We introduce the Attracting Random Walks model, which describes the dynamics of a system of particles on a graph with certain attraction properties. In the model, particles move between adjacent vertices of a graph G, with transition probabilities that depend positively on particle counts at neighboring vertices. From an applied standpoint, the model captures the "rich get richer" phenomenon. We show that the Markov chain underlying the dynamics exhibits a phase transition in mixing time, as the parameter governing the attraction is varied. Namely, mixing is fast in the high-temperature regime, and slow in the low-temperature regime. When G is the complete graph, the model is a projection of the Potts model, whose phase transition is known. On the other hand, when the graph is incomplete, the model is non-reversible, and the stationary distribution is unknown. We demonstrate the existence of phase transition in mixing time for general graphs.