Vanishing of cohomology groups of random simplicial complexes

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The binomial random graph $G(n,p)$ becomes connected with high probability when p is approximately $\frac{\ln n}{n}$. In other words, the value $p = \frac{\ln n}{n}$ is a sharp threshold for the connectedness of $G(n,p)$. Inspired by this classical result, it is interesting to investigate higher-dimensional analogues of both random graphs and connectedness. In particular two different approaches have received considerable attention: random hypergraphs and random simplicial complexes.

We study a model which represents a bridge between these two approaches: random simplicial k-complexes that arise as the downward-closure of random $(k+1)$-uniform hypergraphs, for each integer $k \geq 2$. As connectedness counterpart, we consider $\mathbb{F}_2$-cohomological j-connectedness for each $j \in [k-1]$, that is the vanishing of the cohomology groups with coefficients in the two-element field $\mathbb{F}_2$, of dimension up to j. As it turns out, $\mathbb{F}_2$-cohomological j-connectedness for our model is not a monotone property, so the existence of a single threshold is not guaranteed. By means of sophisticated probabilistic arguments, we determine such threshold, relating the vanishing of the cohomology groups to the disappearance of the last minimal obstruction.

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