

# Covering graphs by monochromatic trees and Helly-type results for hypergraphs

Matija Bucić\*

Dániel Korándi†

Benny Sudakov‡

## Abstract

How many monochromatic paths, cycles or general trees does one need to cover all vertices of a given  $r$ -edge-coloured graph  $G$ ? These problems were introduced in the 1960s and were intensively studied by various researchers over the last 50 years. We establish a connection between this problem and the following natural Helly-type question in hypergraphs. Roughly speaking, this question asks for the maximum number of vertices needed to cover all the edges of a hypergraph  $H$  if it is known that any collection of a few edges of  $H$  has a small cover. We obtain quite accurate bounds for the hypergraph problem and use them to give some unexpected answers to several questions about covering graphs by monochromatic trees raised and studied by Bal and DeBiasio, Kohayakawa, Mota and Schacht, Lang and Lo, and Girão, Letzter and Sahasrabudhe.

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\*Department of Mathematics, ETH, Zürich, Switzerland. Email: [matija.bucic@math.ethz.ch](mailto:matija.bucic@math.ethz.ch).

†Institute of Mathematics, EPFL, Lausanne, Switzerland. Email: [daniel.korandi@epfl.ch](mailto:daniel.korandi@epfl.ch). Research supported in part by SNSF grants 200020-162884 and 200021-175977.

‡Department of Mathematics, ETH, Zürich, Switzerland. Email: [benjamin.sudakov@math.ethz.ch](mailto:benjamin.sudakov@math.ethz.ch). Research supported in part by SNSF grant 200021-175573.