PANCHROMATIC COLORINGS OF HYPERGRAPHS

MARGARITA AKHMEJANOVA, DMITRII SHABANOV

The paper deals with colorings of uniform hypergraphs. Recall that a vertex coloring of a hypergraph H is called panchromatic in r colors if every edge meets every color. In 2002 Kostochka posed the question of finding $p(n,r)$. Formally,

$$p(n,r) = min\{|E(H)| : H \text{ is } n\text{–uniform hypergraph}, \\ H \text{ does not admit a panchromatic } r\text{-coloring}\}.$$

Recent developments in the area was achieved by Cherkashin. His main results:

- a new lower bound on $p(n,r)$ when $r > c\sqrt{n}$: $p(n,r) \geqslant c\frac{r}{n}e^{\frac{n}{r}}$
- improvement upper bound on $p(n,r)$ for $n = o(r^{3/2})$.
- construction of a hypergraph without panchromatic coloring and with $(\frac{r}{r-1} + o(1))^n$ edges for $r = o(\sqrt{n/\ln n})$.

The best known result for the case $r < \frac{n}{2\ln n}$ due to Rozovskaya and Shabanov who showed that for any $r, n \geqslant 2$

$$p(n,r) \geqslant c\frac{1}{r^2}\sqrt{\frac{n}{\ln n}}\left(\frac{r}{r-1}\right)^n$$

The main result of our paper refines the previous estimate as follows.

Theorem 1. *For arbitrary $r \geqslant 2$, there exists $n_0 = n_0(r)$ such that if $n > n_0$ then any n-uniform hypergraph H with the number of edges*

$$|E(H)| \leqslant c(r)\left(\frac{n}{\ln n}\right)^{\frac{r-1}{r}}\left(\frac{r}{r-1}\right)^{n-1}$$

admits panchromatic coloring with r colors.