Machine Learning in mathematical Finance

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Mathematical Challenges in mathematical Finance

- High dimensional stochastic control problems often of a non-standard type (hedging in markets with transaction costs or liquidity constraints).
- High-dimensional inverse problems, where models (PDEs, stochastic processes) have to be selected to explain a given set of market prices optimally.
- High-dimensional prediction tasks (long term investments, portfolio selection).
- High-dimensional feature selection tasks (limit order books).

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Neural Networks

Neural networks in their various topological features are frequently used to approximate functions due ubiquitous universal approximation properties. A neural network, as for instance graphically represented in Figure 1,

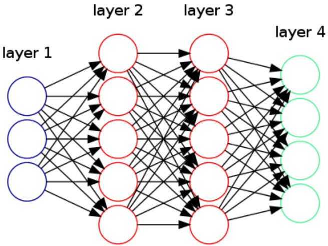

Figure: A 2 hidden layers neural network with 3 input and 4 output dimensions

just encodes a certain concatenation of affine and non-linear functions by composition in a well specified order.

Universal Approximation

- Neural networks appear in the 1943 seminal work by Warren McCulloch and Walter Pitts inspired by certain functionalities of the human brain aiming for articial intelligence (AI).
- Arnold-Kolmogorov Theorem represents functions on unit cube by sums and uni-variate functions (Hilbert's thirteenth problem), i.e.

$$F(x_1, \ldots, x_d) = \sum_{i=0}^{2d} \varphi_i\Big(\sum_{j=1}^{d} \psi_{ij}(x_j)\Big)$$

- Universal Approximation Theorems (George Cybenko, Kurt Hornik, et al.) show that *one hidden layer networks* can *approximate* any continuous function on the unit cube.
- Connections between *deep* neural networks and sparse representations in certain wavelet basis (Helmut Bölcskei, Philipp Grohs et al.) explaining their incredible representation power.

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An example: reservoir computing paradigm

- in many situations input is a time series object of varying length.
- a part of the neural network, which represents the input-output map, is chosen as a *generic dynamical system* (often with physical realization and, of course, with relationship to the input-output map). The goal of this choice is to transform the input into relevant information pieces.
- only the last layer is trained, i.e. a linear regression on the generic network's output is performed.
- this reminds of stochastic differential equations which can be written – in a quite regular way – as linear maps on the input signal's signature, i.e. the collection of all iterated integrals (universal limit theorem of rough path theory).

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Deep Networks in Finance

Recent ideas to use machine learning in Finance

- Deep pricing: use neural networks to constitute efficient regression bases in, e.g., the Longstaff Schwartz algorithm for pricing call-able products like American options.
- Deep hedging: use neural networks to approximate hedging strategies in, e.g., hedging problems in the presence of market frictions (joint work with Hans Bühler, Lukas Gonon, and Ben Wood).
- Deep filtering: use neural networks on top of well selected dynamical systems to approximate laws of signals conditional on "noisy" observation.
- Deep calibration: use machine learning to approximate the solution of inverse problems (model selection) in Finance (joint work with Christa Cuchiero).

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Calibration by machine learning

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Calibration by Machine learning following Andres Hernandez

We shall provide a brief overview of a procedure introduced by Andres Hernandez (2016) as seen from the point of view of Team 3's team challenge project 2017 at UCT:

Algorithm suggested by A. Hernandez

- Getting the historical price data.
- Calibrating the model, *a single factor Hull-White extended Vasiček model* to obtain a time series of (typical) model parameters, here the yield curve, the rate of mean reversion α, and the short rate's volatility σ.
- Pre-process data and generate new combinations of parameters.
- With a new large training data set of (prices,parameters) a neural network is trained.
- The neural network is tested on out-of-sample data.

The data set

- The collected historical data are ATM volatility quotes for GBP from January 2nd, 2013 to June 1st, 2016. The option maturities are 1 to 10 years, 15 years and 20 years. The swap terms from 1 to 10 years, plus 15, 20 and 25 years.
- The yield curve is given 44 points, i.e. it is discretely sampled on 0, 1, 2, 7, 14 days; 1 to 24 months; 3-10 years; plus 12, 15, 20, 25, 30, 40 and 50 years. Interpolation is done by Cubic splines if necessary.

Classical calibration a la QL

Historical parameters

- a Levenberg-Marquardt local optimizer is first applied to minimize the equally-weighted average of squared yield or IV differences.
- calibration is done twice, with different starting points:
  - at first, $\alpha = 0.1$ and $\sigma = 0.01$ are the default choice
  - second the calibrated parameters from the previous day (using the default starting point) are used for the second stage of classical calibration.

