Subcritical random hypergraphs, high-order components, and hypertrees

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Abstract

We will discuss a generalisation of the classical Erdős-Rényi phase transition for the giant component, extending to high-order components in random k-uniform hypergraphs: for $1 \leq j \leq k-1$, two j-sets of vertices are j-connected if there is a sequence of edges between them which consecutively intersect in at least j vertices.

The case $k = 2$ and $j = 1$ corresponds to the standard notion of connectedness in graphs, while the case $j = 1$ has also been well-studied. By contrast, the case $j \geq 2$ presents some significant extra challenges, and even the phase transition threshold was only determined relatively recently.

We discuss the results known in this area so far, and introduce some new results in the subcritical regime, determining the asymptotic size and order of the largest components and showing that with high probability these are *hypertrees*, a natural generalisation of trees in the context of high-order connectedness.