Liquidity in dealer markets

Peter Bank

joint work in progress with
Ibrahim Ekren and Johannes Muhle-Karbe

METE - Mathematics and Economics: Trends and Explorations.

A conference celebrating
METE SONER's 60th birthday and his contributions
to Analysis, Control, Finance and Probability

ETH Zürich, June 4–8, 2018

Financial markets with frictions: Mete's contributions

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Theorem

Mete LOVES frictions.

q.e.d.

Frictions in financial modelling

- Classical Black-Scholes theory: dynamic trading of arbitrary amounts, arbitrarily fast without affect on exogenously given asset prices and without taxes, transaction fees, etc.
- How to account for these nonlinear effects? Formidable challenges at the interfaces between financial modelling, stochastic analysis, and stochastic optimal control
- "Equilibrium models" versus cost specifications
- Illiquidity due to differences in information (Glosten-Milgrom '85, Kyle '85) and/or due to inventory risk (Ho-Stoll '81, Grossman-Miller '88): $<= 3$ period models
- Dynamic equilibrium type models: Back '90, Garleanu-Pedersen-Poteshman '09, Kramkov-Pulido '16, B.-Kramkov '15
- Cost specifications: **Soner**-Shreve '94, Almgren-Chriss '01, Obizhaeva-Wang '13, Roch-**Soner** '13

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Dramatis personae

An FX desk's business as a *ménage a trois...*:

- **Dealers**: compete quoting FX rates, supply currency to their clients; transfer inventory to end-users at a finite rate at fundamental exchange rate, thereby incurring search costs and inventory risk
- **Clients**: demand currency positions from their dealers, get served at their competitive rates
- **"End-users"**: accept positions at fundamental FX rates, can only be contacted at search cost incurred by dealers

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Questions:

- How do the **dealers**' prices (FX rates) match demand with supply? How are they related to fundamentals? What role is played by the dealers' search costs and inventory risk aversion?
- How should **clients** choose their demand to manage their exogenously given risk? What if they internalize their impact? Do they benefit from the dealers' presence?
- Who are the **end-users**?

The dealers' problem

For FX quotes (S_t) and fundamental FX rates (V_t), the dealers servicing their clients' requested positions (K_t) and cumulatively transferring $U_t = \int_0^t u_s \, ds$ to the end-users at costs $\frac{\lambda}{2} u_t^2 dt$ in $t \in [0, T]$, will generate proceeds

$$\int_0^T (-K_t) dS_t - (V_T - S_T) K_T + \int_0^T U_t dV_t - \frac{\lambda}{2} \int_0^T u_t^2 \, dt.$$

Assuming V is a martingale, i.e., ruling out speculation on FX rates trends etc., we get the **dealers' expected proceeds** to be

$$\mathbb{E}\left[\int_0^T (-K_t) dS_t - (V_T - S_T) K_T - \frac{\lambda}{2} \int_0^T u_t^2 \, dt\right].$$

The **dealers' inventory risk** is determined by $U - K$:

$$\frac{1}{2} \mathbb{E}\left[\int_0^T (K_t - U_t)^2 \, dt\right]$$

The dealers' problem

Dealers' target functional with risk aversion $\gamma_d > 0$:

$$J_d(K, u; S) \triangleq \mathbb{E}\left[\int_0^T (-K_t) dS_t - (V_T - S_T) K_T - \frac{\lambda}{2} \int_0^T u_t^2 \, dt\right] - \frac{\gamma_d}{2} \mathbb{E}\left[\int_0^T (K_t - U_t)^2 \, dt\right] \to \max_{K,u}$$

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Observe: Problem can be addressed in two stages.

Stage 1: *Given* K, maximization over u is a quadratic tracking problem

$$\mathbb{E}\left[\frac{\gamma_d}{2}\int_0^T (K_t - U_t)^2\, dt + \frac{\lambda}{2}\int_0^T u_t^2\, dt\right] \to \min_u$$

as solved explicitly in **Soner** et al. '17.

Stage 2: *Given* the optimal transfer policy u^K for *any* K, optimize over K.