Calibration results along time series

The *re-calibration problem* gets visible ... and it is indeed a feasible procedure.

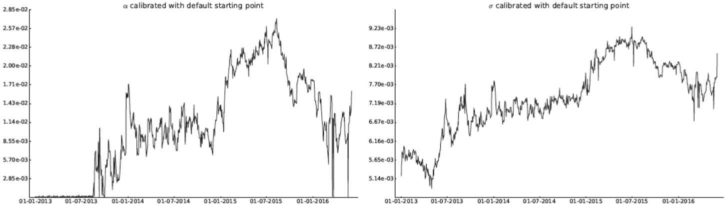


Figure: Calibration using default starting point

How do neural networks enter calibration?

Universal approximation of calibration functionals

- Neural networks are often used to approximate functions due to the universal approximation property.
- We approximate the calibration functional (yields,prices) $\mapsto$ (parameters) which maps (yields, prices) to optimal model parameters by a neural network.

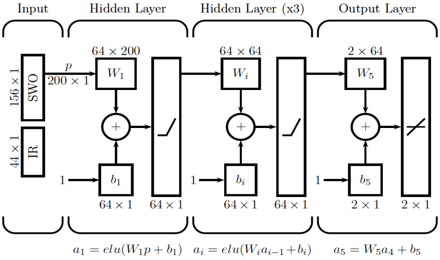
Neural Networks : Training Set Generation

With the calibration history A. Hernandez proceeds by generating the training set

- obtain errors for each calibration instrument for each day,
- take logarithms of of positive parameters, and rescale parameters, yield curves, and errors to have zero mean and variance 1,
- apply a principal component analysis and an appropriate amount of the first modes,
- generate random normally distributed vectors consistent with given covariance,
- apply inverse transformations, i.e. rescale to original mean, variance and exponentiate,
- apply random errors to results.

Neural Networks: Training the network

- With a sample set of 150 thousand training data points, A. Hernandez suggests to train a feed-forward neural network.
  - The architecture is chosen feed-forward with 4 hidden layers, each layer with 64 neurons using an ELU (Exponential Linear Unit)



Neural Networks: testing the trained network

- two neural networks were trained using a sample set produced where the covariance matrix was estimated based on 40% of historical data.
- the second sample set used 73% of historical data.
- for training, the sample set was split into 80% training set and 20% cross-validation.
- the testing was done with the historical data itself (i.e. a backtesting procedure was used to check the accuracy of the data).

Results of A. Hernandez

The following graphs illustrate the results. Average volatility error here just means

$$\frac{\sum_{n=1}^{156} \left| impvol^{mkt} - impvol^{model} \right|}{156} \tag{1}$$

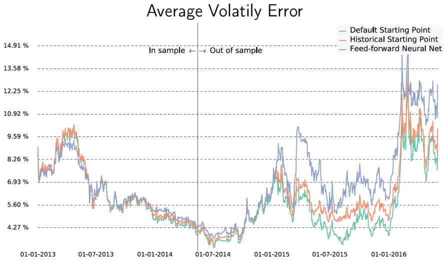
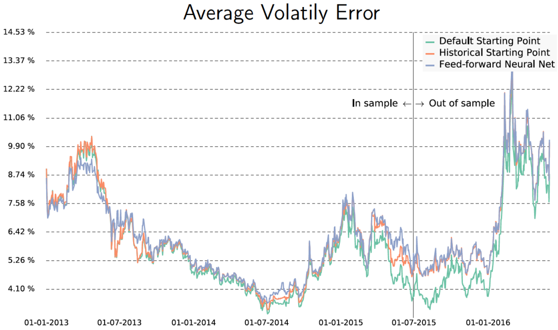


Figure: Correlation up to June 2014



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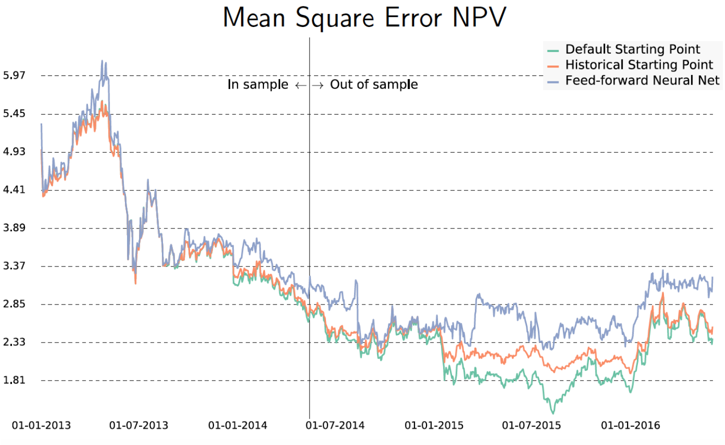

