Cycle lengths in expanding graphs

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Expanders are one of the most central and thoroughly studied notions in modern graph theory. In this talk we will discuss cycle lengths in expanding graphs.

For a constant $\alpha > 0$ a graph G on n vertices is called an α-expander if every vertex subset U of size up to $n/2$ has an external neighborhood whose size is at least $\alpha |U|$. It is known that every α-expander on n vertices contains cycles of lengths logarithmic and linear in n. We prove that cycle lengths in α-expanders are well distributed: for every $0 < \alpha \leq 1$ there exist positive constants a_1, a_2 and $A = O\left(\frac{1}{\alpha}\right)$ such that for every α-expander G on n vertices and every integer $\ell \in [a_1 \log n, a_2 n]$, G contains a cycle whose length is between ℓ and $\ell + A$. Moreover, we show that G has $\Omega\left(\frac{\alpha^3}{\log(1/\alpha)}\right) n$ different cycle lengths.

We also introduce a stronger expansion-type property, guaranteeing the existence of a linearly long interval in the set of cycle lengths. For a constant $\beta > 0$ a graph G on n vertices is called a β-graph if every pair of disjoint sets of size at least βn are connected by at least one edge. We prove that for every $0 < \beta < 1/16$ there exist positive constants b_1, b_2 such that every β-graph G on n vertices contains a cycle of length ℓ for every $\ell \in [b_1 \log n, b_2 n]$.

Joint work with Michael Krivelevich and Rajko Nenadov.