Phase transition for the interchange and quantum Heisenberg models on the Hamming graph

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I will present recent results, obtained jointly with Michał Kotowski and Piotr Miłoś, concerning a family of random permutation models on the 2-dimensional Hamming graph $H(2,n)$, containing the interchange process and the cycle-weighted interchange process with parameter $\theta > 0$. This family contains the random representation of the quantum Heisenberg ferromagnet. It will be shown that in these models the cycle structure of permutations undergoes a *phase transition* – when the number of transpositions defining the permutation is at most cn^2, for small enough $c > 0$, all cycles are microscopic, while for more than Cn^2 transpositions, for large enough $C > 0$, macroscopic cycles emerge with high probability. I will present bounds on the values C, c depending on the parameter θ of the model, which for the interchange process allow to pinpoint exactly the critical time of the phase transition.