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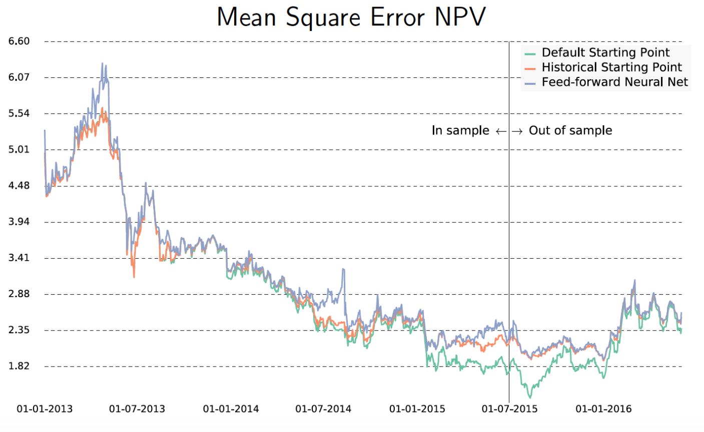

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Towards a Bayesian model

Consider the Hull-White extended Vasiček models (on a space $(\Omega, \mathcal{F}, (\mathcal{G}_t)_{t\geq 0}, \mathbb{P})$):

$$dr_t^{(1)} = (\beta_1(t) - \alpha_1 r_t^{(1)})\, dt + \sigma_1\, dW_t\,,$$

$$dr_t^{(2)} = (\beta_2(t) - \alpha_2 r_t^{(2)})\, dt + \sigma_2\, dW_t\,.$$

We assume that r is is a mixture of these two models with constant probability $\pi \in [0, 1]$, i.e.

$$\mathbb{P}(r_t \leq x) = \pi \mathbb{P}\left(r_t^{(1)} \leq x\right) + (1 - \pi)\mathbb{P}\left(r_t^{(2)} \leq x\right).$$

Of course the observation filtration generated by daily ATM swaption prices and a daily yield curve is smaller than the filtration $\mathbb{G}$, hence the theory of the first part applies.

Bayesian model: setup

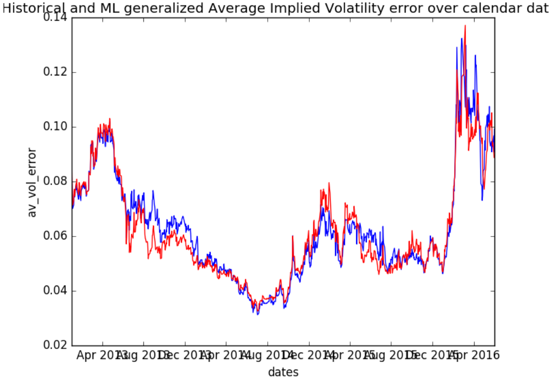
We still have the same set-up (in terms of data):

- $N = 156 + 44 = 200$ input prices (swaptions + yield curve)
- $n = 44 + 4 + 1 = 49$ parameters to estimate. These are $\alpha_1, \alpha_2, \sigma_1, \sigma_2, \pi$ and $\text{yield}_1(t)$ (or, equivalently, $\text{yield}_2(t)$) at 44 maturities (notice that given $\alpha_1, \alpha_2, \sigma_1, \sigma_2, \pi$ there is a one-to-one map between yields and βs).
- Hence, the calibration function is now

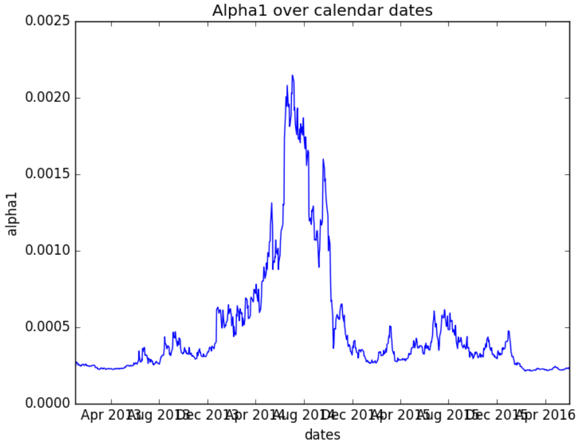
$$\Theta : \mathbb{R}^{200} \longrightarrow \mathbb{R}^{49}, \qquad \begin{pmatrix} \text{SWO1} \\ \text{SWO2} \\ \cdots \\ \text{yield}(0) \\ \text{yield}(1) \\ \cdots \end{pmatrix} \mapsto \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \sigma_1 \\ \sigma_2 \\ \pi \\ \text{yield}_1(0) \\ \text{yield}_1(1) \\ \cdots \end{pmatrix}$$

