Industrial users, inventory holders and speculators

A simple model for commodities markets

Ivar Ekeland, Delphine Lautier, Bertrand Villeneuve

Chaire de Développement Durable, Université Paris-Dauphine

DelbaenFest,.Zurich, September 24, 2012

Commodities markets

- Oil, meat, cereals, minerals, energy,

Here we have a financial market which is strongly connected to the real economy.

Commodities markets

- Oil, meat, cereals, minerals, energy,
- For each of them there is a physical market and a financial market

Here we have a financial market which is strongly connected to the real economy.

Commodities markets

- Oil, meat, cereals, minerals, energy,
- For each of them there is a physical market and a financial market
- On the *physical market*, one trades the commodity itself, either for immediate delivery (spot market) or for later delivery (forward market): this is where producers meet industrial users and consumers

Here we have a financial market which is strongly connected to the real economy.

Commodities markets

- Oil, meat, cereals, minerals, energy,
- For each of them there is a physical market and a financial market
- On the *physical market*, one trades the commodity itself, either for immediate delivery (spot market) or for later delivery (forward market): this is where producers meet industrial users and consumers
- On the *financial market*, one trades money: if I hold a contract for 500 000 barrels of oil with maturity one month it means that in one month's time I will be delivered the current price of 500 000 barrels. This bring in new agents, who are interested not in the commodity itself, but in the risk: speculators or money managers

Here we have a financial market which is strongly connected to the real economy.

Earlier work

How does the financial market influence the physical market ? What is the interplay between industrial users, and consumers on one hand, speculators and inventory holders on the other ? Some of the aspects have been covered in earlier papers:

- Anderson & Danthine (1983)
- Hirshleifer (1988)
- Deaton & Laroque (1992)
- Guesnerie & Rochet (1993)

Ours is the first to study all aspects simultaneously. We introduce a very simple, almost rudimentary, model, and we perform an equilibrium analysis.

The model

- 2 periods, $t = 1$ and $t = 2$. All decisions are taken at $t = 1$ and a source of uncertainty (Ω, P) operates between $t = 1$ and $t = 2$.

The model

- 2 periods, $t = 1$ and $t = 2$. All decisions are taken at $t = 1$ and a source of uncertainty (Ω, P) operates between $t = 1$ and $t = 2$.
- 1 commodity. Produced in quantity ω_1 at $t = 1$ and $\tilde{\omega}_2$ at $t = 2$. At time $t = 1$, ω_1 is observed, but $\tilde{\omega}_2$ is not

The model

- 2 periods, $t = 1$ and $t = 2$. All decisions are taken at $t = 1$ and a source of uncertainty (Ω, P) operates between $t = 1$ and $t = 2$.
- 1 commodity. Produced in quantity ω_1 at $t = 1$ and $\tilde{\omega}_2$ at $t = 2$. At time $t = 1$, ω_1 is observed, but $\tilde{\omega}_2$ is not
- 2 spot markets, at $t = 1$ and $t = 2$. These are *physical* markets: only positive quantities can be traded.

The model

- 2 periods, $t = 1$ and $t = 2$. All decisions are taken at $t = 1$ and a source of uncertainty (Ω, P) operates between $t = 1$ and $t = 2$.
- 1 commodity. Produced in quantity ω_1 at $t = 1$ and $\tilde{\omega}_2$ at $t = 2$. At time $t = 1$, ω_1 is observed, but $\tilde{\omega}_2$ is not
- 2 spot markets, at $t = 1$ and $t = 2$. These are *physical* markets: only positive quantities can be traded.
- 1 futures market. Contracts are bought at $t = 1$ and settled at $t = 2$. This is a *financial* market: negative positions are allowed.

