am Schluss des Textes — die technischer Natur ist und annonciert, nicht beweisen wird — illustrieren will: Inwiefern die Quantentheorie aus dem klassischen Korsett ausbricht. Heute glaube ich, dass er etwas sagen wollte, das dem Gegenteil davon näher ist: Dass auch die feissere Quantentheorie selbst ihr Korsett hat, einfach ein weiteres. Ich kann hier auch falsch liegen. Fast-Namensvetter *Spekkens* hat in der Tat Speckers Parabel in die Nähe des Ausbrechens der Quantentheorie aus dem engeren Korsett gerückt. (Hier diene mir Spekkens' Genialität als Argument: Spekkens kann für *alles* schöne Argumente finden; umso weniger exkludierend sind sie dann, im besten *Feyarbend*schen Sinne.) Zudem hat auch *Xavier Coiteux-Roy*'s von *Claude Crépeau* übernommenes "RGB game" Ähnlichkeit mit dem Rätsel in der Werbungsparabel.

Trotzdem:

Ich denke, dass Specker allein in dieser Geschichte verschlüsselt eine *entgegengesetzte* zusätzliche Nachricht, genauso unbewiesen wie jene am Schluss, und hier spielerisch fabulierend statt wie jene technisch-formal anbringt. *Warum glaube ich das heute?* Wegen eines nicht einmal einminütigen Films, der auf Jürg Fröhlichs Balkon aufgenommen wurde um das Jahr 2010 und von *Adàn Cabello* bekannt gemacht.

Title:
Ernst Specker and the Fundamental Theorem of Quantum Mechanics.

Summary:
Ernst Specker explains his deepest insight into quantum theory — and nobody listens to him.

Film:
https://vimeo.com/52923835

Synopsis:
In the film, there is the main action and a side action. The main line is subtitled, so there is no problem there. Harder to get is the side line: you hear people talking "[...] shrink [...; then Jürg:] ja, ja, das Alter". Not related to this at all is the film's main line: Specker puts away his unfinished dessert for telling us (he is aware he is being filmed, quite obviously) what "according to [him] is something very *fundamental* about quantum mechanics".

Analysis:
What does Specker say in the film? First, he claims *originality* of a thought:

"I have never seen this in print." We ask: Why not? Did you have the idea just now, here on the balcony? If not, why did *you* not print it? After all, you speculate it to be a deep truth about quantum theory. We cannot ask him this now, which is really very sad.

What does he say materially? He says that if something is true *pairwisely*, then in automatically also holds *fully*. This is a property, for instance, *not* shared by independence of events of random variables in probablity theory: If three random variables are such that any *two* of them are independent, it does *not* mean they are all three independent: an example is given by X and Y being independent unbised bits, and $Z := X \oplus Y$. On the other hand, that property *is* valid for orthogonality in a Hilbert space: If a set of vectors is such that they are pairwisely orthogonal, then this is a fully orthogonal set automatically; there is no "higher" orthogonality for sets of three (or more) vectors.

Ah! *This is how the seer in the legend violates quantum mechanics: If, out of three boxes, you can always open two of them, then you can also open all three of them.* This is exactly what turned out to be impossible upon the daugther's passionate intervention. What seems to be a closinng detail of of the story is its main message. It is not about the correlation, this is what I had been blind to see, wearing my "correlation glasses."

In fact, it was a view through these glasses I have reported about here, ten years ago: I talked about a truly mysterious object, the "non-local box" as it was called then, or today rather *Popescu/Rohrlich box*, named after its two inventors. Now this box has an interface with two parties, let us call them *Alice and Bob*, just an in cryptography. Both parties provide an input to their interface of the box, and they get an output from it. If we look at only *one* of the two output bits, this is always an unbiased random bit. If we look at them *both*, then there is an algebraic property always satisfied: *The outputs are equal if and only if both inputs were equal to one.*

What is mysterious about it? In my inaugural lecture at ETH, and at perhaps about a hundred talks since then, I had explained that using an analogy with playing cards: Imagine that each card has a front side that is either red or blue, and the same holds for the card's back side, where *a priori* there is no connection between the front and the back sides of the cards. Now, imagine that pairs of such playings cards are handed to Alice and Bob, each of whom can decide to look at either the front or the back of their card. Imagine further that if both parties decide to look at the front sides of their respective cards, they always see the same colors. The same holds if one of the two looks at the front and the other at the back. Before we go on: What can we conclude from that information? Well, the two front sides have the same color, and each of the back sides has the same color as the front of the *other*. Altogether, because of transitivity of equality, we conclude that the two back sides *also* must have the same color. Imagine now that the opposite happens: When both look at the back side, the colors are different. You say: "This is impossible!". Well yes, it is — with playing cards. Imagine now that we talk about pairs of photons instead of playing cards, that the decision to look at one side or the other corresponds now to measure the polarization of the photons in one direction or another. Then, this behavior becomes possible. Not perfectly, though. Let us go back to the example of the playing cards: Each *fixed coloring* of the cards fails to satisfy the described conditions in at least one out of four conditions; we say that we can "achieve that behavior with probability $3/4 = 75\%$." In the case of photons now, this bound can be "beaten:" roughly 85%. What this now means, this has attracted my thoughts like a magnet for the last dozen years, and I am not the only one. If you look into the sun, it will very fast not appear bright anymore. The more you look at this phenomenon, the brighter it gets, the weirder, the more disturbing. In the end, it turns all your beliefs upside down, beliefs about time, space, and causality. The carpet under the feet of your thinking. Being.

