Powers of Hamiltonian cycles in randomly augmented graphs

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We investigate the existence of powers of Hamiltonian cycles in graphs with large minimum degree to which some additional edges have been added in a random manner. It follows from the theorems of Dirac and of Komlós, Sárközy, and Szemerédi, who confirmed the Posá–Seymour conjecture, that for every $k \geq 1$ and sufficiently large n already the minimum degree $\delta(G) \geq \frac{k}{k+1}n$ for an n-vertex graph G alone suffices to ensure the existence of the k-th power of a Hamiltonian cycle. We show that under essentially the same degree assumption, i.e. for n-vertex graph G with minimum degree at least αn for any $\alpha > \frac{k}{k+1}$, with probability close to one, adding $O(n^{2-2/l})$ random edges yields the existence of the $(kl + r)$-th power of a Hamiltonian cycle. Here, $k \geq 1$, l, $r \geq 1$ are integeres satisfying $l \geq (r + 1)r$. In this instance and for several other choices of k, l, and r we can show that our result is asymptotically optimal.

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