Bootstrap percolation on the Hamming graph

Mohammadreza Bidgoli

Joint work with A. Mohammadian, and B. Tayfeh-Rezaie

June 23, 2019

Abstract

The r-neighbor bootstrap percolation on a graph is an activation process of its vertices. First some vertices are initially activated and then in each phase, any inactive vertex with at least r active neighbors becomes activated. An initially activated set of vertices leading to the activation of all vertices is called a percolating set. The minimum size of a percolating set in a graph G is denoted by $m(G, r)$.

We present upper and lower bounds on $m(K_n^d, r)$, where K_n^d is the Hamming graph, i.e. the Cartesian product of d copies of the complete graph K_n. In particular, we find that $m(K_n^d, r) = \frac{1+o(1)}{(d+1)!}r^d$ if $n > r$ and $r, d \to \infty$ with $d = o(\sqrt{r})$.