The model

- 2 periods, $t = 1$ and $t = 2$. All decisions are taken at $t = 1$ and a source of uncertainty (Ω, P) operates between $t = 1$ and $t = 2$.
- 1 commodity. Produced in quantity ω_1 at $t = 1$ and $\tilde{\omega}_2$ at $t = 2$. At time $t = 1$, ω_1 is observed, but $\tilde{\omega}_2$ is not
- 2 spot markets, at $t = 1$ and $t = 2$. These are *physical* markets: only positive quantities can be traded.
- 1 futures market. Contracts are bought at $t = 1$ and settled at $t = 2$. This is a *financial* market: negative positions are allowed.
- There are 3 prices: 2 spot prices P_1 and $\tilde{P}_2$, and a future price P_F. At time $t = 1$, P_1 and P_F are observed but $\tilde{P}_2$ is not.

The model

- 2 periods, $t = 1$ and $t = 2$. All decisions are taken at $t = 1$ and a source of uncertainty (Ω, P) operates between $t = 1$ and $t = 2$.
- 1 commodity. Produced in quantity ω_1 at $t = 1$ and $\tilde{\omega}_2$ at $t = 2$. At time $t = 1$, ω_1 is observed, but $\tilde{\omega}_2$ is not
- 2 spot markets, at $t = 1$ and $t = 2$. These are *physical* markets: only positive quantities can be traded.
- 1 futures market. Contracts are bought at $t = 1$ and settled at $t = 2$. This is a *financial* market: negative positions are allowed.
- There are 3 prices: 2 spot prices P_1 and $\tilde{P}_2$, and a future price P_F. At time $t = 1$, P_1 and P_F are observed but $\tilde{P}_2$ is not.
- Aim of the paper: determining $P_1, \tilde{P}_2$ and P_F by equilibrium conditions (all markets clear).

Some issues

To make things simple, interest rate is set to 0.

- Market is in *contango* (report) if $P_F > P_1$, and in *backwardation* (déport) if $P_F < P_1$ If inventory is not zero, then arbitrage theory (cash-and-carry) should imply that the market is in contango. However, backwardation is sometimes observed with non-zero inventory (convenience yield).
- If $P_F \neq E\left[\tilde{P}_2\right]$, the futures market is *biased*. Keynes argues that futures markets exhibit a systematic downwards bias: $P_F < E\left[\tilde{P}_2\right]$. because producers and processors of commodities are more prone to hedge their price risk than consumers or speculators, so the latter insure the former.
- Does the existence of a financial market influence prices on the physical markets? Some excellent economists say yes, and other excellent economists say no: a futures contract is a bet on the commodity price, and the speculators do not trade the commodity, just like gamblers do not run on the horsetrack.

The agents

- **Spot traders**, who intervene only on the spot markets.
- Industrial users, or **processors**, who use the commodity to produce other goods which they sell to consumers. Because of the inertia of the production process or because they sell their production forward, they have to decide at $t = 1$ how much to produce at $t = 2$. They cannot store the commodity, so they have to buy all of their input at $t = 2$.
- **Inventory holders**, which have storage capacity, and who can use it to buy the commodity at $t = 1$ and release it at $t = 2$.
- Money managers, or **speculators**, who do not trade on the physical markets, they trade only in futures.

Rational behaviour.

- All agents (except the spot traders) have mean-variance utility: if they make a profit $\tilde{\pi}$ they derive utility:

$$E[\tilde{\pi}] - \frac{1}{2}\alpha \mathrm{Var}[\tilde{\pi}], \text{ with } \alpha = \alpha_I, \alpha_P, \alpha_S$$

- They make optimal decisions at $t = 1$, based on the conditional expectation of $\tilde{P}_2$, which will be determined in equilibrium.
- All of them (except the spot traders) take positions on the futures market, either for hedging or for speculating.