Bayesian model: training

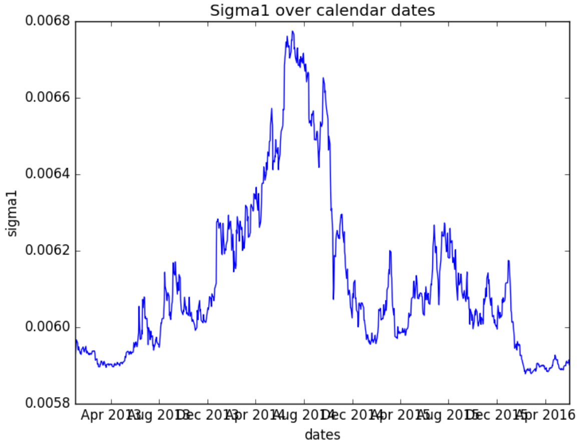
We generated a new training set and trained, tested another neural network with a similar architecture: the quality of the new calibration is the same as the QuantLib calibration and better than previous ML results, in particular out of sample.



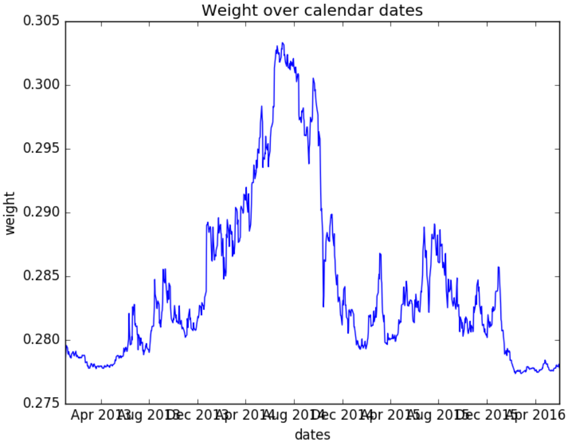
Mixture Model: α_1



Mixture Model: σ_1



Mixture Model: π



- it works to train networks the information of calibration functionals: usually calibration functionals are of a hash function type, i.e. it is easy to calculate prices from given parameters, but it is difficult to re-construct parameters from given prices. Still it is easy to generate training data.
- the "unreasonable effectiveness" is visible by absence of the 'curse of dimension'.
- it will be interesting to train *universal calibrators* of realistic models by offline algorithms which allow to circumvent high-dimensional delicate calibration procedures.

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Frame of ideas

- Many problems in Finance are of filtering nature, i.e. calculating conditional laws of a true signal X_{t+h}, at some point in time $t+h$, given some noisy observation $(Y_s)_{0 \le s \le t}$.
- Such problems often depend in a complicated, non-robust way on the trajectory of Y, i.e. no Lipschitz dependence on Y: regularizations are suggested by, e.g., the theory of regularity structures, and its predecessor, rough path theory. By lifting input trajectories Y to more complicated objects (later called *models*) one can increase robustness to a satisfactory level.
- The idea is to write an abstract theory of expansions as developed by Martin Hairer in a series of papers, understand it as an "expressive" dynamical system and learn the output layer (which is of high regularity).

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- Many solutions of problems in stochastics can be translated to solving fixed point equation on modelled distributions.
- By applying the reconstruction operator the modeled distribution is translated to a real world object, which then depends – by inspecting precisely its continuities – in an at least Lipschitz way on the underlying model, i.e. stochastic inputs.
- The theory of regularity structures tells precisely how 'models' have to be specified such that stochastic inputs actually constitute models: this yields a theory of input specifications.
- Supervised learning: by creating training data (in appropriate input format!) one can learn the input-output map.
- Applications: solutions of stochastic differential equations (Friz, Lyons, Victoir, etc), solutions of correlated filtering problems (Crisan, Friz, etc), solutions of sub-critical stochastic partial differential equations (Hairer, Gubinelli, etc).

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Prediction Tasks

- consider certain noisy observations of a true signal and model them by a corresponding regularity structure (this might be necessary in since there is no reason why non-linear functions of noisy objects should be well defined).
- construct solutions of the optimal filter by solving a fixed point equation on modelled distributions.
- reconstruct the real world filter by the reconstruction operator, which yields – under appropriate regularity conditions – a non-linear, Lipschitz map from the space of observations (the 'models') to the optimal filter.
- Learn this map on regularized noises.

