Concentration Bounds for Degrees of Sets of Vertices in Preferential Attachment Graphs

Jan Dreier

RWTH Aachen University

Abstract

The degrees in preferential attachment graphs are well analyzed. For example Bollobás et al. bound the probability mass function of degrees of certain vertices and Peköz et al. bound the rate of convergence to a limit distribution. The resulting tail bounds, however, have only polynomial accuracy. We complement these results by providing stronger and more general bounds on the probability that the total degree of sets of vertices deviates far from its expected value. This makes our bounds suitable for proving structural and algorithmic properties of preferential attachment graphs.

Our results are the following: Assume the degree of a set of vertices at a time t of the random process is d. If d is sufficiently large then the degree of the same set at a later time n is likely to be in the interval $(1 \pm \varepsilon)d\sqrt{n/t}$ (for $\varepsilon > 0$) for all $n \geq t$. More specifically, the probability that this interval is left is exponentially small in d. Furthermore, the probability that the degree of some set of vertices is a factor a larger than its expected value ranges between $e^{-\Theta(a^2)}$ and $e^{-\Theta(a)}$, and the probability that the degree is a factor a smaller than its expected value is $a^{\Theta(1)}$.

We quickly discuss applications such as bounds on the size of shallow clique minors and first-order model checking in expected FPT time.

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