Monochromatic connected matchings, paths and cycles in 2-edge-colored multipartite graphs

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We solve four similar problems: For every fixed s and large n, we describe all values of $n_1, ..., n_s$ such that for every 2-edge-coloring of the complete s-partite graph with parts of sizes $n_1, ..., n_s$, there exists a monochromatic

(i) cycle with $2n$ vertices,

(ii) cycle with at least $2n$ vertices,

(iii) path with $2n$ vertices, and

(iv) path with $2n + 1$ vertices.

This implies a generalization of the conjecture by Gyárfás, Ruszinkó, Sárközy and Szemerédi that for every 2-edge-coloring of the complete 3-partite graph $K_{n,n,n}$ there is a monochromatic path P_{2n+1}. An important tool is our recent stability theorem on monochromatic connected matchings (A matching M in G is connected if all the edges of M are in the same component of G). We will also talk about exact Ramsey-type bounds on the sizes of monochromatic connected matchings in 2-colored multipartite graphs.

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