Quadratic tracking problem

Theorem (Soner et al. '17)

The dealers' optimal trading rate minimizing

$$\mathbb{E}\left[\frac{\gamma_d}{2}\int_0^T (K_t - U_t)^2\,dt + \frac{\lambda}{2}\int_0^T u_t^2\,dt\right]$$

is

$$u_t^K \triangleq \frac{d}{dt}U_t^K = \frac{\tanh((T-t)/\sqrt{\kappa})}{\sqrt{\kappa}}(\hat{K}_t - U_t^K)$$

where

$$\kappa \triangleq \lambda/\gamma_d \text{ and } \hat{K}_t \triangleq \mathbb{E}\left[\int_t^T K_u \frac{\cosh((T-u)/\sqrt{\kappa})}{\sqrt{\kappa}\sinh((T-t)/\sqrt{\kappa})}\,du \,\middle|\, \mathscr{F}_t\right]$$

⇝ Dealers form a view $\hat{K}$ on expected future demand and trade with the end-users towards this ideal position.

Quadratic tracking problem with terminal constraint

Theorem (Soner et al. '17)

The dealers' optimal trading rate minimizing

$$\mathbb{E}\left[\frac{\gamma_d}{2}\int_0^T (K_t - U_t)^2\,dt + \frac{\lambda}{2}\int_0^T u_t^2\,dt\right]$$

subject to $U_T = K_T$ *is*

$$u_t^K \triangleq \frac{d}{dt}U_t^K = \frac{\coth((T-t)/\sqrt{\kappa})}{\sqrt{\kappa}}(\hat{K}_t - U_t^K)$$

where, as before, $\kappa \triangleq \lambda/\gamma_d$*, but now*

$$\hat{K}_t = \frac{1}{\cosh(\frac{T-t}{\sqrt{\kappa}})}\mathbb{E}\left[K_T \,\middle|\, \mathscr{F}_t\right] + \left(1 - \frac{1}{\cosh(\frac{T-t}{\sqrt{\kappa}})}\right)\mathbb{E}\left[\int_t^T K_s \frac{\sinh(\frac{T-s}{\sqrt{\kappa}})}{(\cosh(\frac{T-t}{\sqrt{\kappa}})-1)\sqrt{\kappa}} \,\middle|\, \mathscr{F}_t\right].$$

Illustration: Deterministic demand expanding midway

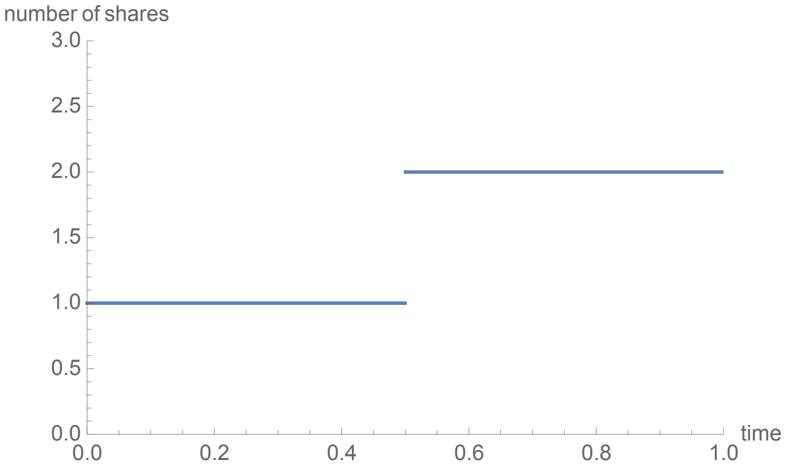


Figure: Demand K with a jump at $t = T/2$ (blue)