The inventory holders

Storage is costly: holding a quantity x costs $\frac{1}{2}Cx^2$. If they buy $x \geq 0$ on the spot market at $t = 1$, resell it on the spot market at $t = 2$, and take a position f_I on the futures market, the resulting profit is:

$$\pi_I(x, f_I) = x(\tilde{P}_2 - P_1) + f_I(\tilde{P}_2 - P_F) - \frac{1}{2}Cx^2.$$

The optimal positions are:

$$x^* = \frac{\max\{P_F - P_1, 0\}}{C}, \quad f_I^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_I \mathrm{Var}[\tilde{P}_2]} - x^*.$$

The storer holds inventory if the futures price is higher than the current spot price.

Processors

Processors decide at time $t = 1$ how much input y to buy at $t = 2$, and which position f_P to take on the futures market. The input y results in an output $y - \frac{\beta}{2}y^2$ (decreasing returns to scale) which is sold at a price P_0. It is assumed that P_0 is known at time $t = 1$. The resulting profit is:

$$\pi_P(y_2, f_P) = P_0\left(y - \frac{\beta}{2}y^2\right) - y\tilde{P}_2 + f_P(\tilde{P}_2 - P_F).$$

The optimal positions are:

$$f_P^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_P \mathrm{Var}[\tilde{P}_2]} + y^*, \quad y^* = \frac{\max\{P_0 - P_F, 0\}}{\beta P_0}.$$

Speculators and spot traders

- **Speculators**. The profit resulting from a futures position f_S is:

$$\pi_S(f_S) = f_S(\tilde{P}_2 - P_F),$$

$$f_S^* = \frac{\mathrm{E}[\tilde{P}_2] - P_F}{\alpha_S \mathrm{Var}[\tilde{P}_2]}.$$

- **Spot traders**.
  If price at time $t = 1, 2$ is P_t, the demands from spot traders are

$$\mu_1 - mP_t \quad \text{and} \quad \tilde{\mu}_2 - m_t P_2$$

To simplify the analysis, we allow negative prices (so that spot traders are paid to hold the commodity).

Clearing the three markets

- **Spot market at $t = 1$.** On the supply side the harvest ω_1 and on the demand side we have the inventory $N_I x^*$ bought by the storers, and the demand of the spot traders.

$$\omega_1 = N_I x^* + \mu_1 - mP_1,$$
$$P_1 = \frac{1}{m}\left(\mu_1 - \omega_1 + N_I x^*\right)$$

- **Futures market.** Positions can be positive or negative:

$$N_S f_S^* + N_P f_P^* + N_I f_I^* = 0.$$
$$P_F = \mathrm{E}[\tilde{P}_2] + \frac{\mathrm{Var}[\tilde{P}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}}\left(N_P y^* - N_I x^*\right)$$

Clearing the three markets

- **Spot market at** $t = 2$. On the supply side, the harvest $\tilde{\omega}_2$, and the inventory $N_I x^*$ sold by the storers, and, on the other side, the input $N_P y^*$ bought by the processors and the demand of the spot traders.

$$\tilde{\omega}_2 + N_I x^* = N_P y^* + \tilde{\mu}_2 - m\tilde{P}_2,$$

$$\tilde{P}_2 = \frac{1}{m}\left(\tilde{\mu}_2 - \tilde{\omega}_2 - N_I x^* + N_P y^*\right)$$

The equilibrium equations

Market Characteristic: $\rho = 1 + m \frac{\mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}}$

Substituting the value for $\tilde{P}_2$ into the equations for P_1 and P_F (which are just numbers, not random variables) we get the system:

$$\begin{aligned} mP_1 - \frac{N_I}{C} \max\{P_F - P_1, 0\} &= \mu_1 - \omega_1 \\ mP_F + \rho \left(\frac{N_I}{C} \max\{P_F - P_1, 0\} - \frac{N_P}{\beta P_0} \max\{P_0 - P_F, 0\} \right) &= E[\tilde{\mu}_2 - \tilde{\omega}_2] \end{aligned}$$

which is a system of two *nonlinear* equations for two unknowns P_1 and P_F. If we can solve this system we derive $\tilde{P}_2$ by substituting.

