Ramsey goodness of trees in random graphs

Pedro Araújo, Luiz Moreira, Matías Pavez-Signé

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For a graph G, we write $G \to \big(K_{r+1}, \mathcal{T}(n,D)\big)$ if every coloring of the edges of G contain a blue K_{r+1} or red copies of every tree with n edges and maximum degree D. In 1977, Chvátal proved that the ramsey number $r(K_{r+1}, T) = rn+1$, i.e. the minimum N such that $K_N \to (K_{r+1}, T)$ is $rn+1$, for every tree T with n edges. Here we work on an analogue problem for the random graph $G(N,p)$ and we proved the following theorem.

Theorem 1. *For every positive integers $r, D \geq 2$ there exists a positive constant C such that if $p = p(N) \gg (\log N/N)^{2/(r+2)}$ and $N \geq rn + C/p$ then, with high probability,*

$$G\left(N,p\right) \to \big(K_{r+1}, \mathcal{T}(n,D)\big).$$

We show that the C/p extra vertices are needed, i.e., for some $c \geq 0$, if $N \leq rn + c/p$, then with high probability $G(N,p) \nrightarrow \big(K_{r+1}, \mathcal{T}(n,D)\big)$. We also have some results concerning values of p not considered in Theorem 1, but we do not not have such sharp result for that.

Moreover, as a byproduct of our main theorem, we proved that the random graph is globaly resilient when it comes to containing linear sized trees. This may be of independent interest.

Theorem 2. *For all $\delta \in (0,1)$ and $D, r \in \mathbb{N}$ with $r \geq 2$, there exists a positive constant $C > 0$ such that the following holds for $G = G(N,p)$ with high probability, given that $pN \geq C$. If G' is a subgraph of G with $e(G') \geq \left(\frac{1}{r} + \delta\right) e(G)$, then G' is $\mathcal{T}(N/r, D)$-universal.*