Given that at some point, you resign and accept the existence of these weird correlations, you are naturally led to some kind of "inverse" thought: "Well, if quantum physics *is* non-local, then why is it not *maximally* so?" Why can the described behavior occur with probability 85% — beating the "classical" 3/4 or, in other words, "violating a *Bell inequality*" —, but not 100%? More specifically (and forgive me to go into that, Jürg), people have tried to find simple information-theoretic arguments why that might be so. As we will see, they were not really "arguments" but elegant, *light* if you will, ways of saying it. In particular, saying it without having to invoke the heavy Hilbert-space mathematical apparatus, how can we characterize the limit of quatum theory in an easier way? Two remarks here: What you find "easy" depends on your taste and background. Second: We cannot give a "reason why" the law is such and such, we just look for a *simple* description of saying the same. Now, certain such "information principles" have been brought forward — information causality, macroscopic locality, non-trivial communication complexity. I focus on one claiming that the general principle of the kind, should it exist, must be *multipartite*: "Local orthogonality" — *pairwise orthogonality (read as: exclusivity) implies full exclusivity.* Imagine here the case of disjointness of sets: If a familiy of sets is such that they are pairwisely disjoint, well then they are as disjoint as it gets. What the authors of that quite recent article now claim is that quantum theory respects this, but not these PR boxes; more precisely, not *pairs of PR boxes*. Let us try to be more specific: We call an *event* a fixed output combination $x_1x_2y_1y_2$ given a fixed input combination $a_1a_2b_1b_2$. What does that mean? It means that the first party (Alice) has given to her interface of the first box the bit a_1 and received the output x_1, etc. An example of such an event would be $(x_1x_2y_1y_2|a_1a_2b_1b_2) = (0000|0000)$: all bits involved are zero. It has a *probability*: If we consider the two PR boxes to be *independent*, then the probability is 1/4: First of all, every combination violating the "PR condition" has probability zero, and the probability weight is equally distributed among the "valid" output combinations: 1/2 each since each *individual* output bit is random and unbiased. Finally, we get $1/4 = (1/2)^2$ since the boxes are independent.

Now we call two such events "orthogonal" or "mutually exclusive" if, roughly speaking, they give a different output to the same input in one of the four interfaces. For instance, the events (0000|0000) and (1110|0011) are orthogonal because of position 1: The input is the same in both cases (namely 0), but the outputs are different. The meaning of this "exclusivity" is that events which are mutually exclusive do not have an overlap and thus cannot have probabilities summing up to more than 1. (Otherwise, the probability of the full set of outcomes, given one input combination, should not exceed 1; but nothing is more probable than *certain*). Now comes the decisive step: *The principle of local orthogonality* postulates that if a set of events is such that they are pairwisely exclusive, then they are fully exclusive and the sum of all their probabilities is not to exceed 1. There is no logical reason this must be the case, but there is rarely a logical reason a natural law must be true, is there? (An exception here is the *second law of thermodynamics* — just kidding; I just wanted to mention it.) So that principle is a law of nature, and it is satisfied within the quantum-mechanical formalism (which, again, is a speculation about nature). The point now is that the principle of *not* followed by the pairs of PR boxes (which are "beyond quantum"): In fact, it is possible to find *five* events that are mutually exclusive: (0000|0000), (1110|0011), (0011|0110), (1101|1011), and (0111|1101). But then, the sum of their probabilities is $5/4 > 1$, in violation of the principle.

In summary, we (and I think it is fair to say that *Adàn Cabello* is the pioneer here) speculate whether Specker has pre-felt, in the text from 1960 — in the same text in which he pre-felt the non-classicality of quantum theory in the form of Kochen/Specker theorem — at the same time this *limit* of quantum theory in the face of speculation coming out naturally of the other result: He gave a lower and an upper bound on "quantum weirdness" at the same time.

In the film, he says that he has never seen this in print, does he not? Well, let us see: "Eine Gesamtheit von Aussagen über ein quantenmechanisches System ist somit genau dann gleichzeitig entscheidbar, wenn je zwei Aussagen der Gesamtheit es sind." There it was, in 1960, *in print*: He had written it himself. I must have overlooked — he must have had forggotten.

Thank you very much!"