Illustration: Deterministic demand expanding midway

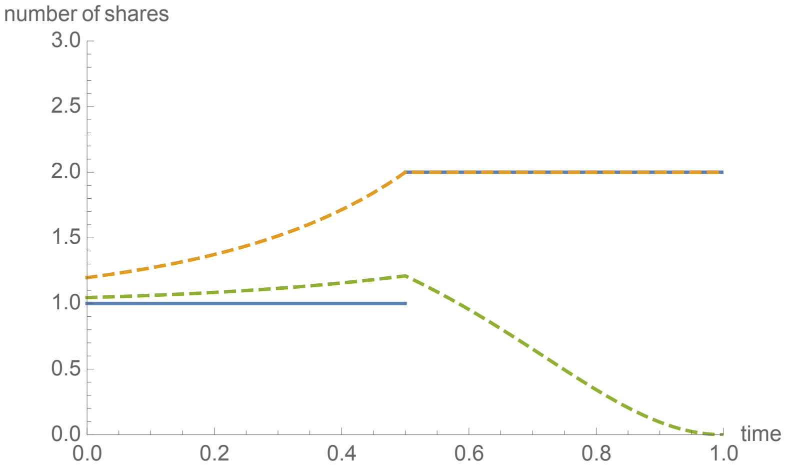


Figure: Demand K with a jump at $t = T/2$ (blue), dealers' unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$

Illustration: Deterministic demand expanding midway

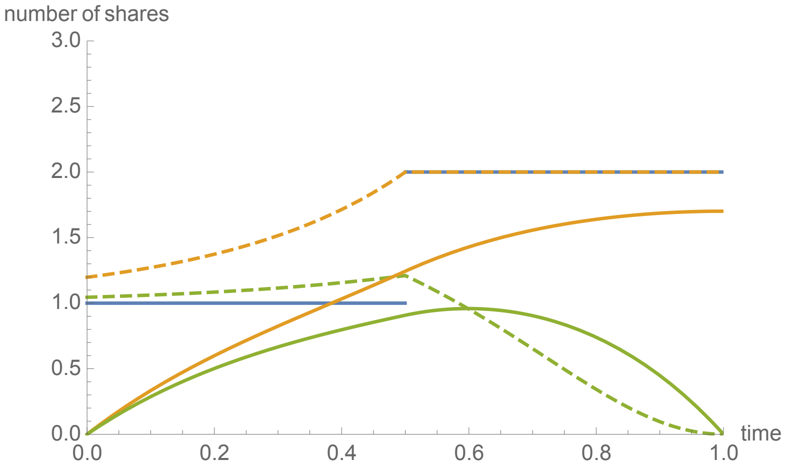
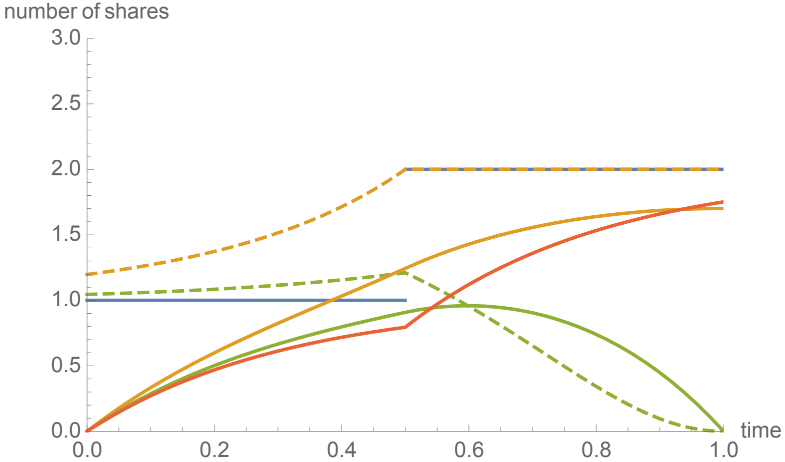


Figure: Demand K with a jump at $t = T/2$ (blue), dealers' unconstrained (orange, dashed) and constrained (green, dashed) target $\hat{K}$, corresponding unconstrained (orange) and constrained (green) transfer policy u^K, and myopic transfer policy (red)



