Resilient degree sequences with respect to Hamiltonicity in random graphs

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Pósa's theorem states that any graph G whose degree sequence $d_1 \leq \ldots \leq d_n$ satisfies $d_i \geq i+1$ for all $i < n/2$ has a Hamilton cycle. This degree condition is best possible. We show that a similar result holds for suitable subgraphs G of random graphs, i.e. we prove a 'resilient' version of Pósa's theorem: if $pn \geq C \log n$ and the i-th vertex degree (ordered increasingly) of $G \subseteq G_{n,p}$ is at least $(i + o(n))p$ for all $i < n/2$, then G has a Hamilton cycle. This is essentially best possible and strengthens a resilient version of Dirac's theorem obtained by Lee and Sudakov.

Chvátal's theorem generalises Pósa's theorem and characterises all degree sequences which ensure the existence of a Hamilton cycle. We show that a natural guess for a resilient version of Chvátal's theorem fails to be true. We formulate a conjecture which would repair this guess, and show that the corresponding degree conditions ensure the existence of a perfect matching in any subgraph of $G_{n,p}$ which satisfies these conditions. This provides an asymptotic characterisation of all degree sequences which resiliently guarantee the existence of a perfect matching.

This is joint work with Padraig Condon, Jaehoon Kim, Daniela Kühn, and Deryk Osthus.