

# Turán theorems for unavoidable patterns

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In this talk, we shall discuss Turán-type theorems for two Ramsey-type problems raised by Bollobás and by Fox and Sudakov. First, for  $t \geq 3$ , we will show that any two-colouring of the complete graph on  $n$  vertices that is  $\delta$ -far from being monochromatic contains an *unavoidable  $t$ -colouring* when  $\delta = \Omega_t(n^{-1/t})$ , where an unavoidable  $t$ -colouring is any two-colouring of a clique of order  $2t$  in which one colour induces either a clique of order  $t$  or two disjoint cliques of order  $t$ . Next, for  $t \geq 3$ , we show that any tournament on  $n$  vertices that is  $\delta$ -far from being transitive contains an *unavoidable  $t$ -tournament* when  $\delta = \Omega_t(n^{-1/\lceil t/2 \rceil})$ , where an unavoidable  $t$ -tournament is the blow-up of a cyclic triangle obtained by replacing each vertex of the triangle by a transitive tournament of order  $t$ . Conditional on a well-known conjecture about bipartite Turán numbers, both results are sharp up to implied constants and hence determine the order of magnitude of the corresponding off-diagonal Ramsey numbers. This is joint work with Bhargav Narayanan.