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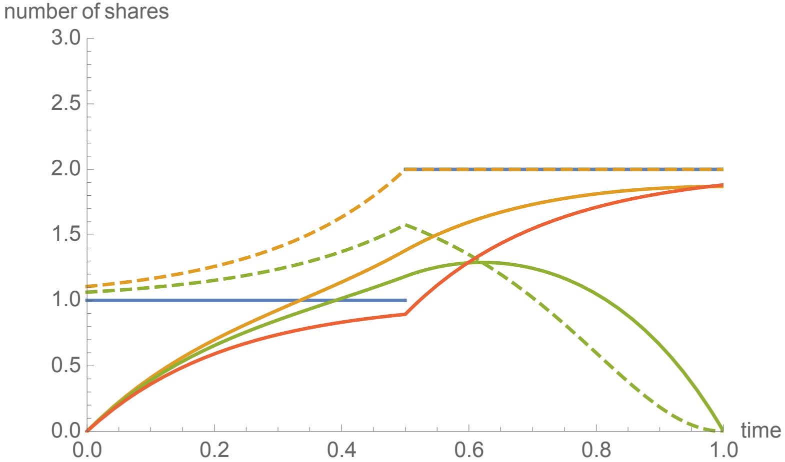

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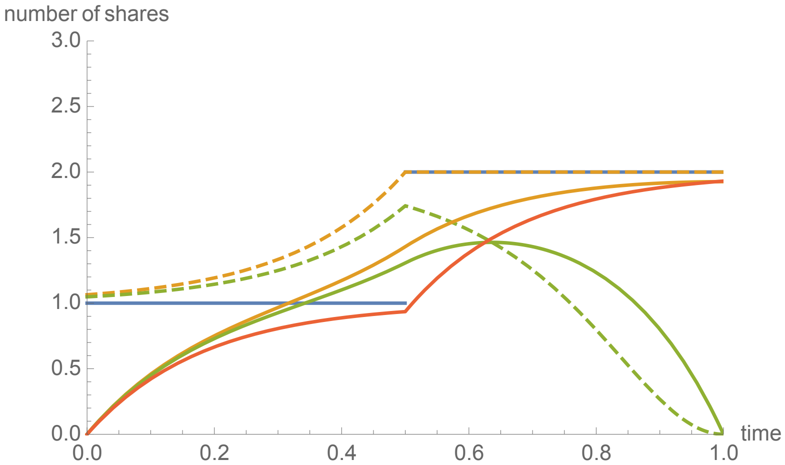

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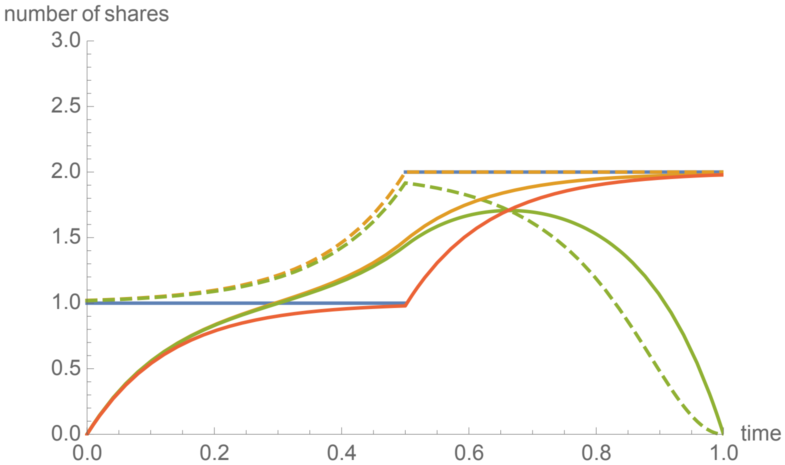

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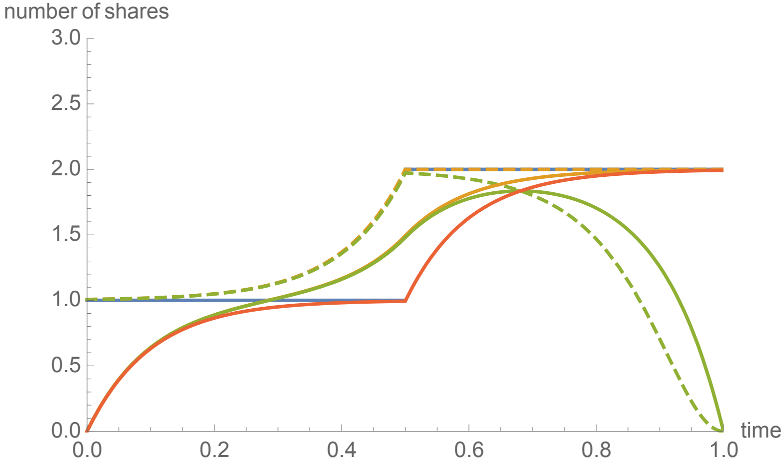