Deep Hedging

- given a generic market situation: scenarios generated by one or many different models fitting aspects of the market environment.
- given transaction costs, liquidity constraints, bid ask spreads. etc.
- given a derivative and a risk objective.
- approximate hedging strategies by deep neural networks of all appropriate factors, which creates a dense subset of admissible strategies,
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Advantages

- particular models play a minor role, very data driven.
- tractability, which is a delicate problem, for high dimensional non-linear PIDEs, does not play a role for setting up the problem: even very high dimensional reinforcement learning problems can be solved in a satisfying way.
- market frictions can be easily included.
- idea: set up a describable set of hedging strategies which allow to ϵ approximate the optimal solution.

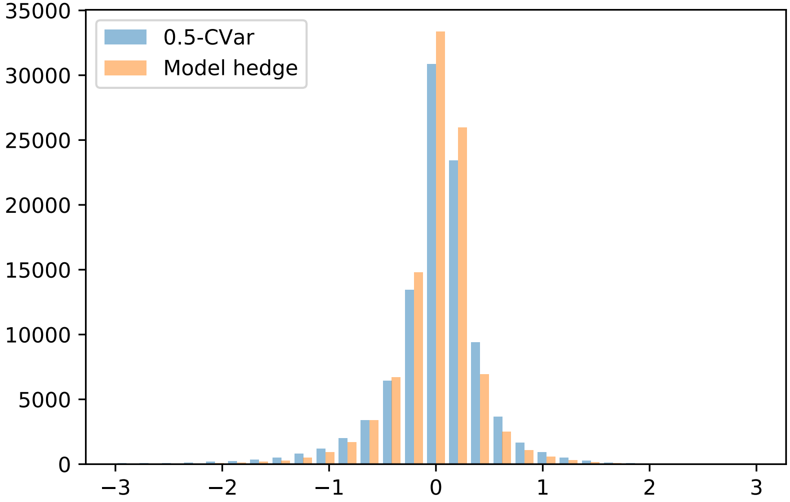


Figure: Comparison of model hedge and deep hedge associated to 50%-expected shortfall criterion.

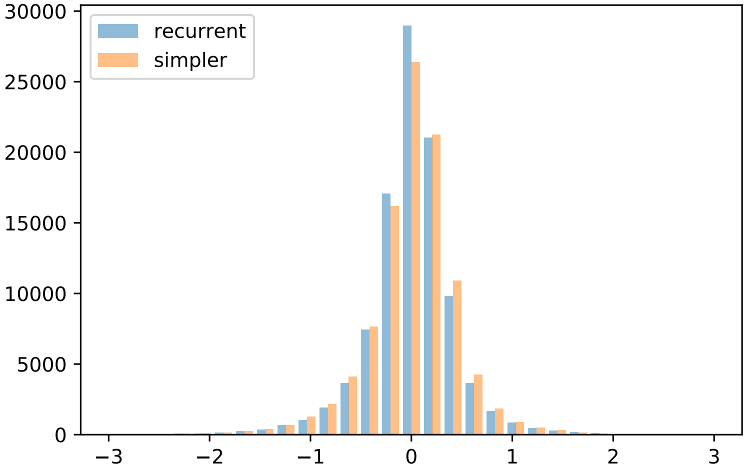
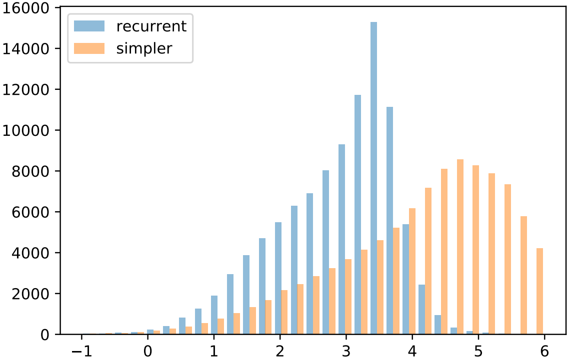


Figure: Comparison of recurrent and simpler network structure (no transaction costs).



	Mean Loss	Price	Realized CVar
recurrent	0.0018	5.5137	-0.0022
simpler	0.0022	6.7446	-0.0

Figure: Network architecture matters: Comparison of recurrent and simpler network structure (with transaction costs and 99%-CVar criterion).

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- Why does it work so well: explain the "unreasonable effectiveness": sparse wavelet representation seem to provide a key.
- How to choose input variables? Universal approximation depends a lot on this: rough paths or regularity structures as a solution concept.
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