Enrico Del Re

Interview with Helmut Hofer and Yakov Eliashberg

Helmut Hofer, professor at the Institute of Advanced Study in Princeton, and Yakov Eliashberg, professor for mathematics at the Stanford University, were awarded the Heinz-Hopf Prize at the ETH for their work in symplectic topology in December 2013.

The Heinz-Hopf Prize is awarded for outstanding scientific work in the field of pure mathematics every two years.

First of all congratulations for winning the Heinz Hopf Prize. Could you introduce the topic you have worked on in your career? The one which earned you this prize.

Eliashberg: This prize was awarded to Helmut and myself for our work in symplectic topology and its application to Hamiltonian dynamics. It is not the only topic we have worked on, but we spend a lot of time with it.

So everybody is used to euclidean geometry. And euclidean is based on measuring distance and angles with a scalar product. If you have two vectors and want to compute the area of the parallelogram which is spanned by this vectors you need a skew symmetric function.

Turns out that the phase space of any mechanical system has a canonical structure of this type and the evolution equations in mechanics preserve this structure.

This topic was more or less started by Ponicarè's interest in celestial mechanics. He was aware that many problems are not integrable and formulated questions one might be able to answer nonetheless, for example how many periodic solutions a system has.

One particular question is now called the "Last geometric theorem of Poincarè".

Not much progress happened after it, until the 80s.

Hofer: The progress started by including some other fields, that happened at the beginning of the 80s. So if you were working in some of the fields which later came together, which for us was the case, I came from analysis, specifically non linear partial differential equations, and he came from geometry, it was interesting to see people coming from different cultures arriving to the same type of problems. The beginning was the merging of ideas from different fields.

You said that you come from different parts of mathematics, but how and when did you decide to work in this specific field, in pure mathematics? Was it a decision you made during your studies or later, or did it just happen coincidentally?

Hofer: I actually started my studies as a physicist. And then the experiments, the measurements, everything was off and it never had anything to do with the theory, there was always this background noise or something went wrong with the machine. And so I went to theoretical physics. There I observed that it is actually very good to know mathematics and learning it I got stuck there. I still like to look at problems in physics but I have a few difficulties arguing as a pure physicist, where sometimes there is no mathematical foundation, but it makes sense from a physical insight.

Eliashberg: I think this division between pure mathematics and applied mathematics is a bit artificial. Good pure mathematics is really in the heart of many important applications. In some sense I think, even if pure mathematicians do not always say this, most of them would really like to see applications of what they are doing.

Application maybe first to other fields of mathematics, then to physics and eventually to the real world around us.

Hofer: Which happens, it usually takes a number of years, like 50. If you look at number theory, some of the people who did number theory were very proud that it had no application, now the encryption business is based on this.

Eliashberg: Also symplectic topology, although it looked pretty theoretical, can have some applications. For example, the question how many periodic solutions of a system has, can have some applications as Helmut explained in his talk.

Hofer: The application would be to help spacecrafts to reach their destination consuming less fuel. Since in celestial mechanics there is a lot of chaos, one small deviation at a certain point can

lead to a large deviation later on. For example to move satellites chaos (coming from the continuous change of gravity) is used. But they use rather classical methods to play around and finding the deviation they need

is like finding a needle in the haystack. Then they can simulate the trajectory due to this deviation with high accuracy. It is to large extend trial and error.

Methods we developed to solve problems in symplectic topology can be used to solve this kind of questions and highlight good picks in the haystack.

Eliashberg: So not working on a specific problem, just trying to develop an area and understand it can lead to interesting results and applications.

Mathematics at the beginning of 19 century started to branch in more and more fields, but now these branches start converging and some unexpected connection between different fields were found. Good mathematics is always inter-related and the convergence will always bring new results.