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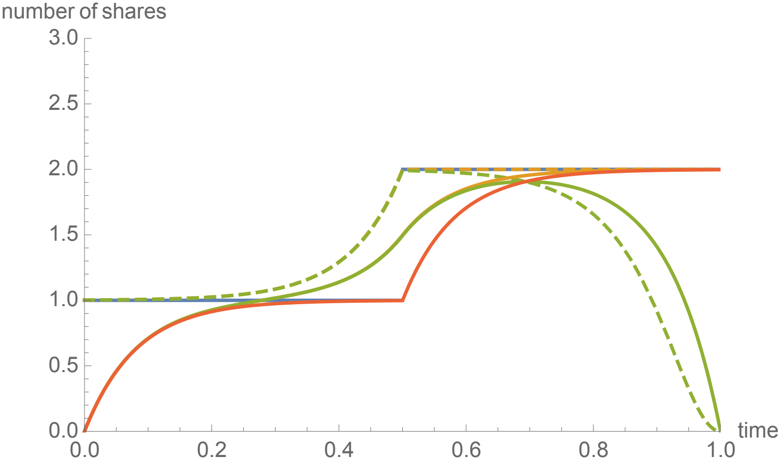

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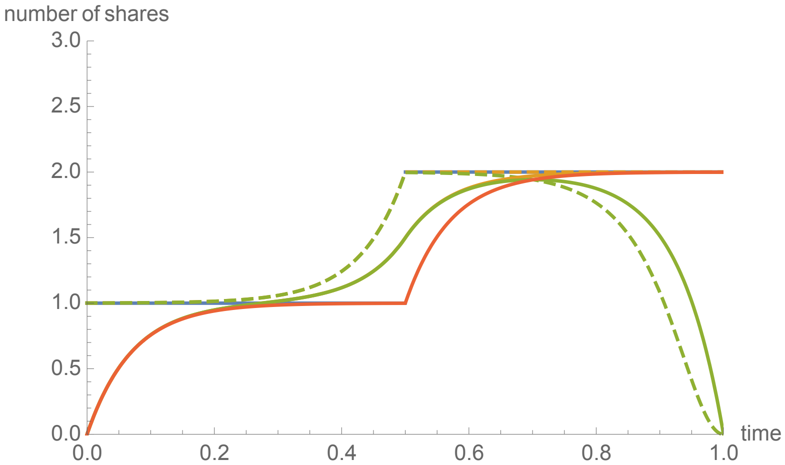

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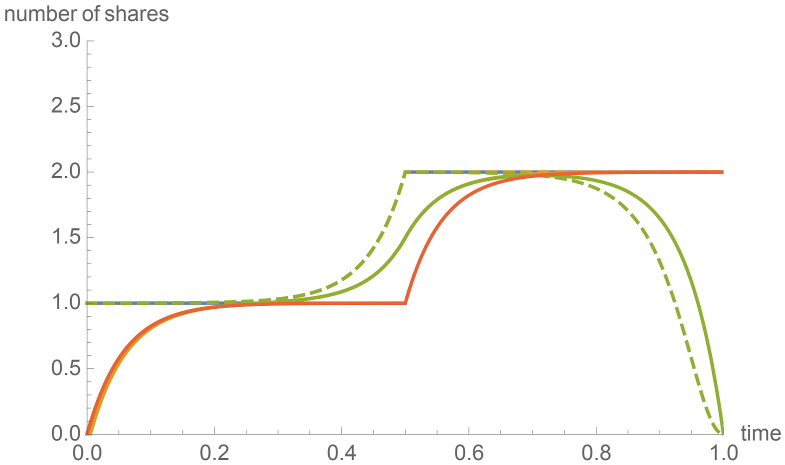