Solving the equilibrium equations

We solve by investigating the piecewise linear map:

$$F\left(P_1, P_F\right) = \left(\begin{array}{c} mP_1 - \frac{N_I}{C} \max\left\{P_F - P_1, 0\right\} \\ mP_F + \frac{\rho N_I}{C} \max\left\{P_F - P_1, 0\right\} - \frac{\rho N_P}{\beta P_0} \max\left\{P_0 - P_F, 0\right\} \end{array} \right)$$

and showing that it is onto. Note that:

$$F\left(P_1, P_F\right) = \left(\begin{array}{c} \mu_1 - \omega_1 \\ E\left[\tilde{\mu}_2 - \tilde{\omega}_2\right] \end{array} \right)$$

are precisely the equilibrium conditions.

The 4 + 1 regimes

- In regions 1 and 2, where $x^* > 0$, the futures market is in contango: $P_F > P_1$
- In regions 3 and 4, where there is no inventory. the futures market is in backwardation: $P_F < P_1$
- When $n_I C x^* > n_P \beta P_0 y^*$ (regions 2 and 3) speculators hold a long position, so $P_F < E\left[\tilde{P}_2\right]$, which is Keynes' downwards bias. When $n_I C x^* < n_P \beta P_0 y^*$ (region 4) speculators are short, and $P_F > E\left[\tilde{P}_2\right]$. The line $n_I C x^* = n_P \beta P_0 y^*$, or

$$P_F = \frac{n_I}{n_I + n_P} P_1 + \frac{n_P}{n_I + n_P} P_0$$

devides region 1 into an upper subregion, where $P_F < E\left[\tilde{P}_2\right]$, and a lower one.

Comparative statics

- Variance: $\mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]$.
- Costs C or β.
- Number of agents of various types: N_I, N_P, N_S.
- Risk aversions: α_I, α_P, α_S.

All subsumed in the market characteristic:

$$\rho = 1 + m \frac{\mathrm{Var}[\tilde{\mu}_2 - \tilde{\omega}_2]}{\frac{N_P}{\alpha_P} + \frac{N_I}{\alpha_I} + \frac{N_S}{\alpha_S}}$$

The optimal number of speculators

We can compute explicitly the indirect utilities of all agents at $t = 1$ wrt P_1 and P_F. Since P_1 and P_F are equilibrium prices, they depend in turn on market characteristics, and we can compute their elasticities $\partial P_1 / \partial \rho$ and $\partial P_2 / \partial \rho$. We find that, in

- in the upper part of region 1

$$\frac{\partial U_I}{\partial \rho} > 0, \quad \frac{\partial U_P}{\partial \rho} > 0$$

- in the lower part of region 1

$$\frac{\partial U_I}{\partial \rho} < 0, \quad \frac{\partial U_P}{\partial \rho} < 0$$

In the upper region, both industrial users and inventory holders will benefit from decreasing the number of speculators (which increases ρ), while in the lower region they would benefit from an increase in N_S. On the frontier, the number of speculators is optimal

Do speculators increase volatility ?

All statements thus far are given conditional on $\mu_1 - \omega_1$.
Do futures markets increase variance?

- Conditional on $\mu_1 - \omega_1$: NO:

$$\tilde{P}_2 = \frac{1}{m}\left(\tilde{\mu}_2 - \tilde{\omega}_2 - N_I x^* + N_P y^*\right)$$

- Non conditional on $\mu_1 - \omega_1$: YES. Inventory holders tranfer risk from $t = 1$ to $t = 2$, and the presence of future markets enables them to hedge their bets, i.e. to transfer more risk than they would otherwise

Future work

Consider an infinite-horizon problem, each agent discounting the future at the rate δ :

$$U(c) = \sum_{t=1}^{\infty} \delta^t u(c_t)$$

and seek Markov strategies which are in equilibrium. We hope this will (among other things) give a rigorous basis for the notion of convenience yield, which up to now we find is rather ad hoc.