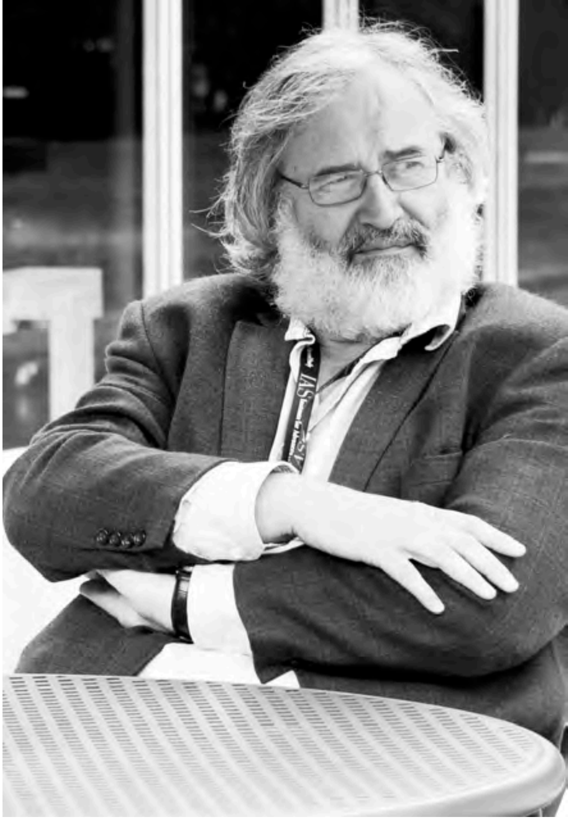
Fermat Theorem's proof is an example for convergence of ideas from very different fields. Another example is the Poincaré conjecture.

Hofer: Therefore students should study different topics. And especially they should throughout understand what they have studied and play with it, use it.

Eliashberg: For me understanding is mostly focused on examples, because if you know the examples you can recreate the theory behind it, and do not have to memorize it.

If you would have to go back, and start studying again, would you still study physics or mathematics? Or is there something you would have liked to do instead?

Hofer: I actually knew when I was 14 years old, that I would like to do research. I do not really want to tinker



Helmut Hofer

something in my life,because life has a certain chaotic component. So if you would start changing some parameters there, it could have gone in all kinds of directions.

I am very happy how things turned out. I mean, you move in some direction and generally life offers you a certain number of possibilities. Then you make a choice and should not think about what you could have done.

Yakov Eliashberg

A lot of people want to have insurance that they are moving in the right direction, so they will never move.

Move forward, life is interesting, no regrets. I would say that is a good recipe for happiness.

Eliashberg: What Helmut said is extremely related to what I feel. Of course you make mistakes, but it is very stupid to say it would be better to do this or that. Okay, you did not do something, even if it was better. So just think what is better to do now.

Of course there are a lot of interesting things I would like to do but do not have the time for. But I try my best in what I can.

What would this other interests, excluding mathematics, be?

Eliashberg: Maybe you already know this joke, that there was a mathematician who was studying integral equations of first kind. As someone asked him what he does in his free time, he responded he is studying integral equations of second kind.

So this joke is actually true?

Eliashberg: No, no. However, as you reach a certain level you have to do some other stuff, not only research. So whenever you have some free time, you use it to think about mathematics. At least this is true for me. But of course this is not my only hobby.

I like to travel, hiking, a lot of interesting things I would like to participate in. For example, in my childhood I played violin and planned to be a professional violinist. However, I changed to mathematics and now I do not play anymore, which is something I do regret a bit.

Hofer: It mostly ends to be a time question. Hobbies which require time are hard to practice. One of my hobbies is fishing, where I can think about mathematics and get something to eat at the end of the day, without wasting time.

Eliashberg: In some sense, thinking in this situations, hiking or fishing, can be good or not so good. If you do not do everything on paper properly it is easy to get overexcited. Then you start doing it on paper and it is not always correct.

Hofer: I calm myself, I learned when I get excited about something I know it is wrong. So when I do research and get maximally bored I know it is right. Because it means I understand it.

Eliashberg: If I feel that I understood something, but not quite, I do not want to spoil this moment, it is an extremely exciting moment.

Hofer: When I was younger I got really excited and then it collapses, so your mood goes from plus infinity to minus infinity. Then again next day. So I figured out that when I get excited it is usually completely nonsense. But I also work differently now. When I was younger I was very problem oriented. I was running against a concrete wall for years until I saw a small crack appear somewhere. Eventually I got through and solved the problem, and although there was happiness I found that this method does not work when you have family. After crashing your head again and again against the same wall you come home and your kids want to talk and play with you, but you are still crashing that wall in your mind.

So I changed my method of working. The method consists in the mindset, that you are like an explorer. In the morning you wake up, take your machete and start clearing some brush in the jungle. In the evening you have cleared some brush, which is obviously some progress. Next morning you do it again. You start understanding more and more, write a paper about what you understand and go on like this. You do not experience the same highs as in my first methods, but it is much more stable.

What hint would you give to mathematics students who would like to do research in mathematics?

Hofer: I would say, do it, but also go out and travel the world, go to different places. And most important, figure out what you are good at.

Eliashberg: I would encourage them to use their time as students to learn as much as possible in different areas. Later on there might not be the time disposal. Working harder than absolutely necessary can help you.

Hofer: But also enjoy your youth, time passes faster when you get older.

Thank you for the Interview. ■