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More on quadratic tracking problem with terminal constraint

Corollary (Soner et al. '17)

A terminal position $K_T \in L^2(\Omega, \mathscr{F}, \mathbb{P})$ *can be attained at finite expected costs, i.e.,*

$$K_T = U_T = \int_0^T u_t \, dt \text{ for some progressive } u \text{ with } \mathbb{E} \int_0^T u_t^2 dt < \infty$$

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if and only if K_T *becomes known sufficiently fast towards the end in the sense that*

$$\int_0^T \frac{\mathbb{E}[(K_T - \mathbb{E}\,[K_T \mid \mathscr{F}_t])^2]}{(T-t)^2} dt < \infty.$$

Equilibrium

Stage 2: Dealers' target functional with risk aversion $\gamma_d > 0$:

$$J_d(K;S) \triangleq \mathbb{E}\left[\int_0^T (-K_t)dS_t - (V_T - S_T)K_T\right]$$
$$- \mathbb{E}\left[\frac{\gamma_d}{2}\int_0^T (K_t - U_t^K)^2\,dt + \frac{\lambda}{2}\int_0^T (u_t^K)^2\,dt\right] \to \max_K$$

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FX quotes (S_t) will generate an **equilibrium** if at these quotes the dealers' optimal supply matches their clients' demand $\mathscr{K}$:

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Theorem

Given clients' demand $\mathscr{K}$, the unique equilibrium quotes $S^{\mathscr{K}}$ are

$$S_t^{\mathscr{K}} \triangleq V_t + \gamma_d \mathbb{E}\left[\int_t^T (\mathscr{K}_s - U_s^{\mathscr{K}})\,ds \,\middle|\, \mathscr{F}_t\right], \quad 0 \le t \le T,$$

where $U^{\mathscr{K}}$ describes the dealers' optimal cumulative transfers to the end-users as determined by **Soner** *et al. '17*

Equilibrium

$$S_t^{\mathscr{K}} = V_t + \gamma_d \mathbb{E}\left[\int_t^T (\mathscr{K}_s - U_s^{\mathscr{K}})\, ds \,\middle|\, \mathscr{F}_t\right], \quad 0 \leq t \leq T,$$

- fundamental value V adjusted for dealers' effective risk
- adjustment in line with asymptotic expansion for small dealer risk aversion in exponential utility setting by Kramkov-Pulido '16 (who do not consider end-users)
- small search costs asymptotics of dealers' surcharge depend on demand regularity:
  - absolutely continuous demand $\mathscr{K} = \int_0^{\cdot} \mu_t^{\mathscr{K}}\, dt$:

$$\int_0^T K_t d(V_t - S_t^{\mathscr{K}}) = \lambda \int_0^T (\mu_t^{\mathscr{K}})^2 dt + o(\lambda) \text{ in } L^1 \text{ as } \lambda \downarrow 0$$

  - diffusive demand $\mathscr{K} = \int_0^{\cdot} (\mu_t^{\mathscr{K}}\, dt + \sigma_t^{\mathscr{K}}\, dW_t)$:

$$\int_0^T \mathscr{K}_t d(V_t - S_t^{\mathscr{K}}) = \sqrt{\lambda \gamma_d} \int_0^T (\sigma_t^{\mathscr{K}})^2 dt + o(\sqrt{\lambda}) \text{ in } L^1 \text{ as } \lambda \downarrow 0$$

- endogenous price impact model *with resilience*, in contrast to B.-Kramkov '15

The clients' problem

How should the clients choose their demand $\mathscr{K}$ given quotes (S_t)?

