The Minrank of Random Graphs over Arbitrary Fields

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Abstract

The minrank of a graph G on the set of vertices $[n]$ over a field $\mathbb{F}$ is the minimum possible rank of an $n \times n$ matrix M over $\mathbb{F}$ with nonzero diagonal entries such that $M_{i,j} = 0$ whenever ij is not an edge of G. We show that the minrank of the random graph $G(n,p)$ over any field $\mathbb{F}$ is on the order of $n \log(1/p)/\log(n)$ with high probability. For the field of real numbers, this settles a problem raised by Knuth in 1994. The proof combines a recent argument of Golovnev, Regev, and Weinstein, who proved the above result for finite fields of size at most $n^{O(1)}$, with tools from linear algebra, including an estimate of Ronyai, Babai, and Ganapathy for the number of zero-patterns of a sequence of polynomials. Joint work with Noga Alon, Lior Gishboliner, Adva Mond, and Frank Mousset.