

# A Generalized Turán Problem and its Applications

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## Abstract

The investigation of conditions guaranteeing the appearance of cycles of certain lengths is one of the most well-studied topics in graph theory. In this paper we consider a problem of this type which asks, for fixed integers  $\ell$  and  $k$ , how many copies of the  $k$ -cycle guarantee the appearance of an  $\ell$ -cycle? Extending previous results of Bollobás–Győri–Li and Alon–Shikhelman, we fully resolve this problem by giving tight (or nearly tight) bounds for all values of  $\ell$  and  $k$ .

We also present a somewhat surprising application of the above mentioned estimates to the study of the graph removal lemma. Prior to this work, all bounds for removal lemmas were either polynomial or there was a tower-type gap between the best known upper and lower bounds. We fill this gap by showing that for every super-polynomial function  $f(\varepsilon)$ , there is a family of graphs  $\mathcal{F}$ , such that the bounds for the  $\mathcal{F}$  removal lemma are precisely given by  $f(\varepsilon)$ . We thus obtain the first examples of removal lemmas with tight super-polynomial bounds. A special case of this result resolves a problem of Alon and the second author, while another special case partially resolves a problem of Goldreich.

This is a joint work with Asaf Shapira.