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Quadratic criterion: Facing exogenous FX exposure (ζ_t), the clients seek to maximize

$$J_c(\mathscr{K}; S) \triangleq \mathbb{E}\left[\int_0^T \mathscr{K}_t \, dS_t\right] - \frac{\gamma_c}{2}\mathbb{E}\left[\int_0^T (\zeta_t - \mathscr{K}_t)^2 dt\right] \to \max_{\mathscr{K}}$$

If (S_t) has drift (μ_t), this amounts to

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Given demand $\mathscr{K}^*$, the equilibrium quotes' $S^{\mathscr{K}^*}$ drift is

$$\mu_t^{\mathscr{K}^*} = -\gamma_d(\mathscr{K}_t^* - U_t^{\mathscr{K}^*})$$

which yields the **equilibrium demand equation**:

$$\mathscr{K}_t^* = \frac{\gamma_d}{\gamma_d + \gamma_c} U_t^{\mathscr{K}^*} + \frac{\gamma_c}{\gamma_d + \gamma_c} \zeta_t, \quad t \in [0, T],$$

where, again, $U^{\mathscr{K}^*}$ is as in **Soner** et al. '17.

Equilibrium demand

The **equilibrium demand equation**:

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The **equilibrium demand equation**:

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is an integral equation for $\mathscr{K}^*$. With

$$k_t \triangleq \mathscr{K}^*_t - \frac{\gamma_c}{\gamma_d + \gamma_c} \zeta_t \text{ and } K_t \triangleq \mathbb{E}\left[\int_t^T \mathscr{K}^*_u \frac{\cosh((T-u)/\sqrt{\kappa})}{\sqrt{\kappa}\cosh((T-t)/\sqrt{\kappa})} du \,\middle|\, \mathscr{F}_t \right]$$

it is equivalent to the *linear forward backward stochastic differential equation* (FBSDE):

$$k_0 = 0,\ dk_t = \left(\frac{\gamma_d}{\gamma_d + \gamma_c} K_t - \frac{\tanh((T-t)/\sqrt{\kappa})}{\sqrt{\kappa}} k_t \right) dt,$$

$$K_T = 0,\ dK_t = \left(\frac{\tanh((T-t)/\sqrt{\kappa})}{\sqrt{\kappa}} K_t - \frac{1}{\kappa}\Big(k_t + \frac{\gamma_c}{\gamma_d + \gamma_c}\zeta_t\Big) \right) dt + dM^K_t$$

for a suitable martingale M^K determined uniquely by the FBSDE.

Equilibrium demand

Theorem

The unique equilibrium demand is given explicitly by

$$\mathscr{K}^*_t = \frac{\gamma_c}{\gamma_d + \gamma_c}\zeta_t + \tilde{U}_t^{\frac{\gamma_d}{\gamma_d+\gamma_c}\zeta}, \quad t \in [0, T]$$

where $\tilde{U}^{\frac{\gamma_d}{\gamma_d+\gamma_c}\zeta}$ *denotes the tracking portfolio from* **Soner** *et al.:*

$$\frac{d}{dt}\tilde{U}_t^{\frac{\gamma_d}{\gamma_d+\gamma_c}\zeta} = \frac{\tanh((T-t)/\sqrt{\tilde{\kappa}})}{\sqrt{\tilde{\kappa}}}\left(\frac{\gamma_d}{\gamma_d+\gamma_c}\zeta_t - \tilde{U}_t^{\frac{\gamma_d}{\gamma_d+\gamma_c}\zeta}\right),$$

for the **aggregate risk tolerance** $1/\tilde{\gamma} = 1/\gamma_d + 1/\gamma_c$*, i.e.,*

$$\tilde{\kappa} \triangleq \lambda/\tilde{\gamma} \text{ and } \tilde{\zeta}_t \triangleq \mathbb{E}\left[\int_t^T \zeta_u \frac{\cosh((T-u)/\sqrt{\tilde{\kappa}})}{\sqrt{\tilde{\kappa}}\sinh((T-t)/\sqrt{\tilde{\kappa}})}\,du \middle| \mathscr{F}_t\right].$$

This balances the clients' demand for immediacy with their tolerance for risk, taking into account also their dealers' risk tolerance and ability of risk transfer to end-users: $\tilde{U}^{\zeta} = U^{\mathscr{K}^*}$.

When do the clients really need their dealers?

Example: Constant target position

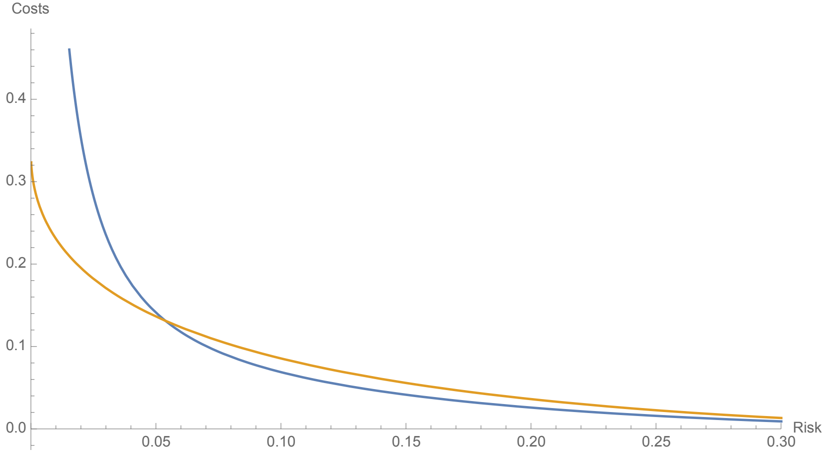
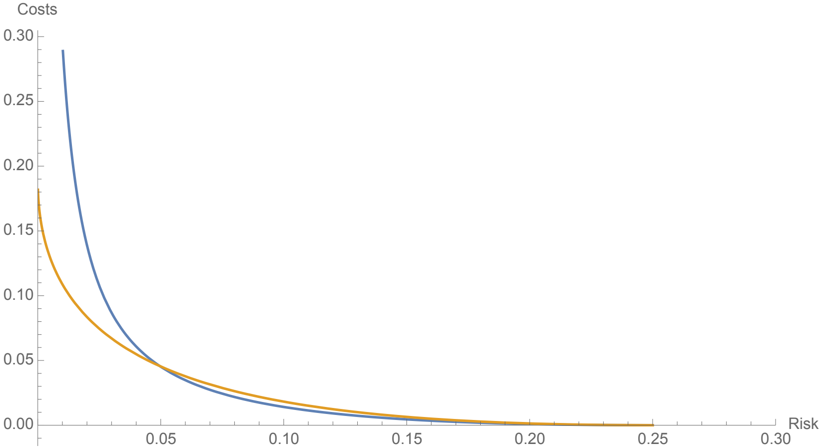


Figure: Risk vs. expected costs for clients' targeting a constant position. Trading through their dealers' and Searching end-users themselves.

When do the clients really need their dealers?

Example: Diffusively fluctuating target position



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In other words: What if the dealers are facing a **large trader**?

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Quadratic criterion: Facing exogenous FX cash flow (ζ_t), the large investor seeks to maximize

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This is still **concave** in $\mathscr{K}$ since $\mathscr{K} \mapsto -\mathbb{E}\left[\int_0^T \mathscr{K}_t \, dS_t^{\mathscr{K}}\right]$ is the dealers' expected profit in equilibrium and thus nonnegative.

$\rightsquigarrow$ **no statistical arbitrage** in this model with **endogenously derived market impact**.

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Remarkably, first order condition for optimality now reads

$$\mathscr{K}_t^* = \frac{\gamma_d}{\gamma_d + \gamma_c/2} U_t^{\mathscr{K}^*} + \frac{\gamma_c/2}{\gamma_d + \gamma_c/2}\zeta_t, \quad t \in [0, T],$$

i.e. the **same equilibrium demand equation** as before, albeit with **half** the clients' risk aversion.

Conclusions

- analyzed dealer market with clients and end-users
- quadratic setting allows for explicit computations following **Soner** et al.'s optimal tracking results
- equilibrium quotes for arbitrary demand take into account legacy position and expected future positions
- optimization of demand with and without impact awareness
- dealers will be used if their search costs and risk aversion is small compared to those of their clients
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Happy $(60 - \varepsilon^{1/8.3125})